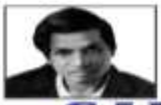

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VOLUME 1

IMPORTANT QUESTIONS – 2 AND 3 MARKS

Applications of Vector Algebra

1. Find the angle between the planes $\vec{r}\cdot(\hat{i}+\hat{j}-2\hat{k})=3$ and $2x-2y+z=2$.

2. Find the length of the perpendicular from the point $(1,-2,3)$ to the plane $x-y+z=5$.

3. Find the image of the point whose position vector is $\hat{i}+2\hat{j}+3\hat{k}$ in the plane $\vec{r}\cdot(\hat{i}+2\hat{j}+4\hat{k})=38$.

4. Find the distance between the planes $\vec{r}\cdot(2\hat{i}-\hat{j}-2\hat{k})=6$ and $\vec{r}\cdot(6\hat{i}-3\hat{j}-6\hat{k})=27$

5. Find the acute angle between the planes $\vec{r}\cdot(2\hat{i}+2\hat{j}+2\hat{k})=11$ and $4x-2y+2z=15$

6. Verify whether the line $\frac{x-3}{-4}=\frac{y-4}{-7}=\frac{z+3}{12}$ lies in the plane $5x-y+z=8$

7. Find the intercepts cut off by the plane $\vec{r}\cdot(6\hat{i}+4\hat{j}-3\hat{k})=12$ on the coordinate axes.

8. Find the direction cosines of the normal to the plane and length of the perpendicular from the origin to the plane $\vec{r}\cdot(3\hat{i}-4\hat{j}+12\hat{k})=5$.

9. Find the shortest distance between the two given straight lines $\vec{r}=(2\hat{i}+3\hat{j}+4\hat{k})+t(-2\hat{i}+\hat{j}-2\hat{k})$ and $\frac{x-3}{2}=\frac{y}{-1}=\frac{z+2}{2}$.

10. Find the acute angle between the straight lines $\frac{x-4}{2}=\frac{y}{1}=\frac{z+1}{-2}$ and $\frac{x-1}{4}=\frac{y+1}{-4}=\frac{z-2}{2}$ and state whether they are parallel or perpendicular.

11. Find the angle made by the straight line $\frac{x+3}{2}=\frac{y-1}{2}=-z$ with coordinate axes.

12. For any vector $\vec{a}$, prove that $\hat{i}\times(\vec{a}\times\hat{i})+\hat{j}\times(\vec{a}\times\hat{j})+\hat{k}\times(\vec{a}\times\hat{k})=2\vec{a}$.

13. If $\hat{a},\hat{b},\hat{c}$ are three unit vectors such that $\hat{b}$ and $\hat{c}$ are non-parallel and $\hat{a}\times(\hat{b}\times\hat{c})=\frac{1}{2}\hat{b}$, find the angle between $\hat{a}$ and $\hat{c}$.

14. Prove that $[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}]=[\vec{a},\vec{b},\vec{c}]^2$.

15. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}$ and $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$. If $c_1=1$ and $c_2=2$, find c_3 such that $\vec{a},\vec{b}$ and $\vec{c}$ are coplanar.

16. If $\vec{a},\vec{b},\vec{c}$ are three vectors, prove that $[\vec{a}+\vec{c},\ \vec{a}+\vec{b},\ \vec{a}+\vec{b}+\vec{c}]=[\vec{a},\vec{b},\vec{c}]$.

17. If $\vec{a}=\hat{i}-\hat{k}$, $\vec{b}=x\hat{i}+\hat{j}+(1-x)\hat{k}$, $\vec{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$, show that $[\vec{a},\vec{b},\vec{c}]$ depends on neither x nor y.

18. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are coplanar, prove that c is the geometric mean of a and b.

19. Let $\vec{a},\vec{b},\vec{c}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, show that $[\vec{a},\vec{b},\vec{c}]^2=\frac{1}{4}|\vec{a}|^2\,|\vec{b}|^2$.

20. If the vectors $\vec{a},\vec{b},\vec{c}$ are coplanar, then prove that the vectors $\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}$ are also coplanar.

21. Show that the four points $(6,-7,0)$, $(16,-19,-4)$, $(0,3,-6)$, $(2,-5,10)$ lie on a same plane.

22. Find the volume of the parallelepiped whose coterminus edges are given by the vectors $2\hat{i}-3\hat{j}+4\hat{k}$, $\hat{i}+2\hat{j}-\hat{k}$ and $3\hat{i}-\hat{j}+2\hat{k}$.

23. Forces of magnitudes $5\sqrt{2}$ and $10\sqrt{2}$ units acting in the directions $3\hat{i}+4\hat{j}+5\hat{k}$ and $10\hat{i}+6\hat{j}-8\hat{k}$, respectively, act on a particle which is displaced from the point with position vector $4\hat{i}-3\hat{j}-2\hat{k}$ to the point with position vector $6\hat{i}+\hat{j}-3\hat{k}$. Find the work done by the forces.

