CHAPTER - 1
RELATIONS AND FUNCTIONS.

1. Let $A = \{1, 2\}$ $B = \{3, 6\}$ and f and g be functions from A to B defined by $f(x) = 3x$ and $g(x) = x^2+2$ show that $f = g$

Solution

Condition for $f = g$

Domain of f = Domain g

Domain of f

$f(1) = 3(1)$

$f(1) = 3$

$f(2) = 3(2)$

$f(2) = 6$

Domain of $f = \{3, 6\}$

Domain of g

$g(1) = 1^2+2$

$g(1) = 3$

$g(2) = 2^2+2$

$= 4+2$

$g(2) = 6$

Domain of $g = \{3, 6\}$

So, $f = g$.

2. Let $f : N \rightarrow N$; $f(x) = 2x$ for all x in N.
check wheather f is a bijective.

f is One-one

If f is one-one function

$f(x) = f(y)$ then $x = y$

$f(x) = 2x$; $f(y) = 2y$

$f(x) = f(y)$

$2x = 2y$

$x = y$

So. f is one-one function

f is onto

$f(x) = y$ in co-domain of f.

$2x = y$

$x = y/2$

$\forall y$ in N, $y/2 \notin N$

So f is not onto function.

$\therefore$ f is not bijective function.

3. Let R be the set of all real numbers. Let $f: R \to R$; $f(x) = \cos x$ and $g = R \to R$; $g(x) = 3x^2$ show that $g \circ f \neq f \circ g$

$g \circ f = g[f(x)]$
$= g[\cos x]$
$g \circ f = 3\cos^2 x$

$f \circ g = f[g(x)]$
$= f[3x^2)$
$f \circ g = \cos 3x^2$

So, $g \circ f \neq f \circ g$.

4. If $n[(A\times B)\cap(A\times C)] = 8$ and $n(B\cap C) = 2$ then find $n(A)$.

We know

$$A\times(B\cap C) = (A\times B)\cap(A\times C)$$

$$n[A\times(B\cap C)] = n[A\times B] \cancel{\times}\, n(A\times C)$$

$$n(A)\times n(B\cap C) = n(A\times B)\,\cancel{\times}\, n(A\times C)$$

$$n(A)\times 2 = 8$$

$$\boxed{n(A) = 4}$$

5. If $n(A) = 2$ and $n(B\cup C) = 3$ then find $n[(A\times B)\cup(A\times C)]$

$$n(A\times B\cup C) = n[(A\times B)\cup(A\times C)]$$

$$n(A)\times n(B\cup C) = n[(A\times B)\cup(A\times C)]$$

$$2\times 3 = n[(A\times B)\cup(A\times C)]$$

$$6 = n[(A\times B)\cup(A\times C)].$$

6. If $f(x) = ax+b$ where a & b are integers $f(-1) = 5$ and $f(3) = 3$ then find the value of a and b.

$$f(x) = ax+b$$

$$f(-1) = a(-1)+b$$
$$= -a+b$$

$$f(-1) = 5$$

$$-a+b = 5 \quad \text{--- (1)}$$

$$f(3) = a(3)+b$$
$$= 3a+b$$

$$f(3) = 3$$

$$3a+b = 3 \quad \text{--- (2)}$$

from (1) & (2)

$$-a+b = 5$$
$$3a+b = 3 \quad (-)$$

$-4a = 2$

$a = 2/-4$

$\boxed{a = -1/2}$

$a = -1/2$ Sub eqn ①

$-1/2 + b = 5$

$b = 5 + 1/2$

$b = 11/2$

Sol:

$a = -1/2$

$b = 11/2$

7. If $f(x) = \frac{3}{x+1} + \frac{1}{x-2}$ find the value of x then $f(x) = 0$

$f(x) = \frac{3}{x+1} + \frac{1}{x-2}$

$0 = \frac{3(x-2) + 1(x+1)}{(x+1)(x-2)}$

$3x - 6 + x + 1 = 0$

$4x - 5 = 0$

$4x = 5$

$\boxed{x = 5/4}$

8) If $f(x) = x^3 + Ax^2 + Bx - 3$; $f(1) = 4$; $f(-1) = -6$ then find value of $2A+B$.

$f(x) = x^3 + Ax^2 + Bx - 3$

$f(1) = 1 + A(1) + B(1) - 3$

$4 = A + B - 2$

$A + B = 6$ — ①

$f(-1) = -1 + A - B - 3$

$-6 = A - B - 4$

$A - B = -2$ —— (2)

from (1) & (2)

$A + B = 6$ (1)

$A - B = -2$

$2A = 4$

$\boxed{A = 2}$

$A = 2$ sub eqn (1)

$2 + B = 6$

$B = 4$

So; $2A + B = 2(2) + 4$

$= 4 + 4$

$\boxed{2A + B = 8}$

9. If $f(x) = \frac{x}{3x+1}$ then find $ff(-1)$

$f \circ f = f[f(x)]$

$= f[x/3x+1]$

$= \frac{x/3x+1}{3\left(\frac{x}{3x+1}\right)+1}$

$= \frac{x/\cancel{3x+1}}{(3x + 3x+1)/\cancel{3x+1}}$

$f \circ f = \frac{x}{6x+1}$

$f \circ f(-1) = \frac{-1}{6(-1)+1}$

$= \frac{-1}{-5}$

$\boxed{ff(-1) = 1/5}$

If $f(x) = 2x-3$ $g(x) = x^2+2$ solve

$fg(x) = gf(x)$

$f \circ g = f[g(x)]$

$= f[x^2+2]$

$= 2(x^2+2) - 3$

$= 2x^2+4-3$

$f \circ g = 2x^2+1$

$g \circ f = g f(x)$

$= g[2x-3]$

$= (2x-3)^2 + 2$

$= 4x^2 - 12x + 9 + 2$

$= 4x^2 - 12x + 11$

Given

$f \circ g = g \circ f$

$4x^2 - 12x + 11 = 2x^2 + 1$

$2x^2 - 12x + 10 = 0$

$x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

$x = 5 ; x = 1$

Sol: $\{5, 1\}$

Sums for Practice:-

1. If $f : R \to R$ and $g : R \to R$ are defined by $f(x) = 2x+3$ and $g(x) = x^2+7$. Find the value of x if $g[f(x)] = 8$

2. If $f(x) = \frac{2x+1}{3x-2}$ then find $f \circ f(2)$.

3. Find the range of the function $f(x) = \frac{x^2 - x}{x^2+2x}$

4. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x+x^3}{1+3x^2}$ then find $f[g(x)]$

