11th STD

INSTANT SUPPLEMENTARY EXAMINATION - JULY - 2023

Reg. No.

Part - III

Mathematics (with answers)

Time Allowed : 3.00 Hours] [Maximum Marks : 90

Instructions :

(1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

PART - I

Note : (i) Answer **all** the questions. **20 × 1 = 20**

(ii) Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer.

1. If $n[(A \times B) \cap (A \times C)] = 8$ and $n(B \cap C) = 2$, then $n(A)$ is :

(a) 6 (b) 4 (c) 8 (d) 16

2. The number of reflexive relations on a set containing n elements is :

(a) $2^{\frac{(n^2+n)}{2}}$ (b) 2^{n^2-n} (c) 2^n (d) 2^{-n}

3. If the difference between the roots of the equation $x^2 + px + 8 = 0$ is 2 then the values of p is :

(a) ± 4 (b) ± 5 (c) ± 6 (d) ± 7

4. The number of roots of $(x+3)^4 + (x+5)^4 = 16$ is :

(a) 4 (b) 2 (c) 3 (d) 0

5. The principal solution of $\sin\theta = -\frac{\sqrt{3}}{2}$ is :

(a) $\theta = \frac{\pi}{6}$ (b) $\theta = \frac{-\pi}{6}$ (c) $\theta = \frac{\pi}{3}$ (d) $\theta = -\frac{\pi}{3}$

6. If $\tan\alpha$ and $\tan\beta$ are the roots of $x^2 + ax + b = 0$, then $\frac{\sin(\alpha+\beta)}{\sin\alpha\,\sin\beta}$ is equal to :

(a) $\frac{b}{a}$ (b) $\frac{a}{b}$ (c) $-\frac{a}{b}$ (d) $-\frac{b}{a}$

7. The number of 5 digit numbers all digits of which are odd is :

(a) 25 (b) 5^5 (c) 5^6 (d) 625

8. ${}^{(n-1)}C_r + {}^{(n-1)}C_{(r-1)}$ is :

(a) ${}^{(n+1)}C_r$ (b) ${}^{(n-1)}C_r$ (c) ${}^{n}C_r$ (d) ${}^{n}C_{r-1}$

9. If a is the arithmetic mean and g is the geometric mean of two numbers, then :

(a) $a \le g$ (b) $a \ge g$ (c) $a = g$ (d) $a > g$

10. If the two straight lines $x + (2k - 7)y + 3 = 0$ and $3kx + 9y - 5 = 0$ are perpendicular then the value of k is :

(a) $k = 3$ (b) $k = \frac{1}{3}$ (c) $k = \frac{2}{3}$ (d) $k = \frac{3}{2}$

11. The value of the determinant of $A = \begin{bmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{bmatrix}$ is :

(a) $-2abc$ (b) abc (c) 0 (d) $a^2 + b^2 + c^2$

12. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, then the values of a and b are :

(1) $a = 4, b = 1$ (2) $a = 1, b = 4$ (3) $a = 0, b = 4$ (4) $a = 2, b = 4$

13. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + x\hat{j} + \hat{k}$, $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$ and $\vec{a} \cdot (\vec{b} \times \vec{c}) = 70$, then the value of x is :

(a) 5 (b) 7 (c) 26 (d) 10

14. $\lim_{x \to 3} \lfloor x \rfloor =$

(a) 2 (b) 3 (c) does not exist (d) 0

15. $\lim_{x \to 0} \frac{a^x - b^x}{x} =$

(a) $\log(ab)$ (b) $\log\left(\frac{a}{b}\right)$ (c) $\log\left(\frac{b}{a}\right)$ (d) $\frac{a}{b}$

16. The number of points in $\mathbb{R}$ in which the function $f(x) = |x - 1| + |x - 3| + \sin x$ is not differentiable is :

(a) 3 (b) 2 (c) 1 (d) 4

17. If $\int f(x)dx = g(x) + c$, then $\int f(x)g'(x)dx$:

(a) $\int (f(x))^2 dx$ (b) $\int f(x)g(x)dx$ (c) $\int f'(x)g(x)dx$ (d) $\int (g(x))^2 dx$

18. $\int \frac{dx}{e^x - 1} =$

(a) $\log|e^x| - \log|e^x - 1| + c$ (b) $\log|e^x| + \log|e^x - 1| + c$

(c) $\log|e^x - 1| - \log|e^x| + c$ (d) $\log|e^x + 1| - \log|e^x| + c$

19. A number is selected from the set $\{1, 2, 3, \ldots, 20\}$. The probability that the selected number is divisible by 3 or 4 is :

