10th STD

INSTANT SUPPLEMENTARY EXAM - JUNE - 2023

Reg. No.

PART - III

Mathematics (With Answers)

Time Allowed : 3.00 Hours] [Maximum Marks : 100

Instructions : (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

Note : This question paper contains **four** parts.

Part - I

Note : (i) Answer **all** the questions. **14 × 1 = 14**

(ii) Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer.

1. If there are 1024 relations from a set A = {1, 2, 3, 4, 5} to a set B, then the number of elements in B is :

(a) 3 (b) 2 (c) 4 (d) 8

2. $7^{4k} \equiv$ _____ (mod 100)

(a) 1 (b) 2 (c) 3 (d) 4

3. The next term of the sequence $\frac{1}{2}, \frac{1}{6}, \frac{1}{10}, \frac{1}{14}$ ... is :

(a) $\frac{1}{15}$ (b) $\frac{1}{16}$ (c) $\frac{1}{18}$ (d) $\frac{1}{20}$

4. $y^2 + \frac{1}{y^2}$ is not equal to :

(a) $\frac{y^4+1}{y^2}$ (b) $\left(y+\frac{1}{y}\right)^2$

(c) $\left(y-\frac{1}{y}\right)^2+2$ (d) $\left(y+\frac{1}{y}\right)^2-2$

5. Graph of a linear equation is a ____.

(a) straight line (b) circle

(c) parabola (d) hyperbola

6. If in ΔABC, DE || BC, AB = 3.6 cm, AC = 2.4 cm and AD = 2.1 cm then the length of AE is :

(a) 1.4 cm (b) 1.8 cm

(c) 1.2 cm (d) 1.05 cm

7. How many tangents can be drawn to the circle from an exterior point?

(a) one (b) two

(c) infinite (d) zero

8. The straight line given by the equation $x = 11$ is

(a) parallel to X axis

(b) parallel to Y axis

(c) passing through the origin

(d) passing through the point (0,11)

9. If the slope of the line PQ is $\frac{1}{\sqrt{3}}$, then the slope of the perpendicular bisector of PQ is :

(a) $\sqrt{3}$ (b) $-\sqrt{3}$

(c) $\frac{1}{\sqrt{3}}$ (d) 0

10. $\tan\theta \operatorname{cosec}^2\theta - \tan\theta$ is equal to :

(a) $\sec\theta$ (b) $\cot^2\theta$

(c) $\sin\theta$ (d) $\cot\theta$

11. The total surface area of a hemisphere is how much times the square of its radius?

(a) π (b) 4 π (c) 3 π (d) 2 π

12. The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is

(a) 60π cm^2 (b) 68π cm^2

(c) 120π cm^2 (d) 136π cm^2

13. The range of the data 8, 8, 8, 8, 8. . . 8 is :

(a) 0 (b) 1 (c) 8 (d) 3

14. The probability a red marble selected at random from a jar containing p red, q blue and r green marbles is

(a) $\frac{q}{p+q+r}$ (b) $\frac{p}{p+q+r}$

(c) $\frac{p+q}{p+q+r}$ (d) $\frac{p+r}{p+q+r}$

Part - II

Note : Answer **any 10** questions. Question No.**28** is **compulsory**. **10 × 2 = 20**

15. A relation $\mathbb{R}$ is given by the set $\{(x, y) / y = x + 3, x \in \{0, 1, 2, 3, 4, 5\}\}$. Determine its domain and range.

16. Check whether $fog = gof$ if $f(x) = x - 6$, $g(x) = x^2$.

17. Find the least number that is divisible by the first ten natural numbers.

18. Find the 8th term of the G.P. 9, 3, 1,…

19. Determine the nature of the roots for the quadratic equations $15x^2 + 11x + 2 = 0$.

20. If $A = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix}$ then verify $(A^T)^T = A$.

21. Check whether AD is bisector of $\angle A$ of ΔABC in which AB = 4 cm, AC = 6 cm, BD = 1.6 cm and CD = 2.4 cm.

22. Find the slope of the line joining the points $(5, \sqrt{5})$ with the origin.

23. Prove that $\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$.

24. The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the cylinder.

25. The volume of a solid right circular cone is 11088 cm^3. If its height is 24 cm, then find the radius of the cone.

26. Find the standard deviation of first 21 natural numbers.

27. A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head.

28. Find the equation of a straight line which is parallel to the line $3x - 7y = 12$ and passing through the point (6, 4).

