51., 52. $G = \{1, -1, i, -i\}$

$e = 1$

i) $i^7 = 1 = e$

$1^1 = 1$

$0(1) = 1$

ii) $i^7 = 1 = e$

$i^4 = 1$

$0(i) = 4$

53., 54. $G = \{1, \omega, \omega^2\}$

$e = 1$

i) $\omega^7 = 1 = e$

$\omega^3 = 1$

$0(\omega) = 3$

ii) $1^1 = 1$

$0(1) = 1$

55., 56. $24 = \{[0], [1], [2], [3]\}$

$e = [0]$

i) $[1] +_4 [1] = [2]$

$[1] +_4 [1] +_4 [1] = [3]$

$[1] +_4 [1] +_4 [1] +_4 [1] = [0] = e$

$[1]^4 = [0] = e$

$0([1]) = 4$

ii) $[2] +_4 [2] = [0] = e$

$[2]^2 = [0] = e$

$0([2]) = 2$

57., 58., 59. $Z_5 - \{[0]\} = \{[1], [2], [3], [4]\}$

$e = [1]$

i) $[1] \cdot_5 [2] \cdot_5 [2] \cdot_5 [2] = [1] = e$

$[2]^4 = [1] = e$

$0([2]) = 4$

ii) $[4] \cdot_5 [5] = [1] = e$

$[4]^2 = [1] = e$

$0([4]) = 2$

iii) $0([1]) = 0\{e\} = 1$

10. Probability Distributions

Blue Print	Part - I 1 Mark		Part - II 6Marks		Part - III 10 Marks		Total Marks
	No. of Qns.	Marks	No. of Qns.	Marks	No. of Qns.	Marks	
	4	4	2	12	1	10	26

Choose the correct answer (Multiple Choice Questions):

I. Book Questions

1. If $f(x) = \begin{cases} kx^2, & 0 < x < 3 \\ 0, & \text{elsewhere} \end{cases}$ is a probability density function then the value of k is *(Jun '08, July '09 & Sep '13)*

a) $\frac{1}{3}$ b) $\frac{1}{6}$ c) $\frac{1}{9}$ d) $\frac{1}{12}$

2. If $f(x) = \frac{A}{\pi} \cdot \frac{1}{16 + x^2}$, $-\infty < x < \infty$ is a p. d. f of a continuous random variable X, then the value of A is

(Mar '06, Mar '07, Sep '08, Sep '09, Mar '11, Sep '11, Mar '14, June '14 & Jun '15)

a) 16 b) 8 c) 4 d) 1

3. A random variable X has the following probability distribution *(Jun '10 & Jun '12)*

X	0	1	2	3	4	5
P(X = x)	1/4	2a	3a	4a	5a	1/4

Then $P(1 \le x \le 4)$ is

a) $\frac{10}{21}$ b) $\frac{2}{7}$ c) $\frac{1}{14}$ d) $\frac{1}{2}$

4. A random variable X has the following probability mass function as follows:

X	−2	3	1
P(X = x)	$\frac{\lambda}{6}$	$\frac{\lambda}{4}$	$\frac{\lambda}{12}$

Then the value of λ is

a) 1 b) 2 c) 3 d) 4

5. X is a discrete random variable which takes the values 0, 1, 2 and $P(X = 0) = \frac{144}{169}$, $P(X = 1) = \frac{1}{169}$, then the value of $P(X = 2)$ is *(March '09, Sep '10 & Jun '13)*

a) $\frac{145}{169}$ b) $\frac{24}{169}$ c) $\frac{2}{169}$ d) $\frac{143}{169}$

6. A random variable X has the following p.d.f *(March '16)*

X	0	1	2	3	4	5	6	7
P(X = x)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

The value of k is

a) $\frac{1}{8}$ b) $\frac{1}{10}$ c) 0 d) -1 or $\frac{1}{10}$

7. Given $E(X + c) = 8$ and $E(X - c) = 12$ then the value of c is .
(Sep '06, Sep '07, Jun '09, Mar '12, Sep '13, Mar '15, Sep '15 & Mar '16)
a) -2 b) 4 c) -4 d) 2

8. X is a random variable taking the values 3, 4 and 12 with probabilities $\frac{1}{3}, \frac{1}{4}$ and $\frac{5}{12}$. Then E(X) is *(Sep '08 & Sep '15)*
a) 5 b) 7 c) 6 d) 3

9. Variance of the random variable X is 4. Its mean is 2. Then $E(X^2)$ is
(Jun '07, Mar '09 & Jun '13)
a) 2 b) 4 c) 6 d) 8

10. $\mu_2 = 20$, $\mu_2' = 276$ for a discrete random variable X. Then the mean of the random variable X is *(Sep '09, Mar '10, Jun '11 & Mar '14)*
a) 16 b) 5 c) 2 d) 1