24. Prove by vector method that an angle in a semi-circle is a right angle.

25. Prove by vector method that the diagonals of a rhombus bisect each other at right angles.

26. Find the magnitude and the direction cosines of the torque about the point $(2,0,-1)$ of a force $2\hat{i}+\hat{j}-\hat{k}$, whose line of action passes through the origin.

27. If D is the midpoint of the side BC of a triangle ABC, show by vector method that $|\overline{AB}|^2+|\overline{AC}|^2=2(|\overline{AD}|^2+|\overline{BD}|^2)$.

28. With usual notations, in any triangle ABC, prove by vector method that $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$.

Two Dimensional Analytical Geometry-II

1. The equation $y=\frac{1}{32}x^2$ models cross sections of parabolic mirrors that are used for solar energy. There is a heating tube located at the focus of each parabola; how high is this tube located above the vertex of the parabola?

The parabolic communication antenna has a focus at $2m$ distance from the vertex of the antenna. Find the width of the antenna $3m$ from the vertex.

2. Identify the type of the conic for the following equations:

(1) $16y^2=-4x^2+64$ (2) $x^2+y^2=-4x-y+4$

3. Show that the absolute value of difference of the focal distances of any point P on the hyperbola is the length of its transverse axis.

4. Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{2b^2}{a}$.

5. Find the vertices, foci for the hyperbola $9x^2 - 16y^2 = 144$.
6. Find the equation of the parabola with vertex $(-1,-2)$, axis parallel to y-axis and passing through $(3,6)$.
7. Find the equation of the parabola with focus $\left(-\sqrt{2},0\right)$ and directrix $x=\sqrt{2}$.
8. Find the length of Latus rectum of the parabola $y^2 = 4ax$.
9. Determine whether the points $(-2,1),(0,0)$ and $(-4,-3)$ lie outside, on or inside the circle $x^2+y^2-5x+2y-5=0$.
10. If $y=4x+c$ is a tangent to the circle $x^2+y^2=9$, find c.
11. The line $3x+4y-12=0$ meets the coordinate axes at A and B. Find the equation of the circle drawn on AB as diameter.
12. Examine the position of the point $(2,3)$ with respect to the circle $x^2+y^2-6x-8y+12=0$.
13. Find the general equation of a circle with centre $(-3,-4)$ and radius 3 units.

Inverse Trigonometric Functions

1. Simplify (i) $\cos^{-1}\left(\cos\left(\frac{13\pi}{3}\right)\right)$ (ii) $\tan^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)$
2. Prove that $\frac{\pi}{2} \le \sin^{-1}x + 2\cos^{-1}x \le \frac{3\pi}{2}$.

   (i) $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2},\ x\in[-1,1]$. (ii) $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2},\ x\in\mathbb{R}$.

3. Find the value of

   (i) $\tan\left(\tan^{-1}\left(\frac{7\pi}{4}\right)\right)$ (ii) $\tan\left(\tan^{-1}(1947)\right)$ (iii) $\tan\left(\tan^{-1}(-0.2021)\right)$
4. Find the value of $\tan^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$. Find the principal value of $\tan^{-1}\left(\sqrt{3}\right)$.
5. Is $\cos^{-1}(-x)=\pi-\cos^{-1}(x)$ true? Justify your answer.
6. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9}+\cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$.
7. Write down the properties of Cosine and sine Function
8. Find the domain of $\sin^{-1}\left(2-3x^2\right)$ Find the principal value of $\sin^{-1}(2)$, if it exists.

Theory of Equations

Theory of Equations

1. Show that the equation $x^9-5x^5+4x^4+2x^2+1=0$ has atleast 6 imaginary solutions.
2. Examine for the rational roots of

   (i) $2x^3-x^2-1=0$ (ii) $x^8-3x+1=0$.

3. Find solution, if any, of the equation

   $2\cos^2 x-9\cos x+4 = 0$.

4. Solve the cubic equation : $2x^3-x^2-18x+9=0$ if sum of two of its roots vanishes.
5. Solve the equation : $x^4-14x^2+45=0$.
6. It is known that the roots of the equation $x^3-6x^2-4x+24=0$ are in arithmetic progression. Find its roots.
7. Prove that a straight line and parabola cannot intersect at more than two points.
8. Show that, if p,q,r are rational, the roots of the equation $x^2-2px+p^2-q^2+2qr-r^2=0$ are rational.
9. Prove that a line cannot intersect a circle at more than two points.
10. Show that the equation $2x^2-6x+7=0$ cannot be satisfied by any real values of x.
11. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

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12. Find the monic polynomial equation of minimum degree with real coefficients having $2-\sqrt{3}i$ as a root.
13. Solve the equation $3x^3-16x^2+23x-6=0$ if the product of two roots is 1.
14. Find the sum of squares of roots of the equation $2x^4-8x^3+6x^2-3=0$.
15. If p is real, discuss the nature of the roots of the equation $4x^2+4px+p+2=0$, in terms of p.

