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TRB-POLYTECHNIC-LECTURER

MATHEMATICS

UNIT-II

COMPLEX ANALYSIS

COMPLEX NUMBERS:

If x and y are real numbers and I denote $\sqrt{-1}$, then z = x + iy is called a complex number. x is called the real part of z and is denoted by Re(z) or R(z). y is called the imaginary part of z and is denoted by Im(z) or I(z). The complex number x – iy is called the conjugate of the complex number z = x + iy and is denoted by $\bar{z}$.

ANALYTIC FUNCTION:

A function f is said to be analytic at a point $z_0 \in C$ if it is differentiable at every z in some ε-neighbourhood of the point z_0. The function f is said to be analytic in a region if it is analytic at every point in the region. The function f is said to be entire if it is analytic in C.

Definition: (Analytic function)

A function w=f(z) is said to have the limit l as z tends to Z_0 if given ε > 0 there exists δ > 0 such that $|Z-Z_0| < \delta$ this implies $|f(z)-l| < \varepsilon$ we write $\underset{z \to z_0}{lt} f(z) = l$.

Theorem: Every isolated singularity of a bounded analytic function is removable.

Limits and Continuity:

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Let u = u(x, y) be a real-valued function of the two real variables x and y. We say that u has the limit u_0 as (x, y) approaches (x_0, y_0) provided that the value of u(x, y) gets close to the value u_0 as (x, y) gets close to (x_0, y_0). We write $\lim_{(x,y)\to(x_0,y_0)} u(x, y) = u_0$.

Example:

1. If $u(x, y) = x^3/(x^2 + y^2)$, then $\lim_{(x,y)\to(x_0,y_0)} u(x,y) = 0$.

2. The function $u\{x, y) = xy/(x^2 + y^2)$ does not have a limit as (x, y) approaches (0,0). If we let (x, y) approach (0, 0) along the x axis, then

$\lim_{(x,0)\to(0,0)} u(x,0) = \lim_{(x,0)\to(0,0)} \frac{(x)(0)}{x^2+0^2} = 0.$ But if we let (x, y) approach (0, 0) along the line y = x,

then $\lim_{(x,x)\to(0,0)} u(x,x) = \lim_{(x,0)\to(0,0)} \frac{(x)(x)}{x^2+x^2} = \frac{1}{2}.$ since the two values are different, the value of the limit is dependent on how (x, y) approaches (0, 0). We conclude that w(x, y) does not have a limit as (x, y) approaches (0, 0).

3. Let f(z) = u(x, v) + iv(x, y) be defined in some neighborhood of z_0. Then f is continuous at z_0 — x_0+ iy_0 iff u and v are continuous at (x_0, y_0).

4. Show that the polynomial function given by $w = P(z) = a_0 + a_1z + a_2 z_2 + \bullet\bullet\bullet + a_n z_n$ is continuous at each point z_0 in the complex plane.

Solution:

Observe that if a_0 is the constant function, then $\lim_{z\to z_0} a_0 = a_0$; and if a_0 # 0, then we can use define with f(z) = a_1z and the choice δ= ε/|ai | to prove that $\lim_{z\to z_0} a_1 z = a_1 z_0$. we obtain

$\lim_{z\to z_0} a_k z^k = a_k z_0{}^k$ for k=0,1,2…..,n.

A finite sum of terms, and we can use the result of equation to obtain

$$\lim_{z\to z_0} P(z) = \lim_{z\to z_0}\left(\sum_{k=0}^{n} a_k z^k\right) = \sum_{k=0}^{n} a_k z_0{}^k = P(z_0).$$

POLYNOMIALS:

1. Every polynomial $p(z)=a_0+a_1z+a_2z^2+\dots\dots\dots+a_nz^n$ is an analytic function.
2. A polynomial of degree one is called exactly n zeros.
3. A zero of order one is called simple zero.
4. Every constant is an analytic function with derivative zero.
5. The simplest non-constant analytic function is z.
6. The constant zero is regarded as a polynomial whose degree is-∞.

Rational Functions:

Definition:

Rational function is a quotient of two polynomials that is h(z)=f(z)/g(z).

Results:

1. The zeros of g(z) are poles of h(z).
2. Rational function without poles must reduces a constant.
3. Derivatives of $h(z) = h'(z) = \frac{g(z)f'(z) - f(z)g'(z)}{[g(z)]^2}$ where $g(z)\neq 0$.