11. Var (4X + 3) is *(Mar '06, Jun '06, Mar '08, Jun '08 & Jun '15)*
a) 7 b) 16 Var (X) c) 19 d) 0

12. In 5 throws of a die, getting 1 or 2 is a success. The mean number of successes is
(Sep '06, Jun '12 & Mar '13)
a) $\frac{5}{3}$ b) $\frac{3}{5}$ c) $\frac{5}{9}$ d) $\frac{9}{5}$

13. The mean of a binomial distribution is 5 and its standard deviation is 2. Then the value of n and p are *(Sep '11, Sep '12 & Sep '15)*
a) $\left(\frac{4}{5}, 25\right)$ b) $\left(25, \frac{4}{5}\right)$ c) $\left(\frac{1}{5}, 25\right)$ d) $\left(25, \frac{1}{5}\right)$

14. If the mean and standard deviation of a binomial distribution are 12 and 2 respectively. Then the value of its parameter p is *(Sep '10, Mar '12, Mar '14 & Sep '14)*
a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{2}{3}$ d) $\frac{1}{4}$

15. In 16 throws of a die getting an even number is considered a success. Then the variance of the successes is *(Jun '07, Mar '10, Mar '11 & Mar '16)*
a) 4 b) 6 c) 2 d) 256

16. A box contains 6 red and 4 white balls. If 3 balls are drawn at random, the probability of getting 2 white balls with out replacement is *(Jun '10, Sep '12 & Sep '14)*
a) $\frac{1}{20}$ b) $\frac{18}{125}$ c) $\frac{4}{25}$ d) $\frac{3}{10}$

17. If 2 cards are drawn from a well shuffled pack of 52 cards, the probability that they are of the same colours with out replacement is *(June '14 & Mar '15)*
a) $\frac{1}{2}$ b) $\frac{26}{51}$ c) $\frac{25}{51}$ d) $\frac{25}{102}$

18. If in a Poisson distribution $P(X = 0) = k$ then the variance is
(Mar '07, Sep '07, Sep '08, Mar '09, Jun '11, Sep '13 & June '14)
a) $\log \frac{1}{k}$ b) $\log k$ c) e^{λ} d) $\frac{1}{k}$

19. If a random variable X follows Poisson distribution such that $E(X^2) = 30$ then the variance of the distribution is *(Jun '08 & Mar '11)*
a) 6 b) 5 c) 30 d) 25

20. The distribution function F(X) of a random variable X is *(March '08, Jun '13 & Sep '14)*
a) a decreasing function b) a non-decreasing function
c) a constant function d) increasing first and then decreasing

21. For a Poisson distribution with parameter $\lambda = 0.25$ the value of the 2^{nd} moment about the origin is *(Sep '09, Sep '11, Sep '12, March '13 & Mar '15)*
a) 0.25 b) 0.3125 c) 0.0625 d) 0.025

22. In a Poisson distribution if $P(X = 2) = P(X = 3)$ then the value of its parameter λ is
(Mar '06, Jun '06, Mar '08, Jun '09, Mar '12 & Jun '15)
a) 6 b) 2 c) 3 d) 0

23. If $f(x)$ is a p.d.f of a normal distribution with mean μ then $\int_{-\infty}^{\infty} f(x)\,dx$ is
(Sep '06, Mar '07, Jun '07 & Sep '10)
a) 1 b) 0.5 c) 0 d) 0.25

24. The random variable X follows normal distribution $f(x) = ce^{\frac{-1/2\,(x-100)^2}{25}}$. Then the value of c is *(Jun '06, Mar '10, Jun '12 & Mar '13)*
a) $\sqrt{2\pi}$ b) $\frac{1}{\sqrt{2\pi}}$ c) $5\sqrt{2\pi}$ d) $\frac{1}{5\sqrt{2\pi}}$

25. If $f(x)$ is a p. d. f of a normal variate X and $X \sim N(\mu, \sigma^2)$ then $\int_{-\infty}^{\mu} f(x)\,dx$ is
a) underfined b) 1 c) .5 d) $-.5$

26. The marks secured by 400 students in Mathematics test were normally distributed with mean 65. If 120 students got more marks above 85, the number of students securing marks between 45 and 65 is *(Sep '07, Jun '10, Jun '11 & Mar '16)*
a) 120 b) 20 c) 80 d) 160

II. COME Book Questions (PTA Questions)

27. A discrete random variable takes *(Sep '11, Jun '13 & Sep '14)*
a) only a finite number of values b) all possible values between certain given limits
c) infinite number of values d) a finite or countable number of values

28. A continuous random variable takes *(Jun '12)*

a) only a finite number of values b) all possible values between certain given limits
c) infinite number of values d) a finite or countable number of values