5. Find the value of 'x' then $f(x) = 6x+1$ and $f(x) = 7$.

6. If $f(x) = 2x-5$ $g(x) = x^2-10$. f and g are function find the value of x if $g[f(x)] = -1$

7. If $f(x) = x^3 - 3x^2 - 2x + 5$ and $g(x) = 2$ then find $g[f(x)]$.

8. If $f(x) = \cancel{x}^2 - 4$ & $g(x) = -2x+1$ find $f[g(2x)]$.

9. If $g(x) = 6x$ and $h(x) = 3x-7$ then find (i) $g\,h(-1)$ (ii) $h\,g(3)$.

10. If $f(x) = 2x+1$ $g(x) = 10-3x$ then find $f[g(2x+1)]$.

11. A function f from $N \to N$ is defined by $f(x) = 3x$. Find the range of f.

12. Given $f(x) = \frac{4x}{x+1}$ with domain $\{-3, -2, 0, 1, 3\}$. Find the range of f.

13. Let $A = Z - \{0\}$ and let $f : A \to A$ be defined by $f(x) = \frac{|x|}{x}$. What type of function is f?

14. Find the images of $-1, -2, 3, 4$ of the function $f : Z \to Z$ is defined by $f(x) = 3x$.

15. Is the function from $N \to N$ given by $f(x) = 2x+1$ onto? Write down the range.

16. Find the value of m if $f \circ g = g \circ f$.

(i) $f(x) = 2x+3$ $\quad g(x) = 5x+m$

(ii) $f(x) = 4x+m$ $\quad g(x) = 2x+5$.

17. Verify that the composition of functions is associative given

(i) $f(x) = 2x+3$, $\quad g(x) = 1-2x$ $\quad h(x) = 3x$

(ii) $f(x) = x+5$ $\quad g(x) = 4x$ $\quad h(x) = x-4$

(iii) $f(x) = x^2$ $\quad g(x) = 3x+5$ $\quad h(x) = x-1$

(iv) $f(x) = x-3$ $\quad g(x) = 2x-5$ $\quad h(x) = 5x$

CHAPTER - 2

NUMBERS AND SEQUENCE.

1. If in an A.P the sum of p terms is equal to q, and sum of q terms is equal to p, then prove that the sum of $p+q$ terms is $-(p+q)$.

In an A.P sum of 'n' terms

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_p = \frac{p}{2}[2a+(p-1)d]$$

But $S_p = q$

$$\frac{p}{2}[2a+(p-1)d] = q$$

$$2ap + p(p-1)d = 2q \quad \text{--- (1)}$$

$$S_q = \frac{q}{2}[2a+(q-1)d]$$

But $S_q = p$

$$\frac{q}{2}[2a+(q-1)d] = p$$

$$2aq + q(q-1)d = 2p \quad \text{--- (2)}$$

(1) - (2)

$$2ap + p(p-1)d = 2q$$
$$2aq + q(q-1)d = 2p$$

$$2a(p-q) + d(p-q)(p+q-1) = -2(p-q)$$

$\div (p-q)$

$$2a + d(p+q-1) = -2 \quad \text{--- (3)}$$

$$S_{p+q} = \frac{p+q}{2}\left[2a + (p+q-1)d\right]$$

$$= \frac{p+q}{\cancel{2}}(-\cancel{2}) \quad \text{by } (3)$$

$$S_{p+q} = -(p+q)$$

Prove that the sum of the latter half of $2n$ terms of an A.P is one third of the sum of $3n$ terms of the same A.P.

Sum of the latter half of $2n$ terms of an A.p

$$= t_{n+1} + t_{n+2} + t_{n+3} + \cdots + t_{2n}$$

$$= (a+nd) + (a+(n+1)d) + \cdots + (a+(2n-1)d)$$

$$= \frac{n}{2}\left[\text{first term} + \text{last term}\right] \qquad \boxed{S_n = \frac{n}{2}(l+a)}$$

$$= \frac{n}{2}\left[a+nd + a+(2n-1)d\right]$$

$$= \frac{n}{2}\left[2a + (3n-1)d\right]$$

$$= \frac{3}{3}\left[\frac{n}{2}(2a+(3n-1)d\right]$$

$$= \frac{1}{3}\left[\frac{3n}{2}(2a+(3n-1)d\right]$$

$= \frac{1}{3}$ sum of $3n$ terms of the same A.p

(3) Find the sum to n terms of the series

$\log a + \log \frac{a^2}{b} + \log \frac{a^3}{b^2} + \cdots$ to n terms

Let $A = \log a + \log \frac{a^2}{b} + \log \frac{a^3}{b^2} + \cdots$ to n terms

$= \log a + \log a^2 - \log b + \log a^3 - \log b^2 + \cdots$ n terms

$= \log a + 2\log a - \log b + 3\log a - 2\log b + \cdots$ n terms

$= (\log a + 2\log a + 3\log a + \cdots$ n terms$) -$

$(\log b + 2\log b + 3\log b + \cdots (n-1)$ terms$)$

$= \log a [1+2+3+ \cdots$ n terms$] - \cancel{\log b}$

$\log b [1+2+3+ \cdots (n-1)$ terms$]$

$= \log a \left[\frac{n(n+1)}{2}\right] - \log b \left[\frac{(n-1)(n-1+1)}{2}\right]$

$= \log a \left[\frac{n(n+1)}{2}\right] - \log b \left[\frac{(n-1)n}{2}\right]$

$= \frac{n}{2}\left[\log a (n+1) - (n-1)\log b\right]$

$= \frac{n}{2}\left[(n+1)\log a - (n-1)\log b\right]$

4. Sum the series to n terms $\frac{1}{\sqrt{3}} + 1 + \sqrt{3} + 3 + \dots$

$a = \frac{1}{\sqrt{3}}$ $\quad r = \frac{t_2}{t_1}$

$= 1/_{1/\sqrt{3}}$

$r = \sqrt{3} > 1$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{1/\sqrt{3}\,(\sqrt{3}^n - 1)}{\sqrt{3} - 1}$$