Part - III

Note : Answer **any 10** questions. Question No.**42** is **compulsory**. **10 × 5 = 50**

29. Let $A=\{x \in \mathbb{W} \mid x < 2\}$, $B = \{x \in \mathbb{N} \mid 1 < x \le 4\}$ and $C = \{3, 5\}$. Verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

30. Find the sum to n terms of the series 3 + 33 + 333 + ...to n terms

31. Rekha has 15 square colour papers of sizes 10 cm, 11 cm, 12 cm,…, 24 cm. How much area can be decorated with these colour papers?

32. Solve the system of linear equations given in three variables $3x - 2y + z = 2$, $2x + 3y - z = 5$, $x + y + z = 6$.

33. Find the square root of the following polynomial $121x^4 - 198x^3 - 183x^2 + 216x + 144$.

34. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that $A^2 - 5A + 7I_2 = 0$.

35. State and prove Pythagoras Theorem.

36. Find the area of the quadrilateral formed by the points (–9, –2), (–8, –4), (2, 2) and (1, –3).

37. Find the equation of the perpendicular bisector of the line joining the points A(–4, 2) and B(6, –4).

38. Prove that $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$

39. A metallic sphere of radius 16 cm is melted and recast into small spheres each of radius 2 cm. How many small spheres can be obtained?

40. Find the co-efficient of variation of 24, 26, 33, 37, 29, 31.

41. Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.

42. Two ships are sailing in the sea on either sides of a lighthouse. The angle of elevation of the top of the lighthouse as observed from the ships are 30° and 45° respectively. If the lighthouse is 200 m high, find the distance between the two ships. ($\sqrt{3}$ =1.732)

Part - IV

Note : Answer **all** the questions. **2 × 8 = 16**

43. (a) Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{3}{5}$ of the corresponding sides of the triangle PQR (Scale factor $\frac{3}{5} < 1$)

(OR)

(b) Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 5 cm. Also, measure the lengths of the tangents.

44. (a) Graph the following linear function $y = \frac{1}{2}x$. Identify the constant of variation and verify it with the graph. Also (i) find y when $x = 9$ (ii) find x when $y = 7.5$.

(OR)

(b) Draw the graph of $y = x^2 - 4$ and hence solve $x^2 - x - 12 = 0$

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ANSWERS

Part - I

1. (b) 2
2. (a) 1
3. (c) $\frac{1}{18}$
4. (b) $\left(y + \frac{1}{y}\right)^2$
5. (a) straight line
6. (a) 1.4 cm
7. (b) two
8. (b) parallel to Y axis
9. (b) $-\sqrt{3}$

10. (d) $\cot\theta$

11. (c) 3π

12. (d) 136π cm^2

13. (a) 0

14. (b) $\dfrac{p}{p+q+r}$

Part - II

15. Given $\mathbb{R} = \{(x, y) / y = x + 3\}$ and $x \in \{0, 1, 2, 3, 4, 5\}$

When $x = 0$, $y = 0 + 3 = 3$ $[\because y = x + 3]$

When $x = 1$, $y = 1 + 3 = 4$

When $x = 2$, $y = 2 + 3 = 5$

When $x = 3$, $y = 3 + 3 = 6$

When $x = 4$, $y = 4 + 3 = 7$

When $x = 5$, $y = 5 + 3 = 8$

$\therefore \mathbb{R} = \{(0, 3), (1, 4), (2, 5), (3, 6), (4, 7), (5, 8)\}$

$\therefore$ Domain of $\mathbb{R} = \{0, 1, 2, 3, 4, 5\}$

[All the first element in $\mathbb{R}$]

Range of $\mathbb{R} = \{3, 4, 5, 6, 7, 8$

[All the second element in $\mathbb{R}$]