29. If X is a discrete random variable then $P(X \geq a) =$ *(Sep '06)*

a) $P(X < a)$ b) $1 - P(X \leq a)$ c) $1 - P(X \lneq a)$ d) 0

30. If X is a continuous random variable then $P(X \geq a) =$ *(March '13)*

a) $P(X < a)$ b) $1 - P(X > a)$ c) $P(X > a)$ d) $1 - P(X \leq a-1)$

31. If X is a continuous random variable then $P(a < X < b) =$ *(Jun '11 & Mar '15)*

a) $P(a \leq X \leq b)$ b) $P(a < X \leq b)$ c) $P(a \leq X < b)$ d) all the three above

32. A continuous random variable X has p. d. f. $f(x)$, then

a) $0 \leq f(x) \leq 1$ b) $f(x) \geq 0$ c) $f(x) \leq 1$ d) $0 < f(x) < 1$

33. A discrete random variable X has probability mass function $p(x)$, then *(Mar '10, June '14 & Sep '15)*

a) $0 \leq p(x) \leq 1$ b) $p(x) \geq 0$ c) $p(x) \leq 1$ d) $0 < p(x) < 1$

34. Mean and variance of binomial distribution are

a) nq, npq b) $np, \sqrt{npq}$ c) np, np d) np, npq

35. Which of the following is or are correct regarding normal distribution curve? *(June '08 & Mar '12)*

i) Symmetrical about the line $X = \mu$ (mean)
ii) Mean = median = mode
iii) Unimodal
iv) Points of inflextion are at $X = \mu \pm \sigma$

a) (i), (ii) only b) (ii), (iv) only c) (i), (ii), (iii) only d) all

36. For a standard normal distribution the mean and variance are *(Jun '07, Mar '08, Mar '09 & Sep '12)*

a) μ, σ^2 b) μ, σ c) 0, 1 d) 1, 1

37. The p.d.f. of the standard normal variate Z is $\varphi(z) =$

a) $\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}z^2}$ b) $\frac{1}{\sqrt{2\pi}} e^{-z^2}$ c) $\frac{1}{\sqrt{2\pi}} e^{\frac{1}{2}z^2}$ d) $\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$

38. If X is a discrete random variable then which of the following is correct? *(Sep '10, Sep '13 & Mar '14)*

a) $0 \leq F(x) < 1$ b) $F(-\infty) = 0$ and $F(\infty) \leq 1$
c) $P[X = x_n] = F(x_n) - F(x_n - 1)$ d) $F(x)$ is a constant function

39. If X is a continuous random variable then which of the following is incorrect? *(Sep '08)*

a) $F'(x) = f(x)$ b) $F(\infty) = 1$; $F(-\infty) = 0$
c) $P[a \leq x \leq b] = F(b) - F(a)$ d) $P[a \leq x < b] \neq F(b) - F(a)$

40. Which of the following are correct? *(Jun '09, Jun '10 & Mar '11)*

i) $E(aX + b) = aE(X) + b$ ii) $\mu_2 = \mu_2' - (\mu_1')^2$
iii) μ_2 = variance iv) $var(aX + b) = a^2 var(X)$

a) all b) (i), (ii), (iii) c) (ii), (iii) d) (i), (iv)

41. Which of the following is not true regarding the normal distribution? *(Sep '09 & June '15)*

a) skewness is zero
b) mean = median = mode
c) the points of inflection are at $X = \mu \pm \sigma$
d) maximum height of the curve is $\frac{1}{\sqrt{2\pi}}$

III. Additional Questions

42. Two dice are rolled and X be the random variable of getting number of '3' s. Then $P(X = 2)$

a) $\frac{25}{36}$ b) $\frac{10}{36}$ c) $\frac{1}{36}$ d) $\frac{6}{36}$

43. A random variable X has the following probability mass function

X	0	1	2	3	4	5	6
$P(X = x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

Then $k = ?$

a) $\frac{1}{13}$ b) $\frac{1}{36}$ c) $\frac{1}{49}$ d) -1 or $\frac{1}{7}$

44. A continuous random variable X follows the probability law, $f(x) = \begin{cases} kx(1-x)^{10}, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$ where $A = \int_0^1 x(1-x)^{10}\,dx$. Then the value of $k =$

a) A b) $\frac{1}{A}$ c) A^2 d) 0

45. If the probability density function of a random variable is given by $f(x) = \begin{cases} k(1-x^2), & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$ then the value of k is

a) $\frac{2}{3}$ b) $\frac{3}{2}$ c) 2 d) 3

46. $F(x) = \frac{1}{\pi}\left(\frac{\pi}{2} + \tan^{-1} x\right) - \infty < x < \infty$ is a distribution function of continuous variable X. Then $P(X \leq 1)$ is *Deletion (Sep '07)*