(a) $\frac{2}{5}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

20. Ten coins are tossed. The probability of getting atleast 8 heads is :

(a) $\frac{7}{64}$ (b) $\frac{7}{32}$ (c) $\frac{7}{16}$ (d) $\frac{7}{128}$

PART - II

Note : Answer **any seven** questions. Question No. **30** is **Compulsory.** **7 × 2 = 14**

21. Let f and g be two functions from R to R defined by $f(x) = 3x - 4$ and $g(x) = x^2 + 3$ Find gof and fog.

22. Solve $23x < 100$ when :

(i) $x \in \mathbb{N}$ (ii) $x \in \mathbb{Z}$

23. Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15°.

24. How many three-digit odd numbers can be formed by using the digits 0, 1, 2, 3, 4, 5 if :

(i) the repetition of digits is not allowed (ii) the repetition of digits is allowed

25. Show that the lines are $3x + 2y + 9 = 0$ and $12x + 8y - 15 = 0$ are parallel lines.

26. Find A + B + C if A, B, C are given by : $A = \begin{bmatrix} \sin^2\theta & 1 \\ \cot^2\theta & 0 \end{bmatrix}$, $B = \begin{bmatrix} \cos^2\theta & 0 \\ -\text{cosec}^2\theta & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

27. Compute : $\lim\limits_{x \to 1} \frac{x^3 - 1}{x - 1}$.

28. Evaluate : $\int \frac{\sin x}{\cos^2 x} dx$

29. A man has 2 ten rupee notes, 4 hundred rupee notes and 6 five hundred rupee notes in his pocket. If 2 notes are taken at random, what are the odds in favour of both notes being of hundred rupee denomination and also its probability?

30. If $y = e^{\sin x}$, find $\frac{dy}{dx}$

PART - III

Note : Answer **any seven** questions. Question No. **40** is **Compulsory**. **7 × 3 = 21**

31. Let A = {a, b, c}, and R = {(a, a) (b, b) (a, c)}. Write down the minimum number of ordered pairs to be included to R to make it :

(i) reflexive (ii) symmetric

(iii) transitive (iv) equivalence.

32. If $\left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)^2 = \frac{9}{2}$, then find the value of $\left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right)$ for $x > 1$.

33. If ${}^nP_r = 720$ and ${}^nC_r = 120$, find n and r.

34. An integer is chosen at random from the first ten positive integers. Find the probability that it is an even number.

35. Show the points $\left(0, \frac{-3}{2}\right)$, $(1, -1)$, $\left(2, -\frac{1}{2}\right)$ are collinear.

36. If the area of the triangle with vertices $(-3, 0)$, $(3, 0)$ and $(0, k)$ is 9 square units, find the value of k.

37. For any vector $\vec{r}$, prove that $\vec{r} = (\vec{r}\cdot\hat{i})\hat{i} + (\vec{r}\cdot\hat{j})\hat{j} + (\vec{r}\cdot\hat{k})\vec{k}$

38. Discuss the continuity of $f(x) = \sqrt{1-x^2}$.

39. If $f'(x) = 3x^2 - 4x + 5$ and $f(1) = 3$, then find $f(x)$.

40. Find the value of tan 165°.

PART - IV

Note : Answer **all** the questions. **7 × 5 = 35**

41. (a) If A × A has 16 elements, $S = \{(a, b) \in A \times A: a < b\}$; $(-1, 2)$ and $(0, 1)$ are two elements of S, then find the remaining elements of S.

(OR)

(b) Evaluate : $\lim\limits_{x\to 3} \dfrac{x^2 - 6x + 5}{x^3 - 8x + 7}$

42. (a) If $\log_2 x + \log_4 x + \log_{16} x = \frac{7}{2}$, find the value of x.

(OR)

(b) Prove that the points whose position vectors $2\hat{i} + 4\hat{j} + 3\hat{k}, 4\hat{i} + \hat{j} + 9\hat{k}$ and $10\hat{i} - \hat{j} + 6\hat{k}$ form a right angled triangle.

43. (a) Prove that ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(OR)

(b) If $\sin y = x \sin(a + y)$, then prove that $\dfrac{dy}{dx} = \dfrac{\sin^2(a+y)}{\sin a}$. Here $a \neq n\pi$.

44. (a) State and prove Napier's Formula.