16. Given $f(x) = x - 6,\ g(x) = x^2$

$fog(x) = f(g(x)) = f(x^2)$ $[\because g(x) = x^2]$

$= x^2 - 6$

[In $f(x) = x - 6$, Replace x by x^2] ...(1)

$gof(x) = g(f(x)) = g(x - 6)$ $[\because f(x) = x - 6]$

$= (x - 6)^2$

[In $g(x) = x^2$, Replace x by $x - 6$]

$= x^2 - 12x + 36$

$[\because (a - b)^2 = a^2 - 2ab + b^2]$ ... (2)

From (1) and (2), $fog(x) \neq gof(x)$

17. The least number that is divisible by the first ten natural numbers is 2520.

> **Hint:**
> 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
> The least multiple of 2 & 4 is 8
> The least multiple of 3 is 9
> The least multiple of 7 is 7
> The least multiple of 5 is 5
> LCM of $8 \times 9 \times 7 \times 5 = 40 \times 63 = 2520$.

18. To find the 8th term we have to use the n^{th} term formula $t_n = ar^{n-1}$

First term $a = 9$, Common ratio $r = \dfrac{t_2}{t_1} = \dfrac{3}{9} = \dfrac{1}{3}$

$$t_8 = 9 \times \left(\frac{1}{3}\right)^{8-1} = 9 \times \left(\frac{1}{3}\right)^7 = \frac{1}{243}$$

Therefore the 8th term of the G.P. is $\dfrac{1}{243}$.

19. $15x^2 + 11x + 2 = 0$ comparing with $ax^2 + bx + c = 0$. Here $a = 15$, $b = 11$, $c = 2$.

$$\Delta = b^2 - 4ac = 11^2 - 4 \times 15 \times 2 = 121 - 120 = 1 > 0.$$

$\therefore$ The roots are real and unequal.

20. If $A = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix}$, $A^T = \begin{bmatrix} 5 & -\sqrt{17} & 8 \\ 2 & 0.7 & 3 \\ 2 & \frac{5}{2} & 1 \end{bmatrix}$

$(A^T)^T = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix} = A$. $\therefore$ verified

21. Given AB = 4 cm, AC = 6 cm, BD = 1.6 cm, CD = 2.4 cm.

By ABT, check

$$\frac{BD}{DC} = \frac{1.6\times10}{2.4\times10} = \frac{\cancel{16}^{\,2}}{\cancel{24}_{\,3}} = \frac{2}{3}$$

$$\frac{AB}{AC} = \frac{\cancel{4}^{\,2}}{\cancel{6}_{\,3}} = \frac{2}{3} \Rightarrow \frac{BD}{DC} = \frac{AB}{AC}$$

$\therefore$ AD is the bisector of ΔABC.

22. If two points are given, slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$\underset{x_1\ \ y_1}{(5, \sqrt{5})}, \underset{x_2\ \ y_2}{(0, 0)}$ $\therefore m = \dfrac{0 - \sqrt{5}}{0 - 5} = \dfrac{\sqrt{5}}{5} = \dfrac{1}{\sqrt{5}}$

23. $\tan^2\theta - \sin^2\theta = \tan^2\theta - \dfrac{\sin^2\theta}{\cos^2\theta}.\cos^2\theta$

$= \tan^2\theta\,(1 - \cos^2\theta) = \tan^2\theta \sin^2\theta$

24. Given that, C.S.A. of the cylinder = 88 sq. cm

$2\pi rh = 88$

$2 \times \dfrac{22}{7} \times r \times 14 = 88$ (h = 14 cm)

$2r = \dfrac{88\times7}{22\times14} = 2$

Therefore, diameter = 2 cm.

25. Let r and h be the radius and height of the cone respectively.

Given that, volume of the cone = 11088 cm^3

$\dfrac{1}{3}\pi r^2 h = 11088$

$$\frac{1}{3}\times\frac{22}{7}\times r^2\times 24 = 11088$$
$$r^2 = 441$$

Therefore, radius of the cone $r = 21$ cm

26. Standard deviation of first n natural number is

$$\sigma = \sqrt{\frac{n^2-1}{12}}$$

$\therefore$ Standard deviation of first 21 natural numbers

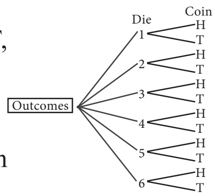
$$\sigma = \sqrt{\frac{21^2-1}{12}} = \sqrt{\frac{441-1}{12}} = \sqrt{\frac{440}{12}} = \sqrt{36.67}$$
$$\sigma = 6.06$$

$\therefore$ Standard deviation of first 21 natural numbers $\sigma = 6.06$.