a) $\frac{3}{4}$ b) $\frac{\pi}{4}$ c) $\frac{1}{2}$ d) $\frac{1}{4}$

47. If $f(x) = \begin{cases} \frac{A}{x}, & 1 < x < e^3 \\ 0, & \text{elsewhere} \end{cases}$ is a probability density function of a continuous random variable X, then the value of A

a) 1 b) 3 c) $\frac{1}{3}$ d) log 3

48. The probability density function of a random variable X is $f(x)\begin{cases} kx^{\alpha-1} e^{-\beta x^{\alpha}}, x, \alpha, \beta > 0 \\ 0 \quad ,\text{elsewhere}\end{cases}$ then the value of $k =$

a) $\frac{\alpha}{\beta}$ b) $\alpha\beta$ c) $\frac{1}{\alpha\beta}$ d) $\frac{\beta}{\alpha}$

49. For the distribution function given by $F(x) = \begin{cases} 0 \; ; & x<0 \\ x^2 \; ; & 0 \le x \le 1 \\ 1 \; ; & x > 1,\end{cases}$ then the p.d.f $f(x)$ is

a) $\begin{cases} 2x, & 0 \le x \le 1 \\ 0 & \text{elsewhere}\end{cases}$ b) $x \, ; x > 0$ c) cannot be determined d) $2x \, ; x < 0$

50. A continuous random variable x has the p.d.f defined by $f(x) = \begin{cases} ce^{-ax}, & 0 < x < \infty \\ 0 & \text{elsewhere}\end{cases}$ Then the value of c

a) 1 b) a c) $\frac{1}{a}$ d) $-a$

51. A random variable X has probability density function $f(x) = \begin{cases} k \;, & 0 < x < 2\pi \\ 0 & \text{elsewhere}\end{cases}$ Then the value of k

a) 2π b) $\frac{1}{\pi}$ c) $\frac{1}{2\pi}$ d) π

52. The mean of the distribution $f(x) = \begin{cases} 3e^{-3x} \;, & 0 < x < \infty \\ 0 \;, & \text{elsewhere}\end{cases}$

a) $\frac{1}{3}$ b) 3 c) $-\frac{1}{3}$ d) -3

53. The mean value of p.d.f $f(x) = \begin{cases} \frac{1}{24} \;, & -12 \le x \le 12 \\ 0 \;, & \text{otherwise}\end{cases}$ is

a) 0 b) 24 c) $\frac{1}{12}$ d) 12

54. In a binomial distribution if $n = 5$, $P(X=3) = 2\,P(X=2)$ then *Deletion (March '07)*

a) $p = 2q$ b) $2p = q$ c) $p = q$ d) $3p = 2q$

55. For a Binomial distribution with mean 2 and variance $\frac{4}{3}$, $p = ?$ *Deletion (June '06)*

a) $\frac{2}{3}$ b) $\frac{1}{3}$ c) $\frac{3}{4}$ d) $\frac{2}{\sqrt{3}}$

56. In a Poisson distribution of $P(X=2) = P(X=3)$ Then mean $\lambda = ?$

a) 1 b) 2 c) 3 d) 4

57. If X is normally distributed with mean 6 and standard deviation 5 and Z is the corresponding normal variate. Then $P(0 \le X \le 8) =$

a) $P(-1.2 < z < .04)$ b) $P(-0.12 < z < 0.4)$ c) $P(-1.2 < z < 0.4)$ d) $P(-0.12 < z < 0.04)$

58. The probability density function of the normal distribution is $f(x) = Ke^{-2x^2+4x}, -\infty < x < \infty$. Then the mean $\mu =$

a) 1 b) 2 c) -2 d) 4

59. For the p.d.f. of the normal distribution $f(x) = Ce^{-x^2+3x}, -\infty < x < \infty$, the mean $\mu =$ *Deletion (March '06)*

a) 3 b) $\frac{2}{3}$ c) $\frac{3}{2}$ d) 6

60. The p.d.f. of the normal distribution is $f(x) = Ke^{-2x^2+4x}, -\infty < x < \infty$. Then the variance $\sigma^2 =$

a) $\frac{1}{4}$ b) $\frac{1}{\sqrt{2}}$ c) 1 d) 2

61. The p.d.f. of the normal distribution is $f(x) = Ce^{-x^2+3x}, -\infty < x < \infty$. Then the variance $\sigma^2 =$

a) $\frac{1}{2}$ b) 2 c) $\frac{3}{2}$ d) $\frac{1}{\sqrt{2}}$

62. For a normal distribution with mean $\mu = 34$ and standard deviation $\sigma = 16$, $P(30 < X < 60) =$ where X is normal variate and Z is the corresponding standard normal variate

a) $P(-0.25 < z < 0.25)$ b) $P(0 < z < 1.625)$
c) $P(0.25 < z < 1.625)$ d) $P(-0.25 < z < 1.625)$

