11th STD

INSTANT SUPPLEMENTARY EXAM - JULY 2023

Reg. No.

PART - III

Business Mathematics And Statistics (with answers)

Time Allowed : 3.00 Hours] **[Maximum Marks : 90**

Instructions :

(1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

PART - I

Note : (i) Answer **all** the questions. **20 × 1 = 20**

(ii) Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer.

1. If $\begin{vmatrix} x & 2 \\ 8 & 5 \end{vmatrix} = 0$ then the value of x is :

(a) $\frac{-5}{6}$ (b) $\frac{5}{6}$ (c) $\frac{-16}{5}$ (d) $\frac{16}{5}$

2. If $A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$ then A^{-1} is :

(a) $\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} -2 & 5 \\ 1 & -3 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & -1 \\ -5 & -3 \end{pmatrix}$ (d) $\begin{pmatrix} -3 & 5 \\ 1 & -2 \end{pmatrix}$

3. The coefficient of fourth term in the expansion of $\left(x - \frac{3}{x}\right)^5$ is ________.

(a) –270 (b) 370 (c) 10 (d) 405

4. The number of ways of selecting 4 players out of 5 is :

(a) 4! (b) 20 (c) 25 (d) 5

5. The slope of the line $7x + 5y - 8 = 0$ is :

(a) $\frac{7}{5}$ (b) $-\frac{7}{5}$ (c) $\frac{5}{7}$ (d) $-\frac{5}{7}$

6. If m_1 and m_2 are the slopes of the pair of lines given by $ax^2 + 2hxy + by^2 = 0$, then the value of $m_1 + m_2$ is :

(a) $\frac{2h}{b}$ (b) $\frac{-2h}{b}$ (c) $\frac{2h}{a}$ (d) $-\frac{2h}{a}$

7. The value of cos (– 480°) is :

(a) $\sqrt{3}$ (b) $-\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

8. $\tan\left(\frac{\pi}{4} - x\right)$ is equal to :

(a) $\left(\frac{1+\tan x}{1-\tan x}\right)$ (b) $\left(\frac{1-\tan x}{1+\tan x}\right)$

(c) 1–tan x (d) 1 + tan x

9. $\lim_{x \to 0} \frac{e^x - 1}{x} =$

(a) e (b) nx^{n-1} (c) 1 (d) 0

10. If $f(x) = \frac{1-x}{1+x}$, $x > 1$, then $f(-x)$ is equal to :

(a) $-f(x)$ (b) $\frac{1}{f(x)}$ (c) $-\frac{1}{f(x)}$ (d) $f(x)$

11. Profit P(x) is maximum when :

(a) MR = MC (b) MR = 0

(c) MC = AC (d) TR = AC

12. $\lim_{\theta \to 0} \frac{\tan\theta}{\theta} =$

(a) 1 (b) ∞ (c) –∞ (d) θ

13. If $q = 1000 + 8p_1 - p_2$, then $\frac{\partial q}{\partial p_1}$ is :

(a) –1 (b) 8

(c) 1000 (d) $1000 - p_2$

14. The annual income on 500 shares of face value ₹100 at 15% is :

(a) ₹7,500 (b) ₹5,000

(c) ₹8,000 (d) ₹8,500

15. The harmonic mean of the numbers 2,3,4 is :

(a) $\frac{12}{13}$ (b) 12 (c) $\frac{36}{13}$ (d) $\frac{13}{36}$

16. Probability that at least one of the events A, B occur is :

(a) $P(A \cup B)$ (b) $P(A \cap B)$

(c) $P(A/B)$ (d) $(A \cup B)$

17. Correlation co-efficient lies between :

(a) 0 to ∞ (b) -1 to $+1$

(c) -1 to 0 (d) -1 to ∞

18. If $\text{cov}(x,y) = -16.5$, $\sigma x^2 = 2.89$, $\sigma y^2 = 100$. Find correlation coefficient.

(a) -0.12 (b) 0.001

(c) -1 (d) -0.97

19. The word CPM means :

(a) Critical Path Method

(b) Crash Project Management

(c) Critical Project Management

(d) Critical Path Management

20. The minimum value of the objective function $Z = x + 3y$ subject to the constraints $2x + y \leq 20$, $x + 2y \leq 20$, $x > 0$ and $y > 0$ is :

(a) 10 (b) 20 (c) 0 (d) 5

PART - II

Note : Answer **any seven** questions. Question No. **30** is **Compulsory.** **7 × 2 = 14**

21. Show that $\begin{bmatrix} 8 & 2 \\ 4 & 3 \end{bmatrix}$ is non - singular.

22. If ${}^{15}C_{3r} = {}^{15}C_{r+3}$, find r.

23. Find the value of 'a' for which the straight lines $3x + 4y = 13$; $2x - 7y = -1$ and $ax - y - 14 = 0$ are concurrent.