27. Sample space

S = {1H,1T,2H,2T,3H,3T,4H,4T, 5H,5T,6H,6T};

n (S) = 12



Let A be the event of getting an odd number and a head.

A = {1H, 3H, 5H}; n(A) = 3; P(A) = $\frac{n(A)}{n(S)} = \frac{3}{12} = \frac{1}{4}$.

28. Equation of the straight line, parallel to $3x - 7y - 12 = 0$ is $3x - 7y + k = 0$

Since it passes through the point (6, 4)

$$3(6) - 7(4) + k = 0$$
$$k = 28 - 18 = 10$$

Therefore, equation of the required straight line is $3x - 7y + 10 = 0$.

Part - III

29. $A \times (B \cup C) = (A \times B) \cup (A \times C)$

A = $\{x \in \mathbb{W} \mid x < 2\}$ = {0, 1} [Whole numbers less than 2]

B = $\{x \in \mathbb{N} \mid 1 < x \leq 4\}$ = {2, 3, 4} [Natural numbers from 2 to 4]

C = {3, 5}

LHS = $A \times (B \cup C)$

$B \cup C$ = $\{2,3,4\} \cup \{3,5\}$ = {2, 3, 4, 5}

$A \times (B \cup C)$ = {(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4),(1, 5)} ...(1)

RHS = $(A \times B) \cup (A \times C)$

$(A \times B)$ = {(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)}

$(A \times C)$ = {(0, 3), (0, 5), (1, 3), (1,5)}

$(A \times B) \cup (A \times C)$ = {(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4),(1, 5)} ...(2)

(1) = (2), LHS = RHS

Hence it is proved.

30. Given series 3 + 33 + 333 + ...+ n terms.

Let S_n = 3 + 33 + 333 + ...+ n terms.

= 3(1 + 11 + 111 + ...+ n terms)[Taking 3 as common]

= $\frac{3}{9}$ (9 + 99 + 999 + ...to n terms)

[Multiplying and dividing by 9 throughout]

= $\frac{1}{3}$[(10 –1) + (100 – 1) + (1000 – 1) + ...n terms]

[$\because$ 9 = 10 – 1, 99 = 100 – 1, 999 = 1000 – 1 and so on]

= $\frac{1}{3}$[(10 + 10^2 + 10^3 + ...n terms) – n]

Hint: Here $a = 10$, $r = 10$ $\therefore S_n = a\frac{(r^n-1)}{r-1}$

[Separating all the first term and second term from the brackets]

$$= \frac{1}{3}\left(\frac{10(10^n-1)}{10-1} - n\right) = \frac{1}{3}\times\left[\frac{10(10^n-1)}{9} - n\right]$$

$$3 + 33 + 333 + ... \text{ to } n \text{ terms} = \frac{10(10^n-1)}{27} - \frac{n}{3}$$

31. Area can be decorated = $10^2 + 11^2 + 12^2 + ...+ 24^2$

$= (1^2 + 2^2 + ...+24^2) - (1^2 + 2^2 + ...+ 9^2)$

$$= \left(\frac{n(n+1)(2n+1)}{6}\right)_{n=24} - \left(\frac{n(n+1)(2n+1)}{6}\right)_{n=9}$$

$$= \frac{\cancel{24}^4\times 25\times 49}{\cancel{6}} - \frac{\cancel{9}^3\times\cancel{10}^5\times 19}{\cancel{6}_{\cancel{3}}}$$

= 4900 – 285 = 4615

$\therefore$ She can decorate 4615 cm^2 area with these colour papers.