IV. Answers

Q.No.	Ans.	Q.No.	Ans.	Q.No.	Ans.	Q.No.	Ans.	Q.No.	Ans.	Q.No.	Ans.
1	c	12	a	23	a	34	d	45	b	56	c
2	c	13	d	24	d	35	d	46	a	57	c
3	d	14	c	25	c	36	c	47	c	58	a
4	b	15	a	26	c	37	d	48	b	59	c
5	b	16	d	27	d	38	c	49	a	60	a
6	b	17	c	28	b	39	d	50	b	61	a
7	a	18	a	29	c	40	a	51	c	62	d
8	b	19	b	30	c	41	d	52	a		
9	d	20	b	31	d	42	c	53	a		
10	a	21	b	32	b	43	c	54	a		
11	b	22	c	33	a	44	b	55	b		

V. Solutions

1. $f(x)$ is a p.d.f then $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_0^3 kx^2 dx = 1$$

$$k\left(\frac{x^3}{3}\right)_0^3 = 1$$

$$k\left(\frac{27}{3} - 0\right) = 1$$

$$k(9) = 1$$

$$\boxed{k = \frac{1}{9}}$$

2. $\int_{-\infty}^{\infty} \frac{A}{\pi} \frac{1}{16 + x^2}\,dx = 1$

$$\frac{A}{\pi} \int_{-\infty}^{\infty} \frac{1}{x^2 + 4^2}\,dx = 1$$

$$\frac{A}{\pi}\,\frac{1}{4}\left[\tan^{-1}\left(\frac{x}{4}\right)\right]_{-\infty}^{\infty} = 1$$

$$\left[\because \int \frac{1}{x^2 + a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C\right]$$

$$\frac{A}{4\pi}\left[\tan^{-1}(\infty) - \tan^{-1}(-\infty)\right] = 1$$

$$\frac{A}{4\pi}\left(\frac{\pi}{2} + \frac{\pi}{2}\right) = 1$$

$$\frac{A}{4\pi}\,\pi = 1$$

$$\boxed{A = 4}$$

3. $\Sigma P(Xi) = 1$

$$\frac{1}{4} + 2a + 3a + 4a + 5a + \frac{1}{4} = 1$$

$$14a = \frac{1}{2}$$

$$\boxed{a = \frac{1}{28}}$$

$P(1 \le X \le 4) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

$= 2a + 3a + 4a + 5a$

$= 14a$

$$\boxed{P(1 \le X \le 4) = \frac{1}{2}}$$

4. $\Sigma P(Xi) = 1$

$$\frac{\lambda}{6} + \frac{\lambda}{4} + \frac{\lambda}{12} = 1$$

$$\lambda \times \frac{(2 + 3 + 1)}{12} = 1$$

$$\boxed{\lambda = 2}$$

5. $\Sigma P(Xi) = 1$

$P(X = 0) + P(X = 1) + P(X = 2) = 1$

$$\frac{144}{169} + \frac{1}{169} + P(X = 2) = 1$$

$$P(X = 2) = 1 - \frac{145}{169}$$

$$\boxed{P(X = 2) = \frac{24}{169}}$$

6. $\Sigma P(Xi) = 1$

$P(X = 0) + P(X = 1) + \ldots\ldots + P(X = 7) = 1$

$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$10k^2 + 9k - 1 = 0$

$(10k - 1)(k + 1) = 0$

$\boxed{k = \frac{1}{10}}$ (or) $\left[\begin{array}{l}\text{If } k = -1 \text{ then } P(X = 1) = -ve \\ P(X = 1) \ne -1 \\ (0 \le P(X) \le 1)\end{array}\right.$

7. $E(X) + c = 8$ ———— (1)

$E(X) - c = 12$ ———— (2)

(1) – (2) ⇒ we get $2c = -4 \Rightarrow c = -2$

$\boxed{c = -2}$ $\quad \because [E(c) = c]$

8.