24. Find the value of $\cos^2 15 - \sin^2 15$.

25. If $z = (ax + b)(cy + d)$, then find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

26. How much will be required to buy 125 shares of ₹25 each at a discount of ₹7?

27. A person purchases tomatoes from each of the 4 places at the rate of 1kg, 2kg, 3kg, and 4kg per rupee respectively. On the average ,how many kilograms has he purchased per rupee?

28. From the following data, calculate the correlation coefficient $\Sigma xy = 120$, $\Sigma x^2 = 90$, $\Sigma y^2 = 640$.

29. Draw the event oriented network for the following data

Events	1	2	3	4	5	6	7
Immediate Predecessors	-	1	1	2,3	3	4,5	5,6

30. Find the equation of the circle with centre $(3, -1)$ and radius is 4 units.

PART - III

Note : Answer **any seven** questions. Question No. **40** is **Compulsory**. **7 × 3 = 21**

31. Solve by using matrix inversion method :

$2x + 5y = 1$

$3x + 2y = 7$

32. Evaluate $\frac{n!}{r!(n-r)!}$ when $n = 5$ and $r = 2$.

33. Prove that : $(\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2 = 4\sin^2\left(\frac{\alpha-\beta}{2}\right)$

34. Evaluate : $\lim_{n\to\infty} \frac{\sum n^2}{n^3}$

35. A dealer has supplied his customer 400 units of a product per week. The dealer gets the product from the manufacturer at a cost of ₹ 50 per unit. The cost of ordering from the manufacturer is ₹75 per order. The cost of holding inventory is 7.5 % per year of the product cost. Find EOQ.

36. A person deposits ₹4,000 in the beginning of every year. If the rate of compound interest is 14%, then ,find the amount after 10 years .$[(1.14)^{10} = 3.707]$.

37. A die is thrown. Find the probability of getting (i) a prime number (ii) a number greater than or equal to 3.

38. The following are the ranks obtained by 10 students in Statistics and Mathematics

Statistics	1	2	3	4	5	6	7	8	9	10
Mathematics	1	4	2	5	3	9	7	10	6	8

Find the rank correlation coefficient.

39. Solve the following LPP, by graphical method.

Maximize $z = 3x_1 + 4x_2$

subject to $x_1 - x_2 \leq -1$;

$-x_1 + x_2 \leq 0$ and $x_1, x_2 \geq 0$

40. If $f(x) = x^n$ and $f'(1) = 5$, then find the value of n.

PART - IV

Note : Answer **all** the questions. **7 × 5 = 35**

41. (a) In an economy there are two industries P_1 and P_2 and the following table gives the supply and the demand position in crores of rupees.

Production sector	Consumption Sector		Final demand	Gross output
	P_1	P_2		
P_1	10	25	15	50
P_2	20	30	10	60

(i) Write the technology matrix

(ii) Test Hawkin's - Simon conditions for the viability of the system.

(OR)

(b) If $\tan\alpha = \frac{1}{3}$ and $\tan\beta = \frac{1}{7}$ then prove that $2\alpha + \beta = \frac{\pi}{4}$.

42. (a) Resolve into partial fraction $\frac{4x+1}{(x-2)(x+1)}$

(OR).

(b) Compute the mean deviation about mean from the following data:

Class Interval	0-5	5-10	10-15	15-20	20-25
Frequency	3	5	12	6	4

43. (a) Find the equation of the circle passing through the points (0,0), (1, 2) and (2,0).

(OR)

(b) Draw the network and calculate the earliest start time, earliest finish time, latest start time and latest finish time of each activity and determine the critical path of the project and duration to complete the project.

Jobs	1-2	1-3	2-4	3-4	3-5	4-5	4-6	5-6
Duration	6	5	10	3	4	6	2	9

44. (a) If $x = a\theta$ and $y = \frac{a}{\theta}$, then prove that $\frac{dy}{dx} + \frac{y}{x} = 0$.

(OR)

(b) Using mathematical induction method, Prove that $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$, $n \in \mathbb{N}$.

45. (a) The demand for a quantity A is $q = 13 - 2p_1 - 3p_2^2$. Find the partial elasticities $\frac{Eq}{Ep_1}$ and $\frac{Eq}{Ep_2}$ when $p_1 = p_2 = 2$

(OR)

(b) Mohan invested ₹29,040 in 15% of ₹100 shares of a company quoted at a premium of 20%. Calculate :

(i) The number of shares bought by Mohan

(ii) His annual income from shares

(iii) The percentage of return on his investment

46. (a) Bag I contains 3 red and 4 blue balls while another Bag II contains 5 red and 6 blue balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from second Bag.