COMPLEX VARIABLES:

The quantity z = x + iy is called a Complex variable, when x and y are two independent real variables. The argand plane in which the variables z are represented by points is called the z-plane.

Function of a Complex Variable:

If z = x + iy and w = u + iv are two complex variables such that there exists one or more values of w, corresponding to each value of z in a certain region R of the z-plane, then w is called a function of z.

(i.e.) w = f(z) = f(x + iy)

When w = u + iv = f(z) = f(x + iy), then u and v are functions of the variables x and y.

Single Valued Function and Multi Valued Function:

If to each value of z, there corresponds one and only one value of w is called a Single-valued function of z, otherwise a multi-valued function of z.

Example: *(i)* $w = \frac{1}{z}$ *and* $w = z^2$ *are called single-valued functions*

(ii) $w = \sqrt{z}$ *is called multi-valued function.*

ANALYTIC FUNCTION [Regular Function or Holomorphic Function]:

A single valued function is said to be analytic at a point if its derivative exists not only at that point but also some neighborhood of that point.

It is said to be analytic in a region R, if it is analytic at every point of R.

Necessary condition for the function f(z) to be analytic [CR-Equations]

The necessary conditions for a complex function w = f(z) = u(x, y) + iv(x, y) to be analytic are $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ That is, $u_x = v_y$ and $u_y = -v_x$

Sufficient condition for the function f(z) to be analytic:

If the partial derivatives u_x, u_y, v_x and v_y are all continuous in R and $u_x = v_y$ and $u_y = -v_x$. Then the function f(z) is analytic in a region R.

Polar Form of C-R Equations:

$\frac{\partial u}{\partial r} = \frac{1}{r}.\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r}.\frac{\partial u}{\partial \theta}$ are the Cauchy-Riemann equations in polar form.

PROBLEM: ***Test whether the function*** $2xy + i(x^2 - y^2)$ ***is analytic or not.***

Solution:

Given $f(z) = 2xy + i(x^2 - y^2)$

$u + iv = 2xy + i(x^2 - y^2)$

Here $u = 2xy$; $v = x^2 - y^2$

$u_x = 2y$; $v_x = 2x$

$u_y = 2x; \quad v_y = -2y$

$\Rightarrow u_x \neq v_y$ and $u_y \neq -v_x$. $\therefore$ C-R equations are not satisfied.

Hence the given function is not analytic.

PROBLEM:

Show that the function $f(z) = \bar{z}$ is nowhere differentiable.

Solution:

Given $f(z) = \bar{z}$

Since $z = x + iy$, $\bar{z} = x - iy$

$\therefore f(z) = u + iv = x - iy$

Here $u = x$; $v = -y$

$u_x = 1; \quad v_x = 0$

$u_y = 0; \quad v_y = -1$

At all points(x,y), $u_x = 1$, $v_y = -1$. Hence C-R equations are not satisfied anywhere.

Therefore, $f(z) = \bar{z}$ is nowhere differentiable.

PROBLEM:

Show that the function $f(z) = |z|^2$ is differentiable at z=0 but not analytic at z=0.

Solution:

Given $f(z) = |z|^2$

Since $z = x + iy$, $\bar{z} = x - iy$

$f(z) = (x + iy)(x - iy) \qquad \because |z|^2 = z\bar{z}$

$= x^2 + y^2$

$f(z) = u + iv$

Here $u = x^2 + y^2$; $v = 0$

$u_x = 2x; \qquad v_x = 0$

$u_y = 2y; \qquad v_y = 0$

At the point (0, 0), the C-R equations are satisfied, so that f(z) is differentiable at z=0. Hence there is no neighborhood of z=0 in which f(z) is differentiable. Therefore, $f(z) = |z|^2$ is not analytic at z=0(orgin).

Problem: ***Find the constants a,b and c if*** $f(z) = x + ay + i(bx + cy)$ ***is analytic.***

Solution:

Given $f(z) = x + ay + i(bx + cy)$

$= u + iv$

Here $u = x + ay$; $v = bx + cy$

$u_x = 1$; $v_x = b$

$u_y = a$; $v_y = c$

Since f(z) is analytic, we have $u_x = v_y$ and $u_y = -v_x$

$\Rightarrow 1 = c$ and $a = -b$

$\Rightarrow c = 1$ and $a = -b$, b may be any value.