32.

$$3x - 2y + z = 2 \quad ...(1)$$
$$2x + 3y - z = 5 \quad ...(2)$$
$$x + y + z = 6 \quad ...(3)$$

Adding (1) and (2),

$$\begin{aligned} 3x - 2y + z &= 2 \\ 2x + 3y - z &= 5 \quad (+) \\ \hline 5x + y &= 7 \quad ...(4) \end{aligned}$$

Adding (2) and (3),

$$\begin{aligned} 2x + 3y - z &= 5 \\ x + y + z &= 6 \quad (+) \\ \hline 3x + 4y &= 11 \quad ...(5) \end{aligned}$$

4 × (4)–(5)

$$\begin{aligned} 20x + 4y &= 28 \\ 3x + 4y &= 11 \quad (-) \\ \hline 17x &= 17 \quad \Rightarrow x = 1 \end{aligned}$$

Substituting $x = 1$ in (4), $5 + y = 7 \Rightarrow y = 2$

Substituting $x = 1$, $y = 2$ in (3), 1 + 2 + z = 6 we get, $z = 3$

Therefore, $x = 1$, $y = 2$, $z = 3$

33. $121x^4 - 198x^3 - 183x^2 + 216x + 144$

	$11x^2 - 9x - 12$
$11x^2$	$121x^4 - 198x^3 - 183x^2 + 216x + 144$
	$\cancel{121x^4}$ (–)
$22x^2 - 9x$	$-198x^3 - 183x^2$
	$-\cancel{198x^3} + 81x^2$ (+) (–)
$22x^2 - 18x - 12$	$-264x^2 + 216x + 144$
	$-\cancel{264x^2} + \cancel{216x} + \cancel{144}$ (+) (–) (–)
	0

$\therefore \sqrt{121x^4 - 198x^3 - 183x^2 + 216x + 144}$

$= |11x^2 - 9x - 12$

34. L.H.S $= A^2 - 5A + 7I_2$

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (9-1) & (3+2) \\ (-3-2) & (-1+4) \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$$

$$7I_2 = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$A^2 - 5A + 7I_2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

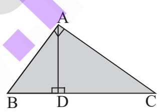
Hence verified.

35. Pythagoras Theorem :

Given : ΔABC, $\angle A = 90^\circ$

To prove : $AB^2 + AC^2 = BC^2$

Construction : Draw $AD \perp BC$



S. No.	Statement	Reason
1.	Compare ΔABC and ΔABD $\angle B$ is common $\angle BAC = \angle BAC$ 90º Therefore, $\Delta ABC \sim \Delta ABD$ Therefore, $\Delta ABC \sim \Delta ABD$ $\frac{AB}{BD} = \frac{BC}{AB}$ $AB^2 = BC \times BD$ ... (1)	Given $= \angle BAC = 90^\circ$ and by construction $\angle BDA = 90^\circ$ By AA similarity
2.	Compare ΔABC and ΔADC $\angle C$ is common $\angle BAC = \angle ADC$ 90º Therefore, $\Delta ABC \sim \Delta ABD$ Therefore, $\Delta ABC \sim \Delta ADC$ $\frac{BC}{AC} = \frac{AC}{DC}$ $AC^2 = BC \times DC$ ... (2)	Given $= \angle BAC = 90^\circ$ and by construction $\angle CDA = 90^\circ$ By AA similarity

Adding (1) and (2) we get

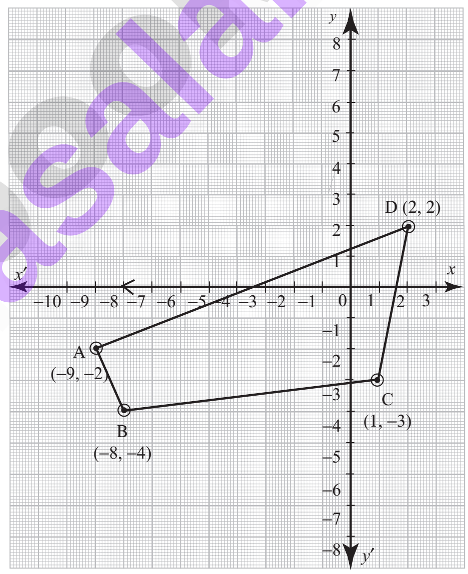
$AB^2 + AC^2 = BC \times BD + BC \times DC$

$= BC(BD + DC) = BC \times BC$

$AB^2 + AC^2 = BC^2$.