X	3	4	12
P(X = x)	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{5}{12}$

$E(X) = \Sigma xiPi$

$= \left(3 \times \frac{1}{3}\right) + \left(4 \times \frac{1}{4}\right) + \left(12 \times \frac{5}{12}\right)$

$= 1 + 1 + 5 \Rightarrow \boxed{E(X) = 7}$

9. $Var(X) = 4 \quad E(X) = 2$

$Var(X) = E(X^2) - [E(X)]^2$

$4 = E(X^2) - 2^2$

$\boxed{8 = E(X^2)}$

10. $Var(X) = E(X^2) - [E(X)]^2$

$\mu_2 = \mu_2^1 - (\mu_1^1)^2$

$20 = 276 - (\mu_1^1)^2$

$(\mu_1^1)^2 = 256 \Rightarrow \boxed{\mu_1^1 = E(X) = 16}$

11. $Var(ax + b) = a^2 Var(X)$

$Var(4X + 3) = 4^2 Var(X) = 16 Var(X)$

12. $n = 5$

$p = \frac{2}{6} = \frac{1}{3}$

$\therefore$ The mean $= np = \frac{5}{3}$

13. $np = 5 \quad ; \quad npq = 4. \quad \because \left[\sqrt{npq} = 2\right.$

$n \times \frac{1}{5} = 5 \quad | \quad 5q = 4$

$\boxed{n = 25} \quad | \quad q = \frac{4}{5} \quad \therefore [q = 1 - p$

$1 - p = \frac{4}{5}$

$p = \frac{1}{5}$

14. $np = 12 \quad ; \quad npq = 4 \quad \because \left[\sqrt{npq} = 2\right.$

$12q = 4$

$\therefore p = 1 - \frac{1}{3} \quad q = \frac{1}{3}$

$\boxed{p = \frac{2}{3}}$

15. $n = 16; \quad p = \frac{3}{6} = \frac{1}{2} \quad [\{2, 4, 6\}$

$q = 1 - p = \frac{1}{2}$

The variance $= npq = 16 \times \frac{1}{2} \times \frac{1}{2}$

$\boxed{n = 4}$

16. $n(s) = 10C_3$

W	R
4	6
↓ 2	↓ 1
$(4C_2)$	$(6C_1)$

$P(X) = \frac{4C_2 \times 6C_1}{10C_3}$

$= \frac{6 \times 6}{120}$

$\boxed{P(X) = \frac{3}{10}}$

17. $n(s) = 52\,C_2$

$P(X) = P$ (Both are Block or Red)

♠ ♣ → 13 13 $(26C_2)$; ♦ ♥ → 13 13 $(26C_2)$

$= P(A \cup B)$

$= P(A) + P(B)$

$= \frac{26\,C_2}{52\,C_2} + \frac{26\,C_2}{52\,C_2} = 2 \times \frac{25 \times 26}{51 \times 52} \Rightarrow \boxed{P(X) = \frac{25}{51}}$

A, B → mutually exclusive events $P(A \cup B) = P(A) + P(B)$

18. $P(X = 0) = k$

$\frac{e^{-\lambda} . \lambda^0}{0!} = k$

$e^{-\lambda} = k$

$-\lambda = \log k$

$\lambda = -\log k$

$\boxed{\lambda = \log\left(\frac{1}{k}\right)}$

19. $E(X^2) = 30$

$Var(X) = E(X^2) - [E(X)]^2 \quad$ [Mean = variance = λ

$\lambda = 30 - \lambda^2 \Rightarrow \lambda^2 + \lambda - 30 = 0$

$\boxed{\lambda = 5}$ (or) $\lambda \neq -6$ (–ve)

20. By using the property, It is a non decreasing function.

21. $Var(X) = E(X^2) - [E(X)]^2$

$\lambda = E(X^2) - \lambda^2$

$E(X^2) = \lambda^2 + \lambda = (0.25)^2 + (0.25)$

$\boxed{E(X^2) = 0.3125}$

22. $P(X = 2) = P(X = 3)$

$\frac{e^{-\lambda} . \lambda^2}{2!} = \frac{e^{-\lambda} . \lambda^3}{3!}$

$1 = \frac{\lambda}{3}$

$\boxed{\lambda = 3}$

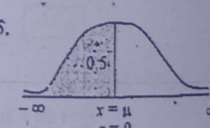
24. $f(x) = C\, e^{\frac{-1}{2} \frac{(x-100)^2}{25}}$

$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2} \frac{(x-\mu)^2}{\sigma^2}}$

here $\left| c = \frac{1}{\sigma\sqrt{2\pi}} \right| \sigma^2 = 25 \quad \therefore \sigma = 5$

$\boxed{c = \frac{1}{5\sqrt{2\pi}}}$

25.