Complex Numbers

1. If $\omega \neq 1$ is a cube root of unity, show that (i) $(1-\omega+\omega^2)^6+(1+\omega-\omega^2)^6=128$.
2. Find the value of $\sum_{k=1}^{8}\left(\cos\frac{2k\pi}{9}+i\sin\frac{2k\pi}{9}\right)$.
3. Find the cube roots of unity.
4. Simplify $\left(\frac{1+\cos 2\theta+i\sin 2\theta}{1+\cos 2\theta-i\sin 2\theta}\right)^{30}$.
5. If $z=(\cos\theta+i\sin\theta)$, show that $z^n+\frac{1}{z^n}=2\cos n\theta$ and $z^n-\frac{1}{z^n}=2i\sin n\theta$.
6. Find the quotient $\frac{2\left(\cos\frac{9\pi}{4}+i\sin\frac{9\pi}{4}\right)}{4\left(\cos\left(\frac{-3\pi}{2}\right)+i\sin\left(\frac{-3\pi}{2}\right)\right)}$ in rectangular form.
7. Find the principal argument $\operatorname{Arg} z$, when $z=\frac{-2}{1+i\sqrt{3}}$.

8. Show that $|z+2-i|<2$ represents interior points of a circle. Find its centre and radius.

9. If $(x_1+iy_1)(x_2+iy_2)(x_3+iy_3)\cdots(x_n+iy_n)=a+ib$, show that

(i) $(x_1^2+y_1^2)(x_2^2+y_2^2)(x_3^2+y_3^2)\cdots(x_n^2+y_n^2)=a^2+b^2$

(ii) $\sum_{r=1}^{n}\tan^{-1}\left(\frac{y_r}{x_r}\right)=\tan^{-1}\left(\frac{b}{a}\right)+2k\pi,\ k\in\mathbb{Z}$.

10. Show that the equation $z^3+2\bar{z}=0$ has five solutions.

11. Find the square roots of (i) $4+3i$ (ii) $-6+8i$ (iii) $-5-12i$.

12. If $|z|=1$, show that $2\le|z^2-3|\le 4$.

13. If z_1, z_2, and z_3 are complex numbers such that $|z_1|=|z_2|=|z_3|=|z_1+z_2+z_3|=1$, find the value of $\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|$.

14. If $z_1=2-i$ and $z_2=-4+3i$, find the inverse of z_1z_2 and $\frac{z_1}{z_2}$.

15. If $\frac{z+3}{z-5i}=\frac{1+4i}{2}$, find the complex number z in the rectangular form

16. Show that (i) $(2+i\sqrt{3})^{10}+(2-i\sqrt{3})^{10}$ is real Simplify $\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3$ into rectangular form

17. A complex number z is purely imaginary if and only if $z=-\bar{z}$

Applications of Matrices and Determinants

1. Solve the system: $x+3y-2z=0,\ 2x-y+4z=0,\ x-11y+14z=0.$

2. In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ? (Use Cramer's rule to solve the problem).

3. Solve the following system of linear equations, using matrix inversion method: $5x+2y=3,\ 3x+2y=5$.

4. Find the inverse of the non-singular matrix $A=\begin{bmatrix}0 & 5\\ -1 & 6\end{bmatrix}$, by Gauss-Jordan method.

5. If $A=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$, show that $A^{-1}=\frac{1}{2}(A^2-3I)$.

6. If $A=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$, show that $A^2-3A-7I_2=O_2$. Hence find A^{-1}.

7. If $F(\alpha)=\begin{bmatrix}\cos\alpha & 0 & \sin\alpha\\ 0 & 1 & 0\\ -\sin\alpha & 0 & \cos\alpha\end{bmatrix}$, show that $[F(\alpha)]^{-1}=F(-\alpha)$.

8. Prove that $\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ is orthogonal.

9. If $A=\begin{bmatrix}4 & 3\\ 2 & 5\end{bmatrix}$, find x and y such that $A^2+xA+yI_2=O_2$. Hence, find A^{-1}.

10. If A is symmetric, prove that adj A is also symmetric.

11. If A is a non-singular matrix of odd order, prove that $|\text{adj } A|$ is positive.

12. Find the inverse of the matrix $\begin{bmatrix}2 & -1 & 3\\ -5 & 3 & 1\\ -3 & 2 & 3\end{bmatrix}$.

13. If $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ is non-singular, find A^{-1}.