PROPERTIES

Property: 1 An analytic function with constant real part is constant.

Property: 2 An analytic function with constant imaginary part is constant.

Property: 3 An analytic function with constant modulus is constant.

Property: 4 If $w = f(z) = u + iv$ is an analytic function, then the curves of the family $u(x,y) = a$ and the curves of the family $v(x,y) = b$ cut orthogonally, where a and b are constants. [or]

When the function $f(z) = u + iv$ is analytic, then the curves u = constant and v = constant are orthogonal.

Result: 1 If f(z) is a regular function of z, then $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$

Result: 2 If f(z) is an analytic function of z, then $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\log|f(z)| = 0$

Result: 3 If f(z) is an analytic function of z in any domain, then prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^p = p^2\left|f'(z)\right|^2|f(z)|^{p-2}$$

Definition: Laplace equation

An expression of the form $\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=0$ is called the Laplace equation in two dimension.

Definition: Harmonic Function

Any function having continuous second order partial derivatives which satisfies the Laplace equation is called the harmonic function.

Definition: Conjugate harmonic function

If u and v are harmonic functions such that u+iv is analytic then each is called the conjugate harmonic function of each other.

Note: If u and v are conjugate harmonic functions then u is conjugate harmonic to v and v is conjugate harmonic to u.

DETERMINATION OF CONJUGATE HARMONIC FUNCTION

Working Rule:

If one of the two conjugate functions be given, the other can be found by using C-R equations. Let u(x, y) be given and v(x, y) can be obtained as follows:

CONSTRUCTION OF ANALYTIC FUNCTION [Milne-Thomson method]

Case(i) If Real part u(x, y) is given,

we shall construct the analytic function f(z) = u + iv as follows:

Step:1 Find u_x, u_y

Step:2 Find $u_x(z,0), u_y(z,0)$

Step:3 Use $\int f'(z)dz = \int u_x(z,0)dz - i\int u_y(z,0)dz$

Then integrating, we get f(z).

Case(ii) If the imaginary part v(x, y) is given,

We shall construct the analytic function f(z) = u + iv as follows:

Step:1 Find v_x, v_y

Step:2 Find $v_x(z,0), v_y(z,0)$

Step:3 Use $\int f'(z)dz = \int v_y(z,0)dz + i\int v_x(z,0)dz$, Integrating, we get f(z).

Problems Using Case(i)

Show that $u(x,y) = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic. Find the harmonic conjugate v and express u + iv as an analytic function of z.

Solution:

Given $u(x,y) = 3x^2y + 2x^2 - y^3 - 2y^2$

$u_x = 6xy + 4x$; $u_y = 3x^2 - 3y^2 - 4y$

$u_{xx} = 6y + 4$; $u_{yy} = -6y - 4$

$u_{xx} + u_{yy} = 6y + 4 - 6y - 4 = 0$

Therefore, u is harmonic.

To find the harmonic conjugate v(x,y)

$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy$ (By exact differential)

$dv = -\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy$ By C-R equations $\mathbf{u_x} = v_y$ and $\mathbf{u_y} = -v_x$

$\boldsymbol{v = -\int \frac{\partial u}{\partial y}dx + \int \left(terms\ of\ \frac{\partial u}{\partial x}\ not\ involving\ x\right)dy}$ $\because$ By exact differential

$= -\int(3x^2 - 3y^2 - 4y)dx + 0$

$= -\frac{3x^3}{3} + 3y^2x + 4yx + C$

$v = 3xy^2 - x^3 + 4yx + C$

Next to find the analytic function f(z)=u+iv

Given $u(x,y) = 3x^2y + 2x^2 - y^3 - 2y^2$

$u_x = 6xy + 4x$; $u_y = 3x^2 - 3y^2 - 4y$

Now $u_x(z,0) = 4z$; $u_y(z,0) = 3z^2$ $\because$ ***put*** $\boldsymbol{x = z, y = 0}$

Using formula, $\int \boldsymbol{f'(z)dz = \int u_x(z,0)dz - i\int u_y(z,0)dz}$ $\because$ By Milne-Thomson method

$= \int 4zdz - i\int 3z^2dz$

$$f(z) = 4\frac{z^2}{2} - \mathrm{i}\frac{3z^3}{3} + C$$

$f(z) = 2z^2 - \mathrm{i}z^3 + C$ is the required analytic function.