$$= \frac{\sqrt{3}^n - 1}{\sqrt{3}(\sqrt{3} - 1)}$$

$$= \frac{\sqrt{3}^n - 1}{3 - \sqrt{3}}$$

5. Find the sum of infinity term 54, 18, 6, 2, ...

$a = 54$ $\quad r = 18/54$

$r = 1/3 < 1$

$$S_\infty = \frac{a}{1 - r}$$

$$= \frac{54}{1 - 1/3}$$

$$= \frac{54}{(3-1)/3}$$

$$= \frac{\overset{27}{\cancel{54}} \times 3}{\cancel{2}}$$

$S_\infty = 27 \times 3$

$\boxed{S_\infty = 81}$

5: Find the sum to infinity of the series

$1+(1+b)r+(1+b+b^2)r^2+\cdots\cdots$

Let $A = 1+(1+b)r+(1+b+b^2)r^2+\cdots\cdots$

$= \frac{1-b}{1-b}\left[1+(1+b)r+(1+b+b^2)r^2+\cdots\right]$

$= \frac{1}{1-b}\left[(1-b)+(1-b)(1+b)r+(1-b)(1+b+b^2)r^2+\cdots\right]$

$= \frac{1}{1-b}\left[1-b+(1-b^2)r+(1-b^3)r^2+\cdots\right]$

$= \frac{1}{1-b}\left[1-b+r-b^2r+r^2-b^3r^2+\cdots\right]$

$= \frac{1}{1-b}\left[(1+r+r^2+\cdots)-(b+b^2r+b^3r^2+\cdots)\right]$

$= \frac{1}{1-b}\left[\frac{1}{1-r}-\frac{b}{1-br}\right]$ $\left[S_\infty = a/_{1-r}\right]$

$= \frac{1}{1-b}\left[\frac{1-br-b(1-r)}{(1-r)(1-br)}\right]$

$= \frac{1}{1-b}\left[\frac{1-\cancel{br}-b+\cancel{br}}{(1-r)(1-br)}\right]$

$= \frac{1}{\cancel{1-b}}\left[\frac{\cancel{1-b}}{(1-r)(1-br)}\right]$

$\boxed{A = \frac{1}{(1-r)(1-br)}}$

6) Find the value of $\sqrt{3\sqrt{3\sqrt{3}}}\ldots$

$$\sqrt{3\sqrt{3\sqrt{3}}}\ldots = 3^{1/2} \times 3^{1/4} \times 3^{1/8} \times \ldots$$

$$= 3^{1/2 + 1/4 + \frac{1}{8} + \ldots}$$

Let $A = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots \infty$

$a = 1/2 \quad r = \frac{1/4}{1/2}$

$r = 1/2 < 1$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{1/2}{1-1/2}$$

$$= \frac{1/2}{1/2}$$

$A = 1$

$$\therefore \sqrt{3\sqrt{3\sqrt{3}}}\ldots = 3^1$$

$$= 3$$

7) Find the value of $\sqrt[3]{9\sqrt[3]{9\sqrt[3]{9}}}\ldots$

$$\sqrt[3]{9\sqrt[3]{9\sqrt[3]{9}}}\ldots = 9^{1/3} \times 9^{1/9} \times 9^{1/27} \times \ldots$$

$$= 9^{\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots}$$

Let $A = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$

$a = 1/3 \quad r = \frac{1/9}{1/3}$

$r = \frac{1}{3} < 1$

$S_\infty = \frac{a}{1-r}$

$A = \frac{1/3}{1-1/3}$

$= \frac{1/3}{2/3}$

$A = 1/2$

$\sqrt[3]{9\sqrt[3]{9\sqrt[3]{9}}}\cdots = 9^{1/2}$

$= (3^2)^{1/2}$

$= 3$

8) The sum of an infinite G.P is 4 and the sum of the cube of the terms is 192. Find the common ratio.

Let G.P

$a + ar + ar^2 + \cdots$

$S_\infty = \frac{a}{1-r}$

$\frac{a}{1-r} = 4$

$$\left(\frac{a}{1-r}\right)^3 = 4^3$$

$$\frac{a^3}{(1-r)^3} = 64 \quad \text{——} \ (1)$$

From conditions (2)

$$a^3 + a^3r^3 + a^3r^6 + \dots = 192$$

$$\frac{a^3}{1-r^3} = 192$$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$$\frac{a^3}{(1-r)(1+r+r^2)} = 192 \quad \text{——} \ (2)$$

(2) / (1)

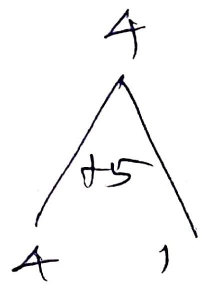
$$\frac{\cancel{a^3}/\cancel{(1-r)}(1+r+r^2)}{\cancel{a^3}/(1-r)^{\cancel{3}\,2}} = \frac{192}{64}$$

$$\frac{(1-r)^2}{1+r+r^2} = 3$$

$$\frac{1-2r+r^2}{1+r+r^2} = 3$$

$$1-2r+r^2 = 3+3r+3r^2$$

$$2r^2+5r+2 = 0$$



$$(r+2)(2r+1) = 0$$

$$r = -2 \qquad 2r = -1$$

$$r = -\tfrac{1}{2}$$

Common ratio -2 (or) $-\tfrac{1}{2}$

9) . The sum of an infinite number of terms of a G.P is 15 and the sum of their squares is 45. Find the sequence.

Let G.P

$a + ar + ar^2 + \cdots$

From 1st condition

$a + ar + ar^2 + \cdots \infty = 15$

$$\frac{a}{1-r} = 15$$

$$\left(\frac{a}{1-r}\right)^2 = 15^2$$

$$\frac{a^2}{(1-r)^2} = 225 \quad \text{--- (1)}$$

From 2nd condition

$a^2 + a^2r^2 + a^2r^4 + \cdots \infty = 45$

$$\frac{a^2}{1-r^2} = 45$$

$$\frac{a^2}{(1+r)(1-r)} = 45 \quad \text{--- (2)}$$

(2) / (1)

$$\frac{\cancel{a^2}/(1+r)\cancel{(1-r)}}{\cancel{a^2}/(1-r)^{\cancel{2}}} = \frac{45}{225}$$

$$\frac{1-r}{1+r} = \frac{1}{5}$$

$$5(1-r) = 1+r$$

$$5-5r = 1+r$$

$$6r = 4$$

$$r = 2/3$$

G.P $r = 2/3$ then

$$\frac{a}{1-2/3} = 15$$

$$\frac{a}{(3-2)/3} = 15$$

$$3a = 15$$

$$\boxed{a = 5}$$

G.P, $5, 5(2/3), 5(2/3)^2, \ldots$

10. A G.P consists of even number of terms. The sum of all the terms is three times that of the odd terms. Find the common ratio.

$$S_{2n} = 3\left[t_1 + t_3 + t_5 + \ldots + t_{2n-1}\right]$$

$$\frac{a(1-r^{2n})}{1-r} = 3\left[a + ar^2 + ar^4 + \ldots + ar^{2n}\right]$$

$$\frac{a(1-r^{2n})}{1-r} = 3\,\frac{a\left[1-(r^2)^n\right]}{1-r^2}$$

$$\frac{\cancel{a}(1-\cancel{r^{2n}})}{1-\cancel{r}} = \frac{3\cancel{a}(1-\cancel{r^{2n}})}{(1+r)\cancel{(1-r)}}$$

$$1 = \frac{3}{1+r}$$

$$r = 3-1$$

$$\boxed{r = 2}$$

Common ratio $= 2$.