(OR)

(b) For the given lines of regression 3X – 2Y = 5 and X – 4Y = 7, Find:

(i) Regression coefficients

(ii) Coefficient of correlation

47. (a) If $u = \log(x^2 + y^2)$, then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

(OR)

(b) Find D_2 and D_6 for the following series 22, 4, 2, 12, 16, 6, 10, 18, 14, 20, 8

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ANSWERS

Part - I

1. (d) $\frac{16}{5}$
2. (a) $\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$
3. (a) –270
4. (b) $\frac{-2h}{b}$
5. (b) $-\frac{7}{5}$
6. (b $\frac{-2h}{b}$
7. (d) $-\frac{1}{2}$
8. (b) $\left(\frac{1-\tan x}{1+\tan x}\right)$
9. (c) 1
10. (b) $\frac{1}{f(x)}$
11. (a) MR = MC
12. (a) 1
13. (b) 8
14. (a) ₹ 7,500
15. (c) $\frac{36}{13}$
16. (a) P(A∪B)
17. (b) –1 to +1
18. (d) – 0.97
19. (a) Critical Path Method
20. (c) 0

Part - II

21. Let $A = \begin{bmatrix} 8 & 2 \\ 4 & 3 \end{bmatrix}$

$$|A| = \begin{bmatrix} 8 & 2 \\ 4 & 3 \end{bmatrix}$$

$$= 24 - 8 = 16 \neq 0$$

∴ A is a non-singular matrix

22. $^{15}C_{3r} = {}^{15}C_{r+3}$

Then by the property,

$$nC_x = nC_y$$

$$\Rightarrow x + y = n, \text{ we have}$$

$$3r + r + 3 = 15$$

$$\Rightarrow r = 3$$

23. Given lines are

$$3x + 4y - 13 = 0$$

$$2x - 7y + 1 = 0, \quad ax - y - 14 = 0$$

Since, the given lines are concurrent,

$$\begin{vmatrix} 3 & 4 & -13 \\ 2 & -7 & 1 \\ a & -1 & -14 \end{vmatrix} = 0$$

$$\Rightarrow 3\begin{vmatrix} -7 & 1 \\ -1 & -14 \end{vmatrix} - 4\begin{vmatrix} 2 & 1 \\ a & -14 \end{vmatrix} - 13\begin{vmatrix} 2 & -7 \\ a & -1 \end{vmatrix} = 0$$

[Expanded along R_1]

$$\Rightarrow 3(98 + 1) - 4(-28 - a) - 13(-2 + 7a) = 0$$

$$\Rightarrow 297 + 112 + 4a + 26 - 91a = 0$$

$$\Rightarrow 435 - 87a = 0$$

$$\Rightarrow 435 = 87a$$

$$\Rightarrow a = \frac{435}{87}$$

$$\Rightarrow a = 5$$

24. $\cos^2 15 - \sin^2 15° = \cos 2 \times 15°$

[Since $\cos^2 A - \sin^2 A = \cos 2A$]

$$= \cos 30° = \frac{\sqrt{3}}{2}$$

25. Given $Z = (ax + b)(cy + d)$

Differentiating partially with respect to 'x',

$$\frac{\partial z}{\partial x} = (cy + d)(a)$$

$$= a(cy + d)$$

Differentiating partially with respect to 'y',

$$\frac{\partial z}{\partial y} = (ax + b)(c)$$
$$= c(ax + b)$$

26. Market value of 1 share $= 25 - 7 = 18$

$\therefore$ Market value of 125 shares $= 125 \times 18$

$= ₹\ 2250$

Hence ₹ 2250 will be required to buy 125 shares.

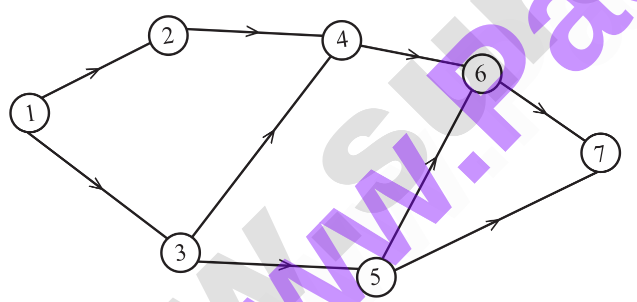
27. Since we are given rate per rupee, harmonic mean will give the correct answer.

$$\text{HM} = \frac{n}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}} = \frac{4}{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}$$
$$= \frac{4 \times 12}{25}$$
$$= 1.92 \text{ kg per rupee.}$$

28. Given $\Sigma xy = 120$, $\Sigma x^2 = 90$, $\Sigma y^2 = 640$

$$\text{Then } r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \, \Sigma y^2}} = \frac{120}{\sqrt{90(640)}} = \frac{120}{\sqrt{57600}}$$
$$= \frac{120}{240} = 0.5$$

29.