Problems Using Case(ii)

Show that $v = e^x(xcosy - ysiny)$ is harmonic function. Find the corresponding analytic function f(z)

Solution:

$$\text{Given } \mathrm{v} = e^x(xcosy - ysiny)$$
$$= e^x xcosy - e^x ysiny$$
$$\mathrm{v_x} = cosy[e^x.1 + xe^x] - ysinye^x$$
$$= e^x cosy + e^x xcosy - ysinye^x$$
$$\mathrm{v_{xx}} = e^x cosy + cosy[e^x.1 + xe^x] - ysinye^x$$
$$= 2e^x cosy + e^x xcosy - e^x ysiny$$
$$\mathrm{v_y} = -e^x xsiny - e^x[ycosy + siny.1]$$
$$= -e^x xsiny - e^x ycosy - e^x siny$$
$$\mathrm{v_{yy}} = -e^x xcosy - e^x[-ysiny + cosy.1] - e^x cosy$$
$$= -e^x xcosy + e^x ysiny - e^x cosy - e^x cosy$$
$$= -2e^x cosy - e^x xcosy + e^x ysiny$$

Now, $v_{xx} + v_{yy} = 0$,

Therefore, v satisfies Laplace's equation.

Next to find f(z):

$$v_x = e^x cosy + e^x xcosy - ysinye^x$$
$$v_x(z,0) = e^z + ze^z = (1+z)e^z$$
$$v_y = -e^x xsiny - e^x ycosy - e^x siny \Longrightarrow v_y(z,0) = 0$$

Using formula, $\int f'(z)dz = \int v_y(z,0)dz + i\int v_x(z,0)dz$

$$f(z) = 0 + i\int(z+1)e^z dz + C \qquad \because \int u\,dv = \mathrm{uv} - \int vdu$$
$$= i[(1+z)e^z - \int e^z\,\mathrm{dz}] + C \qquad \text{Here } u = (1+z)$$
$$= i[(1+z)e^z - e^z] + C \qquad du = dz;\ dv = \int e^z\,\mathrm{dz};$$

$= ie^z \, dz + C$ $\qquad v = e^z$

Definition :(Sequences)

The sequence $\{z_n\}$ is said to be a Cauchy sequence if for every $\varepsilon > 0$ there exists a positive integer N_ε such that if n, m > N_ε, then $|z_n - z_m| < \varepsilon$(or) equivalently, $z_n - z_m$ belongs to the disk $D_r(0)$.

Theorem:

If $\{z_n\}$ is a Cauchy sequence, $\{z_n\}$ converges.

Proof:

Let $z_n = x_n + iy_n$, Using the techniques of both $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences of real numbers. Since Cauchy sequences of real numbers are on convergent,

We know that $\lim_{n\to\infty} x_n = x_0$ and $\lim_{n\to\infty} y_n = y_0$ for some real numbers x_0 and y_0.

This means $\lim_{n\to\infty} z_n = z_0$ where $z_0 = x_0 + iy_0$.

In other words, the sequence $\{z_n\}$ converges to z_0.

Definition:

Let $\{z_n\}$ be a complex sequence. We can form a new sequence $\{S_n\}$, called the **sequence of partial sums**,

(ie.,) $S_1 = z_1$

$S_2 = z_1 + z_2$

:

:

$S_n = z_1 + z_2 + \ldots\ldots + z_n = \sum_{k-1}^{n} z_k$, ……..

The formal expression $\sum_{k-1}^{n} z_k = z_1 + z_2 + \ldots\ldots + z_n + \ldots\ldots$ is called an **infinite series**,

and $z_1, z_2, \ldots$etc., are calles the **terms of the series**. $S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \sum_{k-1}^{n} z_k$,

The infinite series $\sum_{k-1}^{n} z_k$ converges to S.

Theorem: If $\sum_{k-1}^{n} z_k$ is a convergent complex series, then $\lim_{n\to\infty} z_n = 0$.

Example: 1.The series $\sum_{n=1}^{\infty} \frac{(-1)^n + i}{n} = \sum_{n=1}^{\infty}\left[\frac{(-1)^n}{n} + i\frac{1}{n}\right]$ is divergent.

2.The series $\sum_{n=1}^{\infty}(1+i)^n$ is divergent.

Definition: Let $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ be **convergent series**,where a_n and b_n are complex numbers.