11. Let a, b, c be respectively the sums of the first n terms, the next n terms and the next n terms of a G.P show that a, b, c are in G.P.

Sol.

Let A be the first term of A and R be the common ratio of G.P.

a = Sum of the first n terms of the G.P

$$= A + AR + AR^2 + \cdots + AR^{n-1} \quad \boxed{t_n = ar^{n-1}}$$

$$= \frac{A(1-R^n)}{1-R}$$

b = Sum of the next n terms of the G.P

$$= AR^n + AR^{n+1} + \cdots + AR^{2n-1}$$

$$= \frac{AR^n(1-R^n)}{1-R}$$

c = Sum of the next n terms of the G.P

$$= AR^{2n} + AR^{2n+1} + \cdots + AR^{3n-1}$$

$$c = \frac{AR^{2n}(1-R^n)}{1-R}$$

If a, b, c are in G.P

$$b/a = c/b$$

$$b/a = R^n \quad \& \quad c/b = R^n$$

So, a, b, c are in G.P.

12. Find the sum of the first 10 terms if the n^{th} terms of the G.P is $\frac{2^{2n-1}}{3}$.

n^{th} term $t_n = \frac{2^{2n-1}}{3}$

$$t_1 = \frac{2^{2-1}}{3}$$

$$= 2/3$$

$$t_2 = \frac{2^{4-1}}{3}$$

$$= \frac{2^3}{3}$$

$$= 8/3$$

$$r = \frac{t_2}{t_1} = \frac{8/3}{2/3}$$

$$\boxed{r = 4} > 1$$

$$S_n = \frac{a(r^n-1)}{r-1}$$

$$S_{10} = \frac{2/3\,(4^{10}-1)}{4-1}$$

$$= \frac{2}{3\times3}(4^{10}-1)$$

$$\boxed{S_{10} = \frac{2}{9}(4^{10}-1)}$$

If S_1, S_2, S_3 are the sums of the first n natural numbers, their squares and cubes respectively prove that $9S_2^2 = S_3(1+8S_1)$

Proof:

From given information

$$S_1 = 1+2+3+\cdots+n$$

$$S_1 = \frac{n(n+1)}{2}$$

$$S_2 = 1^2+2^2+3^2+\cdots+n^2$$

$$S_2 = \frac{n(n+1)(2n+1)}{6}$$

$$S_3 = 1^3+2^3+3^3+\cdots+n^3$$

$$S_3 = \left[\frac{n(n+1)}{2}\right]^2$$

To prove $9S_2^2 = S_3(1+8S_1)$

LHS: $9S_2^2$

$$S_2^2 = \left[\frac{n(n+1)(2n+1)}{6}\right]^2$$

$$9S_2^2 = \cancel{9}\times\frac{n^2(n+1)^2(2n+1)^2}{\cancel{36}\ 4}$$

$$9S_2^2 = \frac{1}{4}\left[n^2(n+1)^2(2n+1)^2\right] \quad \text{--- (1)}$$

RHS: $S_3(1+8S_1)$

$$8S_1 = \overset{4}{\cancel{8}}\,\frac{n(n+1)}{\cancel{2}}$$

$$= 4n(n+1)$$

$$8S_1 = 4n^2+4n$$

$$1+8S_1 = 1+4n^2+4n$$

$$= 4n^2+4n+1$$

$$= (2n+1)(2n+1)$$

$$= (2n+1)^2$$

4

4

2n 2n

4n 4n

$$S_3(1+8S_1) = \left[\frac{n(n+1)}{2}\right]^2(2n+1)^2$$

$$= \frac{1}{4}n^2(n+1)^2(2n+1)^2 \quad \text{--- (2)}$$

from (1) & (2) LHS = RHS.

14. The sum of certain consecutive odd numbers is $57^2 - 13^2$. Find the numbers.

We know

$$1+3+5+\cdots+l = \left(\frac{l+1}{2}\right)^2 \quad [l \text{ is last term}]$$

So

$$(1+3+5+\cdots+l_1) - (1+3+5+\cdots+l_2) = 57^2 - 13^2$$

$$1+3+5+\cdots+l_1 = 57^2$$

$$\left(\frac{l_1+1}{2}\right)^2 = 57^2$$

$$\frac{l_1+1}{2} = 57$$

$$l_1+1 = 114$$

$$l_1 = 113$$

$$1+3+5+\cdots l_2 = 13^2$$

$$\left(\frac{l_2+1}{2}\right)^2 = 13^2$$

$$\frac{l_2+1}{2} = 13$$

$$l_2+1 = 26$$

$$l_2 = 25$$

$$57^2 - 13^2 = (1+3+5+\cdots+113) - (1+3+5+\cdots+25)$$

$$= 27+29+31+\cdots+113$$

∴ The numbers are

27, 29, 31, ... 113.

15. If 9^{th} term of A.P is zero, prove that its 29^{th} term is double (twice) the 19^{th} term.

In an A.P n^{th} term $t_n = a + (n-1)d$

9^{th} term of A.P $t_9 = a + (9-1)d$

$= a + 8d$

Given $t_9 = 0$

$a + 8d = 0$

$a = -8d$ —— (1)

19^{th} term of A.P $= t_{19} = a + (19-1)d$

$= a + 18d$

$= -8d + 18d$ (by (1))

$t_{19} = 10d$ —— (2)

29^{th} term of A.P $t_{29} = a + (29-1)d$

$= a + 28d$

$= -8d + 28d$ (by (1))

$= 20d$

$= 2 \times 10d$

$= 2 \times t_{19}$ by (2)

$\therefore t_{29} = 2 \times t_{19}$

Hence proved.