30. Equation of circle is

$$(x - h)^2 + (y - k)^2 = r^2$$

Here $(h, k) = (3, -1)$ and $r = 4$

Equation of circle is

$$(x - 3)^2 + (y + 1)^2 = 16$$
$$x^2 - 6x + 9 + y^2 + 2y + 1 = 16$$
$$x^2 + y^2 - 6x + 2y - 6 = 0$$

Part - III

31. The given system can be written as

$$\begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

$$\text{i.e, } AX = B$$
$$X = A^{-1}B$$

$$\text{Where } A = \begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 5 \\ 3 & 2 \end{vmatrix} = -11 \neq 0$$

A^{-1} exists.

$$\text{adj } A = \begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj } A$$
$$= \frac{1}{-11}\begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}$$

$$X = A^{-1}B$$
$$= \frac{1}{-11}\begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \end{bmatrix}$$
$$= -\frac{1}{11}\begin{bmatrix} -33 \\ 11 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$x = 3$ and $y = -1$.

32. When $n = 5$ and $r = 2$,

$$\frac{n!}{r!(n-r)!} = \frac{5!}{2!(5-2)!} = \frac{5!}{2! \times 3!}$$
$$= \frac{5 \times \cancel{4} \times \cancel{3} \times 2 \times 1}{\cancel{2} \times 1 \times \cancel{3} \times \cancel{2} \times 1} = 5 \times 2 = 10$$

33. $(\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2 = 4\sin^2\left(\frac{\alpha-\beta}{2}\right)$

$$\text{LHS} = (\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2$$

$$= \left[-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right]^2 + \left[2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right]^2$$

$$\left[\because \cos\alpha\cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right.$$

$$\left.\text{and}\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right]$$

$$= 4\sin^2\left(\frac{\alpha+\beta}{2}\right)\sin^2\left(\frac{\alpha-\beta}{2}\right) + 4\cos^2\left(\frac{\alpha+\beta}{2}\right)\sin^2\left(\frac{\alpha-\beta}{2}\right)$$

$$= 4\sin^2\left(\frac{\alpha-\beta}{2}\right)\left[\sin^2\left(\frac{\alpha+\beta}{2}\right) + \cos^2\left(\frac{\alpha+\beta}{2}\right)\right]$$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$$= 4\sin^2\left(\frac{\alpha-\beta}{2}\right) = \text{RHS}$$

34. $$\lim_{n\to\infty}\frac{\sum n^2}{n^3} = \lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3}$$

$$= \lim_{n\to\infty}\frac{1}{6}\left\{\left(\frac{n}{n}\right)\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right)\right\}$$

$$= \frac{1}{6}\lim_{n\to\infty}\left\{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right\}$$

$$= \frac{1}{6}(1)(2) = \frac{1}{3}$$

35. R = 400 units per week

C_3 = ₹ 75

C_1 = 7.5% of 50 per year

$$= \frac{7.5}{100\times 52}\times 50 \text{ per week}$$

$$= \frac{0.75\times 50}{52} = 0.072$$

$$\text{EOQ} = \sqrt{\frac{2RC_3}{C_1}} = \sqrt{\frac{2\times 400\times 75}{0.072}}$$

$$= \sqrt{\frac{60{,}000}{0.072}} = \sqrt{833{,}333.33}$$

= 912.87 units per order.

= 913 (*app*) units per order

36. Here a = 4,000; i = 0.14 and n = 10.

$$A = (1+i)\frac{a}{i}[(1+i)^n - 1]$$

$$= (1+0.14)\frac{4000}{0.14}[(1+0.14)^{10} - 1]$$

$$(1.14)\left(\frac{4000}{0.14}\left[(1.14)^{10} - 1\right]\right)$$

$$= 1.14\times\frac{4000}{0.14}(3.707 - 1]$$

$$= 1.14\times\frac{4000}{0.14}(2.707) = 88{,}170.86$$

∴ A ≈ ₹ 88,171

37. S = {1, 2, 3, 4, 5, 6}

(i) Let A = Event of getting a prime number

A = {2, 3, 5} ⇒ n(A) = 3

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(ii) Let B = Event of getting a number greater than or equal to 3