The Cauchy product of the two series is defined to be the **series** $\sum_{n=0}^{\infty} c_n$ where $c_n = \sum_{k=0}^{n} b_{n-k}$.

Theorem:

If the Cauchy product converges, then $\sum_{n=0}^{\infty} c_n = \left(\sum_{n=0}^{\infty} a_n\right)\left(\sum_{n=0}^{\infty} b_n\right)$.

Uniform Convergence:

Definition:

The sequence $\{S_n(z)\}$ converges uniformly to f(z) on the set T if for every $\varepsilon > 0$, there exists a positive integer N_ε (which depends only on ε) such that

if $n \geq N_\varepsilon$, then $|S_n(z) - f(z)| < \varepsilon$ for all z ε T.

If in the preceding, $S_n(z)$ is the nth partial sum of a series $\sum_{k=0}^{\infty} C_k (z-\alpha)^k$.We say that the series converges uniformly to f(z) on the set T.

Power series.

The reader is assumed to be acquainted with the basic properties of convergence of power series a $\sum_{0}^{\infty} a_n z^n$, where $a_n \in c$, $z \in C$. We recall that to any such power series there

is associated a number $\rho \in [0,\infty]$ called its radius of convergence. For $\rho > 0$ the series is absolutely convergent for any $z \in K(0,\rho)$, so we can define the sum function $f : K(0,\rho) \to C$ of the series by

$$f(z) = \sum_{0}^{\infty} a_n z^n, |z| < \rho$$

If $\rho = 0$ the power series is convergent only for $z = 0$ and is of no interest in complex analysis. The power series is divergent for any z satisfying $|z| > \rho$.

Theorem:

Suppose γ is a simple, closed, piecewise smooth path in the region G. Then G contains the interior of γ if and only if $\gamma \sim G_0$.

Proof:

If G contains the interior of γ and D[w,R] is any closed disk in the interior of γ then there is a G-homotopy from γ to C[w,R], and C[w,R] ~Go.

In the other direction we argue by contradiction:

Assume $\gamma \sim G_0$ but G does not contain the interior of γ. So we can find a point w in the interior of γ which is not in G.

Define $g(z) = \frac{1}{z-w}$ for z = w. Now g is holomorphic on G and $\gamma \sim_G$o. so, and we have $\int_\gamma g(z)dz = 0$' On the other hand, choose R > 0 so that D[w,R] is inside γ.There is a homotopy in C\{w} from γ to C[w,R], so Cauchy's Theorem shows that $\int_\gamma g(z)dz = 2\pi i$.

1. An absolutely convergent complex series is convergent.

Proof. For any complex number z we have $|Rez| \le |z|$ and $|Imz| \le |z| \le |Rez|+|Imz|$. Hence, ifP|an| is convergent, then by comparison the real series$\sum Re\ a_n$ and$\sum Im\ a_n$ are absolutely convergent, to x and y say. The theorem now follows from

$$\left|\sum_{n=0}^{N} a_n - x - iy\right| \le \left|\sum_{n=0}^{N} \operatorname{Re} a_n - x\right| + \left|\sum_{n=0}^{N} \operatorname{Im} a_n - y\right| \to 0 \, as N \to \infty$$

Abel's limit theorem:

If $\sum_{n=0}^{\infty} a_n$ converges , then f(z)= $\sum_{n=0}^{\infty} a_n z^n \to f(1)$ as Z approaches1 in such a way that $\frac{|1-z|}{(1-|z|)}$ remains unbounded.

Trigonometric functions:

The complex trigonometric functions are defined in terms of the complex exponential by adapting our earlier formulae

$$\cos z = \frac{e^{iz} + e^{-iz}}{2} = \cos x \cosh y - i \sin x \sinh y \,, \sin z = \frac{e^{iz} - e^{-iz}}{2} = \sin x \cosh y + i \cos x \sinh y \,.$$

The exponential function:

The exponential function is defined to be the power series $\exp z = \sum_{n=0}^{\infty} \frac{z^n}{n!}$.

(or)

The **(complex) exponential function** is exp: C → C, defined for z = x +iy as exp(z) := e^x (cosy+isiny) = $e^x e^{iy}$.