16. If/ m times ~~the~~ m^{th} ~~term of A.P is A,~~ n ~~times~~

16. If m times the m^{th} term of an A.P is equal to n times its n^{th} term, then show that the $(m+n)^{th}$ term of the A.P is zero.

n^{th} term of A.P $t_n = a+(n-1)d$

n times of n^{th} term

$$n t_n = n[a+(n-1)d]$$

$$= an + (n-1)nd$$

$$n t_n = an + n^2 d - nd$$

m^{th} term of A.P

$$t_m = a+(m-1)d$$

m times of m^{th} term

$$m t_m = m[a+(m-1)d$$

$$= ma + (m-1)md$$

$$m t_m = ma + m^2 d - md$$

Given $m t_m = n t_n$

$$ma + m^2 d - md = na + n^2 d - nd$$

$$ma - na + m^2 d - n^2 d - md + nd = 0$$

~~ma~~ ~~nd~~ /=

$$a(m-n) + d(m^2-n^2) - d(m-n) = 0$$

$$a(m-n) + (m+n)(m-n)d - d(m-n) = 0$$

$\div (m-n)$

$$a + (m+n)d - d = 0$$

$$a + [(m+n)-1]d = 0$$

$$\therefore t_{m+n} = 0$$

$\therefore (m+n)^{th}$ term is zero.

17. If a, b, c are in A.P then prove that

$$(a-c)^2 = 4(b^2-a)$$

Sol:

a, b, c are in A.P

$\therefore d = t_2 - t_1 = t_3 - t_2$

$$b-a = c-b$$

$$b+b = c+a$$

$$2b = c+a$$

Square on both sides

$$(2b)^2 = (c+a)^2$$

$$4b^2 = c^2 + 2ca + a^2$$

Add $-4ac$ on both sides

$$4b^2 - 4ac = a^2 + 2ac + c^2 - 4ac$$

$$4(b^2-ac) = a^2 - 2ac + c^2$$

$$4(b^2-ac) = (a-c)^2$$

18. If a, b, c are in A.P then prove that $1/bc$, $1/ca$, $1/ab$ are also in A.P.

Sol:

a, b, c are in A.P

An A.P remains an A.P if each of its terms is divided by a non-zero constant.

$\div abc$

a/abc, b/abc, c/abc

$\frac{1}{bc}, \frac{1}{ac}, \frac{1}{ab}$ are in A.P.

19 If a, b, c, d are in G.P, then prove that

$(b-c)^2 + (c-a)^2 + (d-b)^2 = (a-d)^2$

Sol:

a, b, c, d are in G.P

$\therefore r = b/a = c/b = d/c$

$\frac{b}{a} = c/b$	$c/b = d/c$	$b/a = d/c$
$b^2 = ac$	$c^2 = bd$	$bc = ad$

LHS =

$= (b-c)^2 + (c-a)^2 + (d-b)^2$

$= b^2 - 2bc + c^2 + c^2 - 2ac + a^2 + d^2 - 2bd + b^2$

$= \cancel{ac} - 2bc + \cancel{bd} + \cancel{bd} - \cancel{2ac} + a^2 + d^2 - \cancel{2bd} + \cancel{ac}$

$= -2bc + a^2 + d^2$

$= a^2 - 2bc + d^2$

$= a^2 - 2ad + d^2 \quad [bc = ad]$

$= (a-d)^2$

Hence proved.

If a, b, c, d are in G.P then show that $(a-b+c)(b+c+d) = ab+bc+cd$

Sol:

a, b, c, d are G.P

$r = b/a = c/b = d/c$

$\frac{b}{a} = c/b$	$\frac{c}{b} = d/c$	$b/a = d/c$
$b^2 = ac$	$c^2 = bd$	$bc = ad$

$(a-b+c)(b+c+d) = ab + ac + ad - b^2 - \cancel{bc} - bd + \cancel{bc} + c^2 + cd$

$= ab + \cancel{b^2} + ad - \cancel{b^2} - \cancel{c^2} + \cancel{c^2} + cd$

$= ab + ad + cd$

$= ab + bc + cd$

Hence proved.

20. Find the sum of the first $2n$ terms of the following series

$1^2 - 2^2 + 3^2 - 4^2 + \ldots\ldots$

Sol :

$1^2 - 2^2 + 3^2 - 4^2 + \ldots\ldots$ $2n$ terms

$= 1 - 4 + 9 - 16 + 25 - 36 + \ldots\ldots$ $2n$ terms

$= (1-4) + (9-16) + (25-36) + \ldots\ldots$ n terms

$= -3 + (-7) + (-11) + \ldots\ldots$ n terms

First term $a = -3$

$d = t_2 - t_1$

$= -7 - (-3)$

$d = -4$

$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$1^2 - 2^2 + 3^2 - 4^2 + \ldots$ $2n$ terms

$= \frac{n}{2}[2(-3) + (n-1)(-4)]$

$= n/2[-6 + (n-1)(-4)]$

$= \frac{n}{2}[-6 - 4n + 4]$

$= \frac{n}{2}[-4n - 2]$

$= \frac{n}{2}(-2)[2n+1]$

$= -n(2n+1)$

21). In an Arithmetic series, the sum of first 14 terms is -203 and the sum of the next 11 terms is -572. Find the arithmetic series.

In an A.P

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_{14} = \frac{14}{2}[2a+(14-1)d]$$

$$S_{14} = 7[2a+13d]$$

$$S_{14} = -203$$

$$7[2a+13d] = -203$$

$$2a+13d = -203/7$$

$$2a+13d = -29 \quad \text{--- (1)}$$

Sum of next 11 terms $= -572$

i.e) $S_{25} = S_{14} + (-572)$

$$S_{25} = -203 - 572$$

$$\frac{25}{2}[2a+(25-1)d] = -775$$

$$2a+24d = \frac{\overset{31}{\cancel{-775}} \times 2}{\cancel{25}}$$

$$2a+24d = -62 \quad \text{--- (2)}$$

② − ①

$$2a + 24d = -62$$
$$\underset{(-)}{2a} \underset{(-)}{+} 13d = \underset{(+)}{-29}$$
$$11d = -33$$
$$\boxed{d = -3}$$

$d = -3$ sub equ ①

$$2a + 13(-3) = -29$$
$$2a - 39 = -29$$
$$2a = -29 + 39$$
$$2a = 10$$
$$\boxed{a = 5}$$

∴ Arithmetic Series

$$a + (a+d) + (a+2d) + \cdots$$
$$5 + (5-3) + (5-2\times3) + \cdots$$
$$5 + 2 - 1 \cdots$$

Practice problem :-
In an Arithmetic series the sum of first 11 terms is 44 and that of next 11 terms is 55. Find the Arithmetic series.