26. $P(X > 85) = P(z > z_1)$

$= x$ (say)

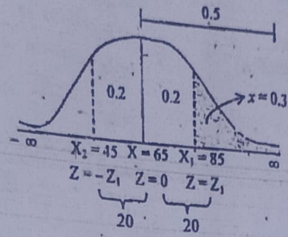
$120 = x \times (400)$

$x = 0.3$

$\therefore P(65 < x < 85) = P(45 < x < 65) = 0.2$

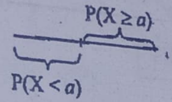
$\therefore$ Total number of students $= 0.2 \times 400$

$= \boxed{80}$



29. $P(X < a) + P(X \geq a) = 1$

$P(X \geq a) = 1 - P(X < a)$



30. $P(X \geq a) = P(X > a)$

32. (It is non – negative for all real X) (i.e., $f(x) \geq 0$)

37. $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\frac{-1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}$

put $Z = \frac{X-\mu}{\sigma}$

$= \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\frac{-1}{2}z^2}$ $\left[\begin{array}{l}\mu = 0 \\ \sigma = 1\end{array}\right.$

$\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}$

42. $P(X = 2) = \frac{n(X = 2)}{n(s)} = \frac{1}{36}$ $\{(3, 3)\}$

43. $\sum Pi = 1$

$k + 3k + 5k + 7k + 9k + 11k + 13k = 1$

$49k = 1 \Rightarrow \boxed{k = \frac{1}{49}}$

44. ($f(x)$ is a p.d.f) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$\int_0^1 kx(1-x)^{10}\, dx = 1$

$k\int_0^1 x(1-x)^{10}\, dx = 1$ $\quad \because \left[A = \int_0^1 x(1-x)^{10}\, dx\right.$

$kA = 1$

$\boxed{k = \frac{1}{A}}$

45. If $f(x)$ is a p.d.f $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$k\int_0^1 (1 - x^2)\, dx = 1$

$k\left[x - \frac{x^3}{3}\right]_0^1 = 1$

$\boxed{k = \frac{3}{2}}$

46. We know that $F(X) = P(X \leq x)$

$\therefore P(X \leq 1) = F(1)$ $\quad \left[F(x) = \frac{1}{\pi}\left(\frac{\pi}{2} + \tan^{-1} x\right)\right]$

$= \frac{1}{\pi}\left(\frac{\pi}{2} + \tan^{-1}(1)\right)$

$= \frac{1}{\pi}\left(\frac{\pi}{2} + \frac{\pi}{4}\right) = \frac{1}{\pi} \times \frac{3\pi}{4}$

$\boxed{P(X \leq 1) = \frac{3}{4}}$

47. $f(x)$ is a p.d.f. $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$A \cdot \int_1^{e^3} \frac{1}{x}\, dx = 1$

$A\,[\log x]_1^{e^3} = 1$ $\quad \because \left[\begin{array}{l}\log e = 1 \\ \log 1 = 0\end{array}\right.$

$A(3\log e - \log 1) = 1$

$3A = 1 \Rightarrow \boxed{A = \frac{1}{3}}$

48. $f(x)$ is a p.d.f. $\int_{-\infty}^{\infty} f(x)\, dx = 1$ $\quad$ Let $\left[\begin{array}{l}t = x^{\alpha} \\ dt = \alpha \cdot x^{\alpha-1} \cdot dx\end{array}\right.$ $\quad \frac{dt}{\alpha} = x^{\alpha-1} \cdot dx$

$k\int_0^{\infty} x^{\alpha-1} e^{-\beta(x^{\alpha})} \cdot dx = 1$

$k\int_{x=0}^{x=\infty} e^{-\beta t} \cdot \frac{dt}{\alpha} = 1$

$\frac{k}{\alpha} \cdot \left[\frac{e^{-\beta t}}{-\beta}\right]_{x=0}^{x=\infty} = 1$

$\frac{-k}{\alpha\beta}\left[e^{-\beta(x^{\alpha})}\right]_0^{\infty} = 1$

$$\frac{-k}{\alpha\beta}\left[e^{-\infty}-e^{0}\right]=1 \qquad \because [e^{-\infty}=0$$

$$\boxed{k=\alpha\beta}$$

49. $F(x)=\begin{cases}0 & ; x<0\\ x^2 & ; 0\le x\le 1\\ 1 & ; x>1\end{cases}$

(differentiate)

$$\frac{d}{dx}(F(x))=\begin{cases}0 & ; x\le 0\\ 2x & ; 0\le x\le 1\\ 0 & ; x>1\end{cases}$$

$$f(x)=\begin{cases}2x & ; 0\le x\le 1\\ 0 & ; \text{(otherwise)}\end{cases}$$

50. $\int_{-\infty}^{\infty} f(x)\,dx=1$

$$c\int_0^{\infty} e^{-ax}\,dx=1$$

$$\frac{-c}{a}\left[e^{-ax}\right]_0^{\infty}=1$$

$$\frac{-c}{a}(0-1)=1 \;\Rightarrow\; \boxed{(c=a)}$$

51. $k\int_0^{2\pi} 1\,dx=1$

$$k[x]_0^{2\pi}=1 \;\Rightarrow\; \boxed{k=\frac{1}{2\pi}}$$

52. $E(X)=\int_0^{\infty} x\,.\,3e^{-3x}\,dx$

$$=3\int_0^{\infty} x\,e^{-3x}\,dx \qquad \left[\begin{matrix}n=1\\ a=3\end{matrix}\right.$$

$$=3\times\frac{1!}{3^2}$$

$$\boxed{E(X)=\frac{1}{3}} \qquad \because \left[\int_0^{\infty} x^n\,.\,e^{-ax}\,dx=\frac{n!}{a^{n+1}}\right.$$