Hence the theorem is proved.

36.



$A(\overset{x_1}{-9}, \overset{y_1}{-2})$, $B(\overset{x_2}{-8}, \overset{y_2}{-4})$, $C(\overset{x_3}{1}, \overset{y_3}{-3})$, $D(\overset{x_4}{2}, \overset{y_4}{2})$

Area of the quadrilateral

$$= \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix} \text{ sq. units}$$

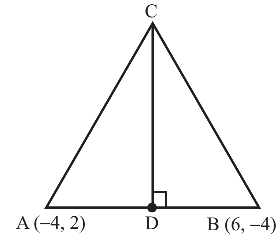
$$= \frac{1}{2}\begin{vmatrix} -9 & -8 & 1 & 2 & -9 \\ -2 & -4 & -3 & 2 & -2 \end{vmatrix} \text{ sq. units.}$$

$$= \frac{1}{2}[(36 + 24 + 2 - 4) - (16 - 4 - 6 - 18)]$$

$$= \frac{1}{2}[58 - (-12)] = \frac{1}{2}(70) = 35 \text{ sq. units}$$

37. Mid Point AB is DC

$$\Rightarrow \text{D is} \left(\frac{-4+6}{2}, \frac{2+(-4)}{2}\right) = \left(\frac{2}{2}, \frac{-2}{2}\right) = (1, -1)$$



$$\text{Slope of AB} = \frac{-4-2}{6-(-4)} = \frac{-6}{10} = \frac{-3}{5}$$

$$\therefore \text{Slope of CD} = \frac{-1}{-3/5} = \frac{5}{3} \quad [\because \text{CD} \perp \text{AB}]$$

$\therefore$ Equation of CD is

$$y-(-1) = \frac{5}{3}(x-1)$$

$$3(y+1) = 5x-5 \Rightarrow 3y+3 = 5x-5$$

$5x - 3y - 8 = 0$ is the required equation of the line.

38. $$\text{L.H.S} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta} \times \frac{\sqrt{1+\sin\theta}}{1+\sin\theta}}$$

[Rationalising the denominator]

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} = \frac{1+\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta + \tan\theta \quad \ldots (1)$$

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{1-\sin\theta}{1+\sin\theta} \times \frac{\sqrt{1-\sin\theta}}{1-\sin\theta}}$$

[Rationalising the denominator]

$$= \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}} = \frac{1-\sin\theta}{\sqrt{1-\sin^2\theta}}$$

$$= \frac{1-\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta - \tan\theta \quad \ldots (2)$$

$$\text{LHS} = (1) + (2) \Rightarrow \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$= \sec\theta + \tan\theta + \sec\theta - \tan\theta = 2\sec\theta =$ R.H.S.

Hence proved

39. Let the number of small spheres obtained be n.

Let r be the radius of each small sphere and R be the radius of metallic sphere.

Here, R = 16 cm, r = 2 cm

Now, $n \times$ (Volume of a small sphere) = Volume of big metallic sphere

$$n\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi r R^3$$

$$n\left(\frac{4}{3}\pi \times 2^3\right) = \frac{4}{3}\pi \times 16^3$$

$$8n = 4096 \Rightarrow n = 512$$

Therefore, there will be 512 small spheres.

40.

x	$d = x - \bar{x}$	d^2
24	–6	36
26	–4	16
33	3	9
37	7	49
29	–1	1
31	1	1
180	$\Sigma d = 0$	112

$$\bar{x} = \frac{\Sigma x}{n} = \frac{180}{6} = 30$$

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{112}{6}} = \sqrt{18.66} = 4.32$$

$\therefore$ Co-efficient of variation C.V = $\frac{\sigma}{x} \times 100$

$$\text{C.V} = \frac{4.32}{30} \times 100\% = 14.4\%$$

41. Two dice rolled once.

$$S = \begin{Bmatrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{Bmatrix}$$

$$n(S) = 36$$

Happening of an even number in the first die is A.