Example:

For all $z, z_1, z_2 \in C$,

(a) $\exp(z_1)\exp(z_2) = \exp(z_1 + z_2)$ (b). $\frac{1}{\exp(z)} = \exp(-z)$

(c).$\exp(z+2\pi i)=\exp(z)$ (d).$|\exp(z)|=\exp(\operatorname{Re} z)$

(e).$\exp(z)\neq 0$ (f). $\frac{d}{dz}\exp(z) = \exp(z)$

CONFORMAL TRANSFORMATION

Closed curves:

The curve is said to be closed if the origin point and the terminus point are coincide.(ie.,) C is closed if Z(a)=Z(b).

Simple closed curve:

The curve C is called simple closed curve if (i). Z(a)=Z(b)

(ii). $Z(t_1) \neq Z(t_2) \Rightarrow t_1 \neq t_2$ where t_1 & $t_2 \varepsilon$ [a,b].

Definition:1 **Conformal Mapping**

A curve C in the complex plane is the continuous function $\gamma : [a,b] \to C$. If $\gamma(t) : x(t) + iy(t)$ then the curve C is determined by the two continuous real valued function x=x(t) & y=y(t) where a≤t≤b. The point z(a) s called the terminus of the curve.

A mapping or transformation which preserves angle in magnitude and in direction between every pair of curves through a point is said to be conformal at a point.

Theorem: Under a conformal mapping orthogonal curves go onto orthogonal curves.

Theorem: Let f be analytic function in the region Ω if $f'(z_0) \neq 0$, $Z_0 \varepsilon \Omega$ then f is conformal at Z_0.

Theorem: W=f(z) is conformal then f(z) is analytic function.

Theorem: Let f be an analytic function in the domain D, and let z_0 be a point in D. If $f'(z_0) \neq 0$, then f is conformal at z_0.

Linear transformations:

A transformation $L : \Re^n \to \Re^m$ defined on all of $\Re^n$ is linear if L(X+Y) =L(X)+L(Y) for all X and Y in $\Re^n$ and L(aX)=aL(X) for all X in $\Re^n$ and real numbers a.

Definition: Cross Ratio:

For the set of four distinct points z_1, z_2, z_3, z_4 the cross ratio denoted by (z_1, z_2, z_3, z_4) if defined by $(z_1, z_2, z_3, z_4) = \frac{(z_1 - z_2)(z_3 - z_4)}{(z_1 - z_4)(z_3 - z_2)}$.

If the one of the four points is ∞, the fctors containing it should be omitted.

Theorem: Let z_2, z_3, z_4 be any three distinct points then there exists a linear transformation which maps ,these points into $(1,0,\infty)$ respectively. Then the bilinear transformation is,

$$w = \frac{(z-z_3)(z_2-z_4)}{(z-z_4)(z_2-z_3)}.$$

Results:

1. The cross ratio (z_1, z_2, z_3, z_4) is real iff the four points lies on the **st.line** (or) **circle.**

2. The bilinear transformation which maps the three points z_1, z_2, z_3 in Z-plane to three points w_1, w_2, w_3 in w-plane is given by

$(w, w_1, w_2, w_3) = (z, z_1, z_2, z_3)$ (ie.,) $w = \frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)} = \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$

Definition:2 Isogonal transformation

The transformation preserves the angle in magnitude but not in direction between every pair of curves through a point is said to be isogonal at a point.

Definition:3 Critical point

A point at which $f'(z) = 0$ is called a critical point of the transformation. i.e., at the critical point, the transformation w = f(z) is not conformal.

Remark: Critical points of a transformation are obtained by $\frac{dw}{dz} = 0$ and $\frac{dz}{dw} = 0$.

Problem: 1 **Find the critical points of the transformation** $w = z + \frac{1}{z}$.

Solution: **Given** $w = z + \frac{1}{z}$

$$\frac{dw}{dz} = 0 \Rightarrow 1 - \frac{1}{z^2} = 0 \qquad \because d\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

$$\Rightarrow \frac{1}{z^2} = 1$$

$$\Rightarrow z^2 = 1$$

$$\Rightarrow z = \pm 1$$

The critical points are -1 and 1.

Complex integration:

Definition:

The line integrals $\int_{\gamma} f(z)dz = \int_{a}^{b} f(z(t))z'(t)dt$ and f = u+iv, z = x+iy you will realize that $\int_{\gamma} f(z)dz$ is the line integral $\int_{\gamma} udx - vdy + i\int_{\gamma} vdx + udy$, but we will not use this. It is, however, very important that the complex integral is independent of the parameterizations of the arc γ.

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