53. $E(X)=\int_{-12}^{12} x\,.\,\frac{1}{24}\,dx=\frac{1}{24}\int_{-12}^{12} x\,dx$ ∵ [x – is an odd function)

$$=0$$

54. $n=5$;

$$P(X=3)=2P(X=2)$$

$$5C_3\,.\,p^3\,.\,q^2=2\times 5C_2\times p^2\times q^3$$

$$p^3q^2=2p^2q^3 \qquad [5C_3=5C_2.$$

$$\boxed{p=2q}$$

55. $\boxed{np=2}$; $npq=\frac{4}{3}$

$$2q=\frac{4}{3}$$

$$q=\frac{2}{3}$$

$$1-p=\frac{2}{3}$$

$$\boxed{p=\frac{1}{3}}$$

56. $P(X=2)=P(X=3)$

$$\frac{e^{-\lambda}\,.\,\lambda^2}{2!}=\frac{e^{-\lambda}\,.\,\lambda^3}{3!}$$

$$1=\frac{\lambda}{3} \;\Rightarrow\; \boxed{\lambda=3}$$

57. $P(0\le X\le 8)=P(-1.2<z<0.4)$

$$z=\frac{X-\mu}{\sigma}=\frac{X-6}{5}$$

(i) $X=0\Rightarrow z=\frac{0-6}{5}=-1.2$

(ii) $X=8\Rightarrow z=\frac{8-6}{5}=0.4$

58. $f(x)=k\,e^{-2x^2+4x}$

$$=k\,e^{-2(x^2-2x)}$$

$$=k\,e^{-2(x^2-2x+1-1)}$$

$$=k\,e^{-2(x-1)^2+2}$$

$$=ke^2\,e^{-2(x-1)^2}$$

$$=ke^2\,e^{\frac{-(x-1)^2}{1/2}}$$

$$f(x)=ke^2\,e^{\frac{-1}{2}\frac{(x-1)^2}{(1/4)}} \qquad \text{———— (1)}$$

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\,e^{\frac{-1}{2}\frac{(X-\mu)^2}{\sigma^2}} \qquad \text{———— (2)}$$

From (1) and (2) $\boxed{\mu=1}$

59. $f(x)=Ce^{-(x^2-3x)}$

$$=Ce^{-[x^2-3x+(3/2)^2-(3/2)^2]}$$

$$=Ce^{-(x-3/2)^2+9/4}$$

$$f(x)=Ce^{9/4}\,.\,e^{-(x-3/2)^2} \qquad \text{———— (1)}$$

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\,e^{\frac{-1}{2}\cdot\frac{(X-\mu)^2}{\sigma^2}} \qquad \text{———— (2)}$$

From (1) and (2) $\boxed{\mu=\frac{3}{2}}$

60. $f(x)=k\,e^{-2x^2+4x}$

$$f(x)=ke^2\,.\,e^{\frac{-1}{2}\frac{(x-1)^2}{(1/4)}} \qquad \text{———— (1)} \qquad \because \text{[Ref:- Qn. No- 58]}$$

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\,e^{\frac{-1}{2}\frac{(X-\mu)^2}{\sigma^2}} \qquad \text{———— (2)}$$

From (1) and (2) $\boxed{\sigma^2=\frac{1}{4}}$

61. $f(x)=Ce^{-x^2+3x}$

$$f(x)=Ce^{9/4}\,e^{-(x-3/2)^2}$$

$$=Ce^{9/4}\,e^{\frac{-1}{2}\left(\frac{x-3/2}{1/2}\right)^2} \qquad \text{———— (1)} \qquad \because \text{[Ref:- Qn. No- 59]}$$

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\,e^{\frac{-1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} \qquad \text{———— (2)}$$

From (1) and (2) $\boxed{\sigma^2=\frac{1}{2}}$

62. $\therefore P(30<X<60)$

$$=P(-0.25<z<1.625)$$

$$Z=\frac{X-\mu}{\sigma}=\frac{X-34}{16}$$

(i) If $X=30\Rightarrow Z=\frac{30-34}{16}=-0.25$

(ii) If $X=60\Rightarrow Z=\frac{60-34}{16}=\frac{26}{16}=1.625$