$$A = \begin{Bmatrix} (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{Bmatrix}$$

$$n(A) = 18$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{18}{36}$$

Happening of a total of face sum is 8 is B.

$$B = \{(2, 6), (3,5), (4, 4), (5, 3), (6, 2)\}$$

$$n(B) = 5$$

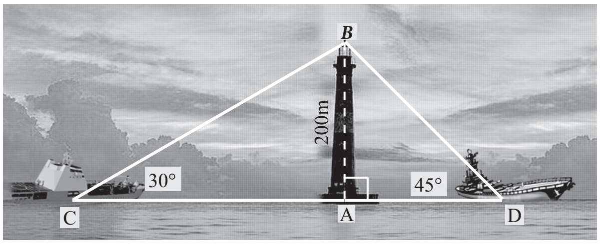
$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$$

$$(A \cap B) = \{(2, 6), (4, 4), (6, 2)\}$$

$$n(A \cap B) = 3 \Rightarrow P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{36}$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{18+5-3}{36}$$
$$= \frac{\cancel{20}^{5}}{\cancel{36}_{9}} = \frac{5}{9}$$

42. Let AB be the lighthouse. Let C and D be the positions of the two ships.



Then, AB = 200 m. $\angle ACB = 30°$, $\angle ADB = 45°$

In the right angled ΔBAC, $\tan 30° = \frac{AB}{AC}$

$$\frac{1}{\sqrt{3}} = \frac{200}{AC} \Rightarrow AC = 200\sqrt{3} \quad ...(1)$$

In the right angled ΔBAD, $\tan 45° = \frac{AB}{AD}$

$$1 = \frac{200}{AD} \Rightarrow AD = 200 \quad ...(2)$$

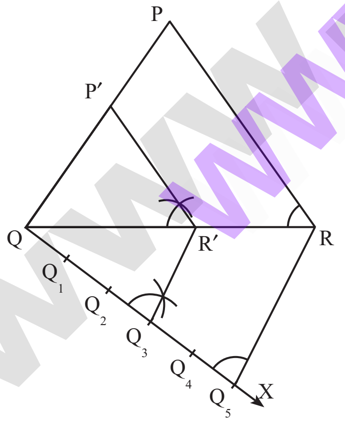
Now, $CD = AC + AD = 200\sqrt{3} + 200$ [by (1) and (2)]

$CD = 200(\sqrt{3} + 1) = 200 \times 2.732 = 546.4$

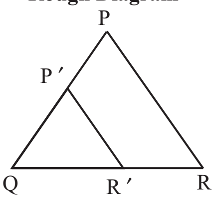
Distance between two ships is 546.4 m.

Part - IV

43. (a) Given a triangle PQR we are required to construct another triangle whose sides are $\frac{3}{5}$ of the corresponding sides of the triangle PQR.



Rough Diagram



Steps of construction

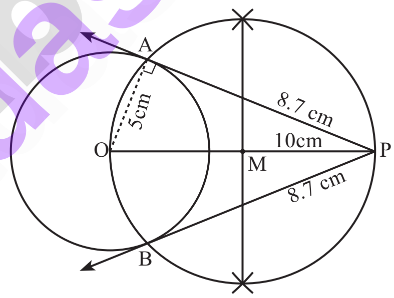
1. Construct a ΔPQR with any measurement.
2. Draw a ray QX making an acute angle with QR on the side opposite to vertex P.
3. Locate 5 (the greater of 3 and 5 in $\frac{3}{5}$) points. Q_1, Q_2, Q_3, Q_4 and Q_5 on QX so that $QQ_1 = Q_1Q_2 = Q_2Q_3 = Q_3Q_4 = Q_3Q_4 = Q_4Q_5$
4. Join Q_5R and draw a line through Q_3 (the third point, 3 being smaller of 3 and 5 in $\frac{3}{5}$) parallel to Q_5R to intersect QR at R′.
5. Draw line through R′ parallel to the line RP to intersect QP at P′.

Then, $\Delta P'QR'$ is the required triangle each of whose sides is three - fifths of the corresponding sides of ΔPQR.