22. A geometric series consists of four terms and has a positive common ratio. The sum of first two terms is 8, and the sum of the last two terms is 72. Find the series

Let the Geometric series

$a + ar + ar^2 + ar^3$; $r > 0$

Given

$a + ar = 8$ | $ar^2 + ar^3 = 72$

$a(1+r) = 8$ —① | $ar^2(1+r) = 72$

$a(1+r) \times r^2 = 72$

$8 \times r^2 = 72$

$r^2 = 9$

$r = \pm 3$

$r = 3$ $r > 0$

$r = 3$ then sub ①

$a(1+3) = 8$

$a \times 4 = 8$

$\boxed{a = 2}$

Geometric series

$2 + 2\times3 + 2\times3^2 + 2\times3^3$

$2 + 6 + 18 + 54$.

Practice:

A geometric series consists of four terms and a positive common ratio. The sum of the first two terms is 9 and sum of last two terms is 36. Find the series.

23. If S_1, S_2 and S_3 are the sum of first n, $2n$ and $3n$ terms of ~~the~~ a geometric series respectively, then prove that $S_1(S_3 - S_2) = (S_2 - S_1)^2$.

From given information

$$S_1 = \frac{a(1-r^n)}{1-r} \quad r<1$$

$$S_2 = \frac{a(1-r^{2n})}{1-r} \quad r<1$$

$$S_3 = \frac{a(1-r^{3n})}{1-r} \quad r<1$$

To prove $S_1(S_3-S_2) = (S_2-S_1)^2$

$$S_3 - S_2 = \frac{a(1-r^{3n})}{1-r} - \frac{a(1-r^{2n})}{1-r}$$

$$= \frac{a}{1-r}\left[\cancel{1} - r^{3n} - \cancel{1} + r^{2n}\right]$$

$$= \frac{a}{1-r}\left[r^{2n} - r^{3n}\right]$$

$$= \frac{ar^{2n}}{1-r}(1-r^n)$$

$$S_1(S_3-S_2) = \frac{a(1-r^n)}{1-r} \times \left(\frac{ar^{2n}}{1-r}(1-r^n)\right)$$

$$S_1(S_3-S_2) = \frac{a^2r^{2n}(1-r^n)^2}{(1-r)^2} \quad \text{--- (1)}$$

$$S_2 - S_1 = \frac{a(1-r^{2n})}{1-r} - \frac{a(1-r^n)}{1-r}$$

$$= \frac{a}{1-r}\left[\cancel{r} - r^{2n} \cancel{-1} + r^n\right]$$

$$= \frac{a}{1-r}\left(r^n - r^{2n}\right)$$

$$S_2 - S_1 = \frac{a}{1-r} \times r^n \left(1 - r^n\right)$$

$$(S_2 - S_1)^2 = \frac{a^2}{(1-r)^2} r^{2n} \left(1 - r^n\right)^2 \quad \text{---} \ (2)$$

from (1) & (2)

$$S_1(S_3 - S_2) = (S_2 - S_1)^2$$

Hence proved.

Simultaneous Equations:-

Word Problems:-

1. In $\triangle ABC$, $\angle C$ is 20° greater than $\angle A$. The sum of $\angle A$ and $\angle C$ is twice $\angle B$. Find three angles.

Let x be the angle of $\angle A$

y be the angle of $\angle B$

z be the angle of $\angle C$

In a $\triangle$ sum of all angles is 180°

$$x+y+z = 180^\circ \quad \text{—} \ (1)$$

From 1st condition $z = x+20^\circ$ — (2)

From 2nd condition

$$x+z = 2y \quad \text{—} \ (3)$$

equ (3) sub equ (1)

$$y+2y = 180$$

$$3y = 180^\circ$$

$$y = 60^\circ$$

$y = 60^\circ$ & $z = x+20^\circ$ sub equ (1)

$$x+60^\circ+x+20^\circ = 180^\circ$$

$x = 50^\circ$

$$2x+80^\circ = 180^\circ$$

$$2x = 180-80^\circ$$

$$2x = 100$$

$x = 50°$ $y = 60°$ Sub Equ ①

$$50 + 60° + z = 180$$
$$z = 180 - 110°$$
$$z = 70°$$

∴ Result $\angle A = 50°$ $\angle B = 60°$ $\angle C = 70°$

2. The sum of three numbers is 24. Among them One number is equal to half of the sum of Other two numbers but four times the difference Of them. Find the numbers.

Let x, y, z be the three numbers.

From 1st condition

$$x + y + z = 24 \quad \text{--- ①}$$

From 2nd condition

Let $x = \frac{1}{2}(y+z)$

$$2x = y + z \quad \text{--- ②}$$

From 3rd condition

$$x = 4(y - z)$$
$$x = 4y - 4z \quad \text{--- ③}$$

Equ ② Sub Equ ①

$$x + 2x = 24$$
$$3x = 24$$
$$\boxed{x = 8}$$

From ② + ③

$2x \times 4 = 8x$
$= 8 \times 8$
$= 64$

$4 \times$ ② $\Rightarrow$ $4y + 4z = 64$

③ $\Rightarrow$ $4y - 4z = 8$

$8y = 72$

$\boxed{y = 9}$

x & y value sub ②

$2 \times 8 = 9 + z$

$16 = 9 + z$

$z = 16 - 9$

$z = 7$

∴ Three numbers are

8, 9, 7

3. The sum of the digits in a three digit number is 24. Twice the tenth digit is equal to sum of the digits in ~~three / at~~ the other two places. If 198 is added with the number, then the digits will be in the reversed order. Find the number.

Let xyz be the three digit number

From 1st condition

$x + y + z = 24$ —— ①

From 2nd condition

$2y = x + z$ —— ②

From 3^{rd} condition

$$xyz + 198 = zyx$$

(ie) $100x + \cancel{10y} + z + 198 = 100z + \cancel{10y} + x$

$$100x - x - 100z + z = -198$$

$$99x - 99z = -198$$

$\div 99$ $\quad x - z = -2 \quad$ — (3)

from (1) & (2)

$$y + 2y = 24$$

$$3y = 24$$

$$\boxed{y = 8}$$

from (2) & (3)

$$x + z = 16 \quad (2y = 2 \times 8)$$

$$x - z = -2 \quad (+)$$

$$2x = 14$$

$$\boxed{x = 7}$$

$x = 7$; $y = 8$ sub Equ (2)

$$7 + z = 16$$

$$z = 9$$

$\therefore$ Required number 789.