B = {3, 4, 5, 6} ⇒ n(B) = 4

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

38. Let R_X is considered for the ranks of Statistics and R_Y is considered for the ranks of mathematics.

R_X	R_Y	$d = R_X - R_Y$	d^2
1	1	0	0
2	4	–2	4
3	2	1	1
4	5	–1	1
5	3	2	4
6	9	–3	9
7	7	0	0
8	10	–2	4
9	6	3	9
10	8	2	4
			$\Sigma d^2 = 36$

The rank correlation is given by

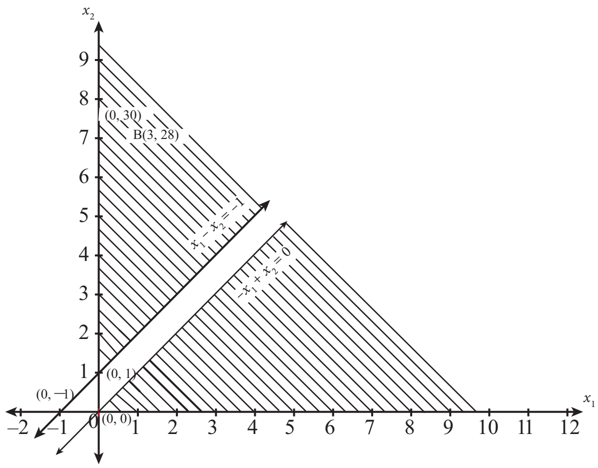
$$\rho = 1 - \frac{6\sum d^2}{N(N^2-1)}$$

$$= 1 - \frac{6(36)}{10(10^2-1)}$$

$$= 1 - 0.218$$

$$\therefore \rho = 0.782$$

39.



Since both the decision variables x_1, x_2 are non-negative ,the solution lies in the first quadrant of the plane.

Consider the equations $x_1 - x_2 = -1$ and $-x_1 + x_2 = 0$.

$x_1 - x_2 = -1$ is a line passing through the points (0, 1) and (–1, 0).

$-x_1 + x_2 = 0$ is a line passing through the point (0,0).

Now we draw the graph satisfying the conditions $x_1 - x_2 \leq -1$; $-x_1 + x_2 \leq 0$ and $x_1, x_2 \geq 0$.

There is no common region(feasible region) satisfying all the given conditions. Hence the given LPP has no solution.

40.

$$f(x) = x^n$$
$$\therefore f'(x) = nx^{n-1}$$
$$f'(1) = n(1)^{n-1}$$
$$f'(1) = n$$
$$f'(1) = 5 \text{ (given)}$$
$$\Rightarrow \quad n = 5$$

Part - IV

41. (a) $a_{11} = 10$ $\quad a_{12} = 25$ $\quad x_1 = 50$

$a_{21} = 20$ $\quad a_{22} = 30$ $\quad x_2 = 60$

$$b_{11} = \frac{a_{11}}{x_1} = \frac{10}{50} = \frac{1}{5};$$
$$b_{12} = \frac{a_{12}}{x_2} = \frac{25}{60} = \frac{5}{12}$$
$$b_{21} = \frac{a_{21}}{x_1} = \frac{20}{50} = \frac{2}{5};$$
$$b_{22} = \frac{a_{22}}{x_2} = \frac{30}{60} = \frac{1}{2}$$

The technology matrix is $B = \begin{bmatrix} \frac{1}{5} & \frac{5}{12} \\ \frac{2}{5} & \frac{1}{2} \end{bmatrix}$

$$I - B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{5} & \frac{5}{12} \\ \frac{2}{5} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & -\frac{5}{12} \\ -\frac{2}{5} & \frac{1}{2} \end{bmatrix}$$, elements of main diagonal are positive

$$\text{Now, } |I - B| = \begin{vmatrix} \frac{4}{5} & -\frac{5}{12} \\ -\frac{2}{5} & \frac{1}{2} \end{vmatrix} = \frac{7}{30} > 0$$

Main diagonal elements of |I – B| are positive and |I – B| is positive

∴ The problem has a solution

$$\text{adj } |I - B| = \begin{bmatrix} \frac{1}{2} & \frac{5}{12} \\ \frac{2}{5} & \frac{4}{5} \end{bmatrix}$$

$$(I - B)^{-1} = \frac{1}{|I - B|} \text{ adj } (I - B)$$

(Since $|I - B| \neq 0$, $(I - B)^{-1}$ exist)

$$= \frac{1}{\frac{7}{30}} \text{ adj } (I - B) = \frac{30}{7} \begin{bmatrix} \frac{1}{2} & \frac{5}{12} \\ \frac{2}{5} & \frac{4}{5} \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 15 & \frac{25}{2} \\ 12 & 24 \end{bmatrix}$$