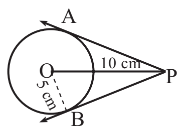
(OR)

(b) Radius = 5 cm

The distance between the point from the centre is 10 cm.



Rough diagram



Construction:

Step 1 : With O as centre, draw a circle of radius 5 cm.

Step 2 : Draw a line OP = 10 cm.

Step 3 : Draw a perpendicular bisector of OP which cuts OP at M.

Step 4 : With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.

Step 5 : Join AP and BP. AP and BP are the required tangents. Thus length of the tangents are PA and PB = 8.7 cm.

Verification: In the right triangle $\angle POA$;

$$PA = \sqrt{OP^2 - OA^2}$$
$$PA = \sqrt{10^2 - 5^2}$$
$$= \sqrt{100 - 25} = \sqrt{75}$$
$$\cong 8.7 \text{ cm (approximately)}$$

44. (a)

x	2	4	6	8
$y = \frac{1}{2}x$	1	2	3	4

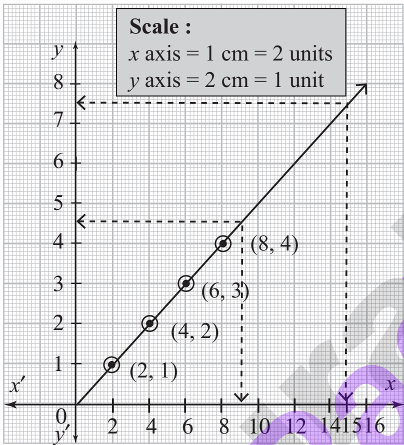
From the table above table we observe that as x increases y also increase. Therefore it is in direct variation.

We get $y \propto x$ (i.e.) $y = kx \Rightarrow \frac{y}{x} = k$, where k is a constant of proportionality.

From the table we find $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{1}{2} = k$.

$\therefore$ We get $k = \frac{1}{2}$.

Plot the points (2, 1), (4, 2), (6, 3) and (8, 4) and join them.



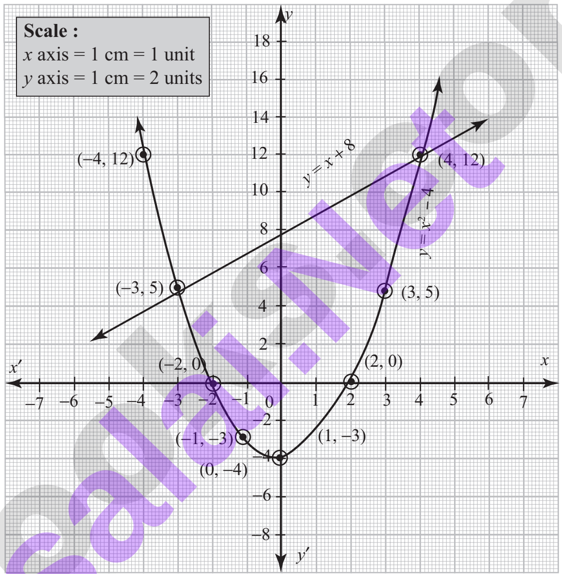
$\therefore$ The relation $y = \frac{1}{2}x$ forms a straight line is exhibited in the graph.

From the graph we find (i) when $x = 9$, $y = 4.5$ (ii) when $y = 7.5$, $x = 15$

(OR)

(b)

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
-4	–4	–4	–4	–4	–4	–4	–4	–4	–4
$x^2 - 4$	12	5	0	–3	–4	–3	0	5	12



To solve

$$\begin{array}{rcl} x^2 - x - 12 & = & 0 \\ \cancel{x^2} + 0x - 4 & = & y \\ \cancel{x^2} - x - 12 & = & 0 \\ (-)\ \ (+)\ \ (+) & & (-) \\ \hline x + 8 & = & y \\ y & = & x + 8 \end{array}$$

x	–4	–3	–2	–1	0	1	2	3	4
8	8	8	8	8	8	8	8	8	8
$x - 8$	4	5	6	7	8	9	10	11	12

Point of intersection (–3, 5), (4, 12) solution of $x^2 - x - 12 = 0$ is –3, 4

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