4. The sum of the digits in a three digit number is 15. If 99 is subtracted the digits will be in the reverse order. The digit in the ten's place is equal to $\frac{2}{3}$ times the sum of the digits in the hundred's place and unit's place. Find the number.

Let xyz be the three digit number.

From 1^{st} condition

$$x+y+z = 15 \quad \text{--- (1)}$$

From 2^{nd} condition

$$xyz - 99 = zyx$$

$$100x + \cancel{10y} + z - 99 = 100z + \cancel{10y} + x$$

$$99x - 99z = 99$$

$$x - z = 1 \quad \text{--- (2)}$$

From 3^{rd} condition

$$y = \frac{2}{3}(x+z) \quad \text{--- (3)}$$

$$3y = 2x + 2z$$

$$2x - 3y + 2z = 0 \quad \text{--- (4)}$$

From (1) & (4)

$$(1) \times 3 \Rightarrow \quad 3x + \cancel{3y} + 3z = 45$$

$$(4) \Rightarrow \quad 2x - \cancel{3y} + 2z = 0$$

$$5x + 5z = 45$$

$$x + z = 9 \quad \text{--- (5)}$$

From (1) & (5)

$$y + 9 = 15$$

$$\boxed{y = 6}$$

~~From (3) & (5)~~

~~$y = \frac{2}{3}(9)$~~

~~$y = 2 \times 3$~~

~~$y =$~~

From (2) & (5)

$$x - z = 1$$

$$x + z = 9 \quad (+)$$

$$2x = 10$$

$$\boxed{x = 5}$$

$x = 5$ sub equ (5)

$$5 + z = 9$$

$$z = 9 - 5$$

$$\boxed{z = 4}$$

$\therefore$ Required number 564

Practice:

1. In $\triangle ABC$ the sum of the first two angles is twice the third angle. $\angle B$ is 5° greater than $\angle C$. Find the angles.

2. In a workshop, the pay bill of workers in 3 successive weeks were Rs 1200, Rs 1130 and Rs 1160. If 5 men, 5 women and 6 children worked in the first week. 4 men, 6 women and 5 children worked in the second week, 4 men, 7 women and 4 children worked in the third week find the wage paid for each worker.

3. 4 pens, 12 note books, 6 ball pens cost Rs 160. 3 pens ~~2f~~ note books and 1 ball pen cost Rs 66. 3 pens, ~~6 ball pens~~ 6 note books and 4 balls pens cost Rs 94. Find the cost of each.

4. A bag contains ten, five and two rupees currencies. The total number of currencies is 20 and the total value of money is Rs 125. If the second and the third sorts of curriences are interchanged the value will be decreased by Rs 6. Find the number of currency in each sort.

Solve the equations

1. $x+2y+3z=14$; $3x+y+2z=11$; $2x+3y+z=11$
2. $x+y=3$; $y+z=-5$; $z+x=2$
3. $3x-2y+z=0$; $4x+6y-3z=13$; $x-2y+2z=-4$
4. $x+2y+3z=10$; $x-2y+4z=3$; $x+y-3z=2$
5. $2x-2y+4z=-12$; $3x+2y+2z=19$; $-x+y-z=3$.

Find the G.C.D of the given pair of polynomials.

1. $x^3-3x^2+4x-12$; $x^4+x^3+4x^2+4x$
2. x^3+4x^2-5 ; x^3-3x+2
3. $x^3+6x^2+11x+6$; $x^3+9x^2+27x+27$
4. $x^4+2x^3+x^2-1$; x^4+x^2+1

Rational Expression Practice sums :

1. Simplify

1. $\frac{a+b}{a-b} \times \frac{a^3-b^3}{a^3+b^3}$ (2) $\frac{x^2-16}{x-2} \times \frac{x^2-4}{x^3+64}$

3. $\frac{p^2-1}{p} \times \frac{p^3}{p-1} \times \frac{1}{p+1}$ (4) $\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \div \left(\frac{1}{a} - \frac{1}{b}\right)$

5. $\frac{x^2-2x-8}{x-2} \times \frac{4x-8}{x^2-4x-12} \div \frac{x^2-7x+12}{x^2-9x+18}$.

6. $\frac{y}{x^2-y^2} - \frac{x}{x-y}$ (7) $\frac{m}{m+1} + \frac{1}{m+1} + \frac{1}{m^2-1}$

(8) $\frac{1}{2x+3y} + \frac{1}{2x-3y} - \frac{4x}{4x^2-9y^2}$ (9) $\frac{x+a}{x-a} - \frac{x-a}{x+a} + \frac{4ax}{a^2-x^2}$

10. $\frac{x}{x^2-9x+20} + \frac{x}{x^2-8x+15} - \frac{x}{x^2-7x+12}$

Square root practice sums.

b) Find the square root of following Polynominals.

1. $a^4 - 2a^3 + \frac{3}{2}a^2 - \frac{1}{2}a + \frac{1}{16}$
2. $\frac{1}{4}x^4 - 3x^3 + 13x^2 - 24x + 16$
3. $9x^4 - 18x^3 + 33x^2 - 24x + 16$.
4) $ax^4 + bx^3 + 109x^2 - 60x + 36$.
5). $m - nx + 28x^2 + 12x^3 + 9x^4$

Find the unknown values.

1. $p + qx + 10x^2 + 12x^3 + 9x^4$
2. $25x^4 - 40x^3 - 34x^2 + ax + b$
3. $9x^4 + 12x^3 + 40x^2 + ax + b$

5. Problems Solving using Quadratic Equation.

5. The sum of a number and its square is 90. Find the number.

Let x be the required number.

From given information

$$x + x^2 = 90$$

$$x^2 + x - 90 = 0$$

$$(x+10)(x-9) = 0$$

$$x = -10 \mid x = 9$$

$\therefore$ So required number -10 (or) 9.

-90: $+10$, -9 (sum $+1$)

6. The height of right circular cone is 7 cm greater than its radius. The slant height is 8 cm greater than its radius. Find the CSA of cone.

Let radius of the cone is x cm

$\therefore r = x$ cm

Slant Height $l = (x+8)$ cm

From given information

Height $h = (x+7)$ cm

slant height $l = \sqrt{r^2 + h^2}$

$$l^2 = r^2 + h^2$$

$$(x+8)^2 = x^2 + (x+7)^2$$

$$\cancel{x^2} + 16x + 64 = \cancel{x^2} + x^2 + 14x + 49$$

$$x^2 + 14x - 16x + 49 - 64 = 0$$

$$x^2 - 2x - 15 = 0$$

$$(x-5)(x+3) = 0$$

-15: -5, $+3$

$x = 5 \mid x = -3$ not possible.

