



## SARVA SAKTHI TUITION CENTER

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### TEST 2 - 2023

### EXPECTED QUESTIONS – 5 MARKS – VOLUME 1

1. Investigate for what values of  $\lambda$  and  $\mu$  the system of linear equations  
 $x + 2y + z = 7$ ,  $x + y + \lambda z = \mu$ ,  $x + 3y - 5z = 5$   
 has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

(OR)

Show that  $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5 = -\sqrt{3}$ .

2. Solve the equation

$$(x-2)(x-7)(x-3)(x+2)+19=0.$$

(OR)

Solve, by Cramer's rule, the system of equations

$$x_1 - x_2 = 3, 2x_1 + 3x_2 + 4x_3 = 17, x_2 + 2x_3 = 7.$$

3. Find the equation of the tangent and normal to the circle  $x^2 + y^2 - 6x + 6y - 8 = 0$  at  $(2, 2)$ .

(OR)

A semielliptical archway over a one-way road has a height of  $3m$  and a width of  $12m$ . The truck has a width of  $3m$  and a height of  $2.7m$ . Will the truck clear the opening of the archway? (Fig. 5.6)

4. A tunnel through a mountain for a four lane highway is to have an elliptical opening. The total width of the highway (not the opening) is to be  $16m$ , and the height at the edge of the road must be sufficient for a truck  $4m$  high to clear if the highest point of the opening is to be  $5m$  approximately. How wide must the opening be? (OR)

Show that the lines  $\vec{r} = (-\hat{i} - 3\hat{j} - 5\hat{k}) + s(3\hat{i} + 5\hat{j} + 7\hat{k})$  and  $\vec{r} = (2\hat{i} + 4\hat{j} + 6\hat{k}) + t(\hat{i} + 4\hat{j} + 7\hat{k})$  are coplanar. Also, find the non-parametric form of vector equation of the plane containing these lines.

5. Prove that  $\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}$ ,  $-1 < x < 1$ . (OR)

Solve  $\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right) = \sin\left\{\cot^{-1}\left(\frac{3}{4}\right)\right\}$ .

6. Find the parametric form of vector equation of a straight line passing through the point of intersection of the straight lines  $\vec{r} = (\hat{i} + 3\hat{j} - \hat{k}) + t(2\hat{i} + 3\hat{j} + 2\hat{k})$  and  $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+3}{4}$ , and perpendicular to both straight lines. (OR)

Solve :  $2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}} = \frac{b}{a} + \frac{6a}{b}$ .

7. If  $z = x + iy$  is a complex number such that  $\operatorname{Im}\left(\frac{2z+1}{iz+1}\right) = 0$ , show that the locus of  $z$  is  $2x^2 + 2y^2 + x - 2y = 0$ . (OR)

Show that the line  $x - y + 4 = 0$  is a tangent to the ellipse  $x^2 + 3y^2 = 12$ . Also find the coordinates of the point of contact.

8. Solve, by Cramer's rule, the system of equations

$$\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0, \frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0, \frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0 \quad (\text{OR})$$

If  $A = \frac{1}{9} \begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$ , prove that  $A^{-1} = A^T$ .

9. Show that the points  $1$ ,  $\frac{-1}{2} + i\frac{\sqrt{3}}{2}$ , and  $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$  are the vertices of an equilateral triangle. (OR)

If  $z = x + iy$  and  $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$ , show that  $x^2 + y^2 = 1$ .

10. Find the condition that the roots of cubic equation  $x^3 + ax^2 + bx + c = 0$  are in the ratio  $p : q : r$ . (OR)

A variable plane moves in such a way that the sum of the reciprocals of its intercepts on the coordinate axes is a constant. Show that the plane passes through a fixed point