$X = (I - B)^{-1} D$, where $D = \begin{bmatrix} 35 \\ 42 \end{bmatrix}$

$$= \frac{1}{7} \begin{bmatrix} 15 & \frac{25}{2} \\ 12 & 24 \end{bmatrix} \begin{bmatrix} 35 \\ 42 \end{bmatrix} = \begin{bmatrix} 150 \\ 204 \end{bmatrix}$$

∴ The output of industry P_1 should be ₹150 crores and P_2 should be ₹204 crores.

(OR)

(b) $$\tan(2\alpha+\beta) = \frac{\tan 2\alpha + \tan\beta}{1-\tan 2\alpha . \tan\beta}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2\times\frac{1}{3}}{1-\frac{1}{9}} = \frac{3}{4}$$

$$\tan(2\alpha+\beta) = \frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}} = \frac{\frac{25}{28}}{\frac{25}{28}} = 1 = \tan\frac{\pi}{4}$$

$$2\alpha+\beta = \frac{\pi}{4}$$

42. (a) $$\frac{4x+1}{(x-2)(x+1)} = \frac{A}{x-2}+\frac{B}{x+1}$$

$$\Rightarrow \frac{4x+1}{(x-2)(x+1)} = \frac{A(x+1)+B(x-2)}{(x-2)(x+1)}$$

$$\Rightarrow 4x+1 = A(x+1)+B(x-2) \quad \ldots (1)$$

Putting $x = -1$ in (1) we get,

$$-4+1 = 0+B(-1-2)$$

$$\Rightarrow -3 = -3B$$

$$\Rightarrow \boxed{B = 1}$$

Putting $x = 2$ in (1) we get

$$8+1 = A(2+1)+0$$

$$\Rightarrow 9 = 3A$$

$$\Rightarrow \boxed{A = 3}$$

$$\frac{4x+1}{(x-2)(x+1)} = \frac{3}{x-2}+\frac{1}{x+1}$$

(OR)

(b)

Class Interval	Mid x	Frequency f	fx	$\|D\| = \|X-13\|$	$f\|D\|$
0 – 5	2.5	3	7.5	10.5	31.5
5 – 10	7.5	5	37.5	5.5	27.5
10 – 15	12.5	12	150	0.5	6
15 – 20	17.5	6	105	4.5	27
20 – 25	22.5	4	90	9.5	38
		30	390		130

$$\overline{X} = \frac{\Sigma f x}{N} = \frac{390}{30} = 13$$

Mean deviation about mean

$$= \frac{\Sigma f|D|}{N} = \frac{130}{30} = 4.33$$

43. (a) Let the equation of the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

The circle passes through the point (0, 0)

$$c = 0 \quad \text{... (1)}$$

The circle passes through the point (1, 2)

$$1^2 + 2^2 + 2g(1) + 2f(2) + c = 0$$

$$2g + 4f + c = -5 \quad \text{... (2)}$$

The circle passes through the point (2, 0)

$$2^2 + 0 + 2g(2) + 2f(0) + c = 0$$

$$4g + c = -4 \quad \text{... (3)}$$

Solving (1), (2) and (3), we get

$$g = -1, f = -\frac{3}{4} \text{ and } c = 0$$

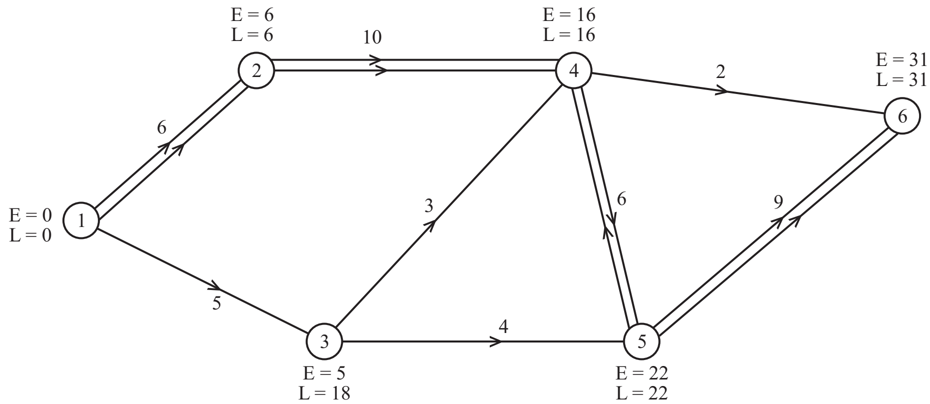
∴ The equation of the circle is

$$x^2 + y^2 + 2(-1)x + 2\left(\frac{-3}{4}\right)y + 0 = 0$$

$$\text{i. e., } 2x^2 + 2y^2 - 4x - 3y = 0$$

(OR)