$\therefore$ Radius $r = 5$ cm

Slant height $l = 5+8$

$= 13$ cm.

CSA of cone $= (\pi r l)$ sq. cm

$= \frac{22}{7} \times 5 \times 13$

$= \pi \times 65$

$= 65\pi$ sq. cm.

7. The age of the father is square of the age of his daughter Ramya. Five years hence, the father is three times as old as Ramya. Find their present age.

Sol:

	Ramya's Age	Father's Age
Present age.	x	x^2 [1st condition]
5 years Hence (After five years)	$x+5$	x^2+5

From 2nd condition

$x^2+5 = 3(x+5)$

$x^2+5 = 3x+15$.

$x^2-3x-15=0$

$(x-5)(x+2)=0$

$x-5=0$ | $x+2=0$

$x=5$ | $x=-2$: not possible.

-15

$-5 \quad +2$

Ramya's Present Age $= 5$ yrs

Father's Present Age $= 25$

8. A motor boat whose speed is 15 km/hr in still water goes 30 km downstream and comes back in 4 hrs 30 min. Determine the speed of water.

Let x be the speed of water.

Distance = 30 km

Total time taken = $4\frac{1}{2}$ hrs. (4 hrs 30 min)

$= 9/2$ hrs.

We know

Distance = Speed × time.

Let T_1 be the time taken for 30 km downstream

$$30 = (15+x) \times T_1$$

$$T_1 = \frac{30}{15+x}$$

Let T_2 be the time taken for 30 km upstream

$$30 = (15-x) \times T_2$$

$$T_2 = \frac{30}{15-x}$$

$$T_1 + T_2 = 4\frac{1}{2} \text{ hrs.}$$

$\cancel{15+x}$

$$\frac{30}{15+x} + \frac{30}{15-x} = 9/2$$

$$\frac{30(15-x) + 30(15+x)}{(15+x)(15-x)} = 9/2$$

$$\frac{450 \cancel{-30x} + 450 \cancel{+30x}}{225 - x^2} = 9/2$$

$$\cancel{900}^{100} \times \frac{2}{\cancel{9}} = 225 - x^2$$

$$200 = 225 - x^2$$

$$x^2 = 225 - 200$$

$$x^2 = 25$$

$$x = 5$$

$\therefore$ Speed of water = 5 km/hr.

9. Sum of the square of 3 consecutive number is 194. Find them

Let three consecutive number $x, x+1, x+2$

From the condition

$$x^2 + (x+1)^2 + (x+2)^2 = 194$$

$$x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = 194$$

$$3x^2 + 6x + 5 = 194$$

$$3x^2 + 6x + 5 - 194 = 0$$

$$3x^2 + 6x - 189 = 0$$

$$\div 3 \quad x^2 + 2x - 63 = 0$$

$$-63 \to 9, -7$$

$$(x-7)(x+9) = 0$$

$x - 7 = 0$ | $x + 9 = 0$

$x = 7$ | $x = -9$ not possible

$\therefore$ Three numbers are 7, 8, 9.

10. The Perimeter of a rectangle is 36 cm its area is 80 Sq. cm. Find its dimension.

Let 'l' be the length of rectangle

'b' be the breadth of rectangle.

From 1st condition $2(l+b) = 36$

$$l+b = 18$$

$$b = 18-l$$

From 2nd condition $l \times b = 80$

$$l \times (18-l) = 80$$

$$18l - l^2 = 80$$

$$l^2 - 18l + 80 = 0$$

80: −10, −8 (−18)

$$(l-10)(l-8) = 0$$

$$l = 10 \mid l = 8$$

If $l = 8$ then $b = 10$

If $l = 10$ then $b = 8$

∴ Dimension of rectangle (10 cm, 8 cm) (or) 8 cm, 10 cm.

11. The area of a rectangular land is 240 m². If 8 m is decreased from its length it will become a square. Determine the length and breadth of the land.

Let l be the length of rectangular land.

b be the breadth of rectangular land.

From 1^{st} condition

$$l \times b = 240 \quad \text{—— (1)}$$

If 8cm decreased by its length it will be ~~area~~ Square.

So, $b = l - 8$ [Square all side Equal] —— (2)

Eqn (2) sub Eqn (1)

$$l \times (l-8) = 240$$

$$l^2 - 8l = 240$$

$$l^2 - 8l - 240 = 0$$

$$(l-20)(l+12) = 0$$

$$l = 20 \quad \left[l = -12 \text{ not Possible.} \right.$$

$2 | 240$
$2 | 120$
$2 | 60$
$2 | 30$
$3 | 15$
5

8×30

20

If $l = 20$ then $b = 20 - 8$

$b = 12$

$\therefore$ Dimension of lands

$l = 20$ m $b = 12$ m.

12. The sum of ages of a father and the son is 45 yrs. Five years hence the product of their ages will be 600. Find their present ages.

	Father's age	Son's age.
Present age.	x	y
Hence 5 yrs	$x+5$	$y+5$

From 1st condition

$$x+y = 45$$

$$y = 45 - x \quad \text{—— (1)}$$

From 2nd condition

$$(x+5)(y+5) = 600$$

$$(x+5)(45-x+5) = 600$$

$$(x+5)(50-x) = 600$$

$$50x - x^2 + 250 - 5x = 600$$

$$-x^2 + 45x + 250 = 600$$

$$x^2 - 45x + 600 - 250 = 0$$

$$x^2 - 45x + 350 = 0$$

$$(x-35)(x-10) = 0$$

350: -10, -35; -45

$2 \,|\, 350$, $5 \,|\, 175$, 35

$x = 35$ | $x = 10$ not possible for father's age.

If $x = 35$ then $y = 45 - 35$

$$y = 10$$

∴ Father's age (present) = 45 yrs

Son's age (present) = 10 yrs.

13. The Product of age of a man, 6 years before and 10 years later is 960. Find his present age.

Let x be the present age of man.

6 years before his age $= x-6$

10 years Later his age $= x+10$

From the given information

$$(x-6)(x+10) = 960$$

$$x^2+10x-6x-60 = 960$$

$$x^2+4x-60-960 = 0$$

$$x^2+4x-1020 = 0$$

$$(x+34)(x-30) = 0$$

$x = -34$ not Possible | $x = 30$

$$\begin{array}{r|l} 2 & 1020 \\ 2 & 510 \\ 3 & 255 \\ 5 & 85 \\ & 17 \end{array}$$

$\therefore$ Present age of man is 30 yrs.