ST. ANNE'S ACADEMY
(MATHS & PHYSICS TUITION CENTRE)
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Unit Test (Chapter 1)
CLASS – XII - MATHEMATICS

Time Allowed : 2 Hrs Marks : 60

**I. Answer ALL questions.** **6x2 = 12**

1) Find the inverse (if it exists) of the following:

$$\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$$

2) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it to a row-echelon form.

3) State Rouché - Capelli theorem.

4) If $A$ is a non-singular matrix of odd order, prove that $|\text{adj } A|$ is positive.

5) If $\text{adj}(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find $A^{-1}$.

6) If $A^T A^{-1}$ is symmetric, then show that, $A^2 = (A^T)^2$

**II. Answer ALL questions.** **6x3 = 18**

7) Verify the property $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$.

8) Find the inverse of the non-singular matrix $A = \begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$, by Gauss-Jordan method.

9) Four men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

10) Solve, by Cramer's rule, the system of equations
$x_1 - x_2 = 3, 2x_1 + 3x_2 + 4x_3 = 17, x_2 + 2x_3 = 7.$

11) Investigate for what values of $\lambda$ and $\mu$ the system of linear equations
$x + 2y + z = 7,\ x + y + \lambda z = \mu,\ x + 3y - 5z = 5$
has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

**-Please Turn Over-**

12) If $A=\begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB=I_2$, then, show that $B=\left(\cos^2\frac{\theta}{2}\right)A^T$

**III. Answer ALL questions.** **6x5 =30**

13) If $A=\frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find $a,b$ and $c$, and hence $A^{-1}$.

14) If $A=\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the products $AB$ and $BA$ and hence solve the system of equations $x-y+z=4, x-2y-2z=9, 2x+y+3z=1$.

15) The upward speed $v(t)$ of a rocket at time $t$ is approximated by $v(t)=at^2+bt+c$, $0\le t\le 100$ where $a,b$, and $c$ are constants. It has been found that the speed at times $t=3, t=6$, and $t=9$ second are, 64, 133, and 208 miles per second respectively. Find the speed at time $t=15$ seconds. (Use Gaussian elimination method.)

16) Find the value of $k$ for which the equations $kx-2y+z=1$, $x-2ky+z=-2$, $x-2y+kz=1$ have (i) no solution (ii) unique solution (iii) infinitely many solution

17) By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8+O_2\rightarrow CO_2+H_2O$.

18) If the system of equations $px+by+cz=0$, $ax+qy+cz=0$, $ax+by+rz=0$ has a non-trivial solution and $p\neq a, q\neq b, r\neq c$, prove that $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}=2$.