(b)



$E_1 = 0$

$E_2 = 0 + 6 = 6$

$E_3 = 0 + 5 = 5$

$E_4 = 6 + 10 = 16$

$E_5 = (5 + 4)$ or $(16 + 6)$

Whichever is maximum

$= 22$

$E_6 = (16 + 2)$ or $(22 + 9)$

Whichever is maximum

$= 31$

$L_6 = 31$

$L_5 = 31 - 9 = 22$

$L_4 = 22 - 6 = 16$ (or) $(31 - 2)$

whichever is minimum

$L_3 = 22 - 4 = 18$

$L_2 = 16 - 10 = 6$

$L_1 = 6 - 6 = 0$

Activity	Duration	EST	EFT = EST + t_{ij}	LST = LFT – t_{ij}	LFT
1 - 2	6	0	6	6 – 6 = 0	6
1 - 3	5	0	5	18 – 5 = 13	18
2 - 4	10	6	16	16 – 10 = 6	16
3 - 4	3	5	8	16 – 3 = 13	16
3 - 5	4	5	9	22 – 4 = 18	22
4 - 5	6	16	22	22 – 6 = 16	22
4 - 6	2	16	18	31 – 2 = 29	31
5 - 6	9	22	31	31 – 9 = 22	31

Since EFT and LFT is same on 1 – 2, 2 – 4, 4 – 5 and 5 – 6, the critical path is 1 – 2 – 4 – 5 – 6 and duration time taken is 31 days.

44. (a)

$$x = a\theta \qquad y = \frac{a}{\theta}$$

$$\frac{dx}{d\theta} = a \qquad \frac{dy}{d\theta} = \frac{-a}{\theta^2}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$= \frac{\left(\frac{-a}{\theta^2}\right)}{a}$$

$$= -\frac{1}{\theta^2}$$

$$= -\frac{y}{x}$$

i.e. $\frac{dy}{dx} + \frac{y}{x} = 0$

Aliter :

Take $xy = a\theta . \frac{a}{\theta}$

$$xy = a^2$$

Differentiating with respect to x,

$$x\frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx} + \frac{y}{x} = 0$$

(OR)

(b) Let the given statement P(n) be defined as

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2} \text{ for } n \in \mathbb{N}$$

Step 1: Put $n = 1$

LHS P(1) = 1

RHS P(1) = $\frac{1(1+1)}{2} = 1$

LHS = RHS for $n = 1$

∴ P(1) is true

Step 2: Let us assume that the statement is true for $n = k$.

i.e., P(k) is true

$$1 + 2 + 3 + \ldots + k = \frac{k(k+1)}{2} \text{ is true}$$

Step 3: To prove that P($k + 1$) is true

P(k + 1) = 1 + 2 + 3 + … + k + (k + 1)

$$= P(k)+k+1 = \frac{k(k+1)}{2}+k+1$$

$$= \frac{k(k+1)+2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$$

$\therefore$ P$(k+1)$ is true

Thus if P(k) is true, then P$(k+1)$ is also true.

$\therefore$P(n) is true for all $n \in \mathbb{N}$

Hence $1+2+3+\ldots+n = \dfrac{n(n+1)}{2}$, $n \in \mathbb{N}$

45. (a) Given $q = 13-2p_1-3p_2^2$

Differentiating partially with respect to 'p_1' we get,

$$\frac{\partial q}{\partial p_1} = 0-2-0=-2$$

$$\frac{Eq}{Ep_1} = \frac{-p_1}{q}\left(\frac{\partial q}{\partial p_1}\right)$$

$$= \frac{-p_1}{13-2p_1-3p_2^2}(-2)$$

$$= \frac{2p_1}{13-2p_1-3p_2^2}$$

$\dfrac{Eq}{Ep_1}$ when $p_1 = p_2 = 2$ is $\dfrac{2(2)}{13-2(2)-3(2^2)}$

$$\frac{Eq}{Ep_1} = \frac{4}{13-4-12} = \frac{4}{-3} = -\frac{4}{3}$$

Differentiating 'q' partially with respect to 'p_2' we get,

$$\frac{\partial q}{\partial p_2} = 0-0-6p_2$$

$$\frac{Eq}{Ep_2} = -\frac{p_2}{q}\left(\frac{\partial q}{\partial p_2}\right)$$

$$= \frac{-p_2}{13-2p_1-3p_2^2}(-6p_2)$$

$$= \frac{6p_2^2}{13-2p_1-3p_2^2}$$

$\dfrac{Eq}{Ep_2}$ when $p_1 = p_2 = 2$ is

$$\frac{Eq}{Ep_2} = \frac{6(2)^2}{13-2(2)-3(2^2)}$$

$$= \frac{24}{13-4-12} = \frac{24}{-3} = -8$$

(OR)

(b)

Investment = ₹ 29,040

Market value of 1 share = 100 + 20 = 120

(i) No. of shares $= \dfrac{\text{Investment}}{\text{Market value of 1 share}}$

$= \dfrac{29040}{120} = 242$

(ii) Annual income = No. of shares × FV × Rate percentage

$= 242 \times 100 \times \dfrac{15}{100}$ = ₹ 3630

(iii) Percentage return on the shares $= \dfrac{\text{Income}}{\text{Investment}} \times 100$

$= \dfrac{3630}{29040} \times 100 = 12.5\%$

$= 12\frac{1}{2}\%$

46. (a) Let E_1 be the event of choosing the first bag, E_2 the event of choosing the second bag and A be the events of drawing a red ball.

Then $P(E_1) = P(E_2) = \dfrac{1}{2}$

Also $P(A/E_1)$ = P (drawing a red ball from Bag I) $= \dfrac{3}{7}$

and $P(A/E_2)$ = P (drawing a red ball from Bag II) $= \dfrac{5}{11}$

Now, the probability of drawing a ball from Bag II, being given that it is red, is $P(E_2/A)$.

By using Baye's theorem, we have

$$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{\sum_{i=1}^{2} P(E_i)P(A/E_i)}$$

$$= \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)} = \frac{\frac{1}{2}\cdot\frac{5}{11}}{\left(\frac{1}{2}\times\frac{3}{7}\right)+\left(\frac{1}{2}\times\frac{5}{11}\right)} = \frac{35}{68}$$

(OR)

(b) (i) First convert the given equations Y on X and X on Y in standard form and find their regression coefficients respectively.

Given regression lines are

$$3X - 2Y = 5 \quad \text{... (1)}$$

$$X - 4Y = 7 \quad \text{... (2)}$$

Let the line of regression of X on Y is

$$3X - 2Y = 5$$

$$3X = 2Y + 5$$

$$X = \frac{1}{3}(2Y + 5)$$

$$X = \frac{1}{3}(2Y + 5)$$

$$X = \frac{2}{3}Y + \frac{5}{3}$$

∴ Regression coefficient of X on Y is

$$b_{xy} = \frac{2}{3} (<1)$$

Let the line of regression of Y on X is

$$X - 4Y = 7$$

$$-4Y = -X + 7$$

$$4Y = X - 7$$

$$Y = \frac{1}{4}(X - 7)$$

$$Y = \frac{1}{4}X - \frac{7}{4}$$

∴ Regression coefficient of Y on X is

$$b_{yx} = \frac{1}{4} (<1)$$

(ii) **Coefficient of correlation**

Since the two regression coefficients are positive then the correlation coefficient is also positive and it is given by

$$r = \sqrt{b_{yx} \cdot b_{xy}}$$

$$= \sqrt{\frac{2}{3} \cdot \frac{1}{4}}$$

$$= \sqrt{\frac{1}{6}}$$

$$= 0.4082$$

$$\therefore r = 0.4082$$

47. (a) $u = \log(x^2 + y^2)$

$$\frac{\partial u}{\partial x} = \frac{1}{x^2+y^2}(2x) = \frac{2x}{x^2+y^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\left(x^2+y^2\right)\cdot 2 - 2x\cdot 2x}{(x^2+y^2)^2} = \frac{2(y^2-x^2)}{(x^2+y^2)^2}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{(x^2+y^2)\cdot 2 - 2y\cdot 2y}{(x^2+y^2)} = \frac{2(x^2-y^2)}{(x^2+y^2)^2}$$

$$\therefore \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

(OR)

(b) Here $n = 11$ observations are arranged into ascending order

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22

$$D_2 = \text{size of } 2\left(\frac{n+1}{10}\right)^{th} \text{ value}$$

$$D_6 = \text{size of } 6\left(\frac{n+1}{10}\right)^{th} \text{ value}$$

D_2 = size of 2.4th value ≈ size of 2nd value = 4

D_6 = size of 7.2th value ≈ size of 7th value =14

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