(OR)

(b) Evaluate : $\displaystyle\int \frac{\sin x + \cos x}{\sin x - \cos x}\, dx$

45. (a) Prove that $\sqrt[3]{x^3+6} - \sqrt[3]{x^3+3}$ is approximately equal to $\frac{1}{x^2}$ when x is sufficiently large.

(OR)

(b) Using factor theorem, prove that $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8\,abc$.

46. (a) If $\theta + \phi = \alpha$ and $\tan\theta = k \tan\phi$, then prove that $\sin(\theta - \phi) = \frac{k-1}{k+1}\sin\alpha$.

(OR)

(b) If $y = e^{\tan^{-1}x}$, show that $(1 + x^2)y'' + (2x - 1)y' = 0$

47. (a) Find the points on the line $x + y = 5$, that lie at a distance 2 units from the line $4x + 3y - 12 = 0$

(OR)

(b) A die is rolled. If it shows an odd number, then find the probability of getting 5.

ANSWERS

PART - I

1. (b) 4

2. (b) $2^{n^2 - n}$

3. (c) ± 6

4. (a) 4

5. (d) $\theta = -\frac{\pi}{3}$

6. (c) $-\frac{a}{b}$

7. (b) 5^5

8. (c) ^nC_r

9. (b) $a \geq g$

10. (a) $k = 3$

11. (c) 0

12. (b) $a = 1, b = 4$

13. (c) 26

14. (c) does not exist

15. (b) $\log\left(\frac{a}{b}\right)$

16. (b) 2

17. (a) $\int (f(x))^2\,dx$

18. (c) $\log|e^x - 1| - \log|e^x| + c$

19. (c) $\frac{1}{2}$

20. (d) $\frac{7}{128}$

PART - II

21. We have, $(g \circ f)(x) = g(f(x)) = g(3x-4) = (3x-4)^2 + 3 = 9x^2 - 24x + 19$

$(f \circ g)(x) = f(g(x)) = f(x^2+3) = 3(x^2+3) - 4 = 3x^2 + 5$.

22. Given $23x < 100$

(i) when x is a natural number $23x < 100$

$$\Rightarrow x < \frac{100}{23} \Rightarrow x < 4.348 \Rightarrow x = \{1, 2, 3, 4\}$$

(ii) when x is an integer $x < 4.348$

$\Rightarrow x = \{\cdots\cdots -3, -2, -1, 0\ 1, 2, 3, 4\}$

Hence solution set is $\{\cdots\cdots -3, -2, -1, 0\ 1, 2, 3, 4\}$

23. Let s be the length of the arc of a circle of radius r subtending a central angle θ. Then $s = r\theta$

We have, $\theta = 15° = 15 \times \frac{\pi}{180} = \frac{\pi}{12}$ radians

So that, $s = r\theta$ gives $s = 5 \times \frac{\pi}{12} = \frac{5\pi}{12}$ cm

24. **(i)** **repetition of digits is not allowed**

hundreds	tens	unit
4	4	3

Since we need 3 – digit odd numbers, the unit place can be filled in 3 ways using the digits 1, 3 or 5.

Hundred's place can be filled in 4 ways (excluding 0 and the number used for one's place) since repetition is not allowed.

Ten's place can be filled 4 ways including 0.

∴ By fundamental principle of multiplication, number of required 3 digit odd numbers

$= 4 \times 4 \times 3 = 48$

(ii) **the repetition of digits is allowed**

hundreds	tens	unit
5	6	3

The unit place can be filled in 3 ways using the digits 1, 3, or 5 since we need 3 digit odd numeric.

Hundreds place can be filled in 5 ways excluding 0 and repetition of digits is allowed.

Tens place can be filled in 6 ways.

∴ By fundamental principle of multiplication, required number of 3-digit odd numbers

$= 5 \times 6 \times 3 = 30 \times 3 = 90$.

25. If the equation of two lines are in general form as $a_1 x + b_1 y + c = 0$ and $a_2 x + b_2 y + c_2 = 0$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \text{ or } a_1 b_2 = a_2 b_1$$

Given lines are $3x + 2y + 9 = 0$ and $12x + 8y - 15 = 0$

$$\frac{3}{12} = \frac{2}{8} \Rightarrow \frac{1}{4} = \frac{1}{4}$$

Hence the given lines are parallel..

26. By the definition of sum of matrices, we have

$$A + B + C = \begin{bmatrix} \sin^2\theta + \cos^2\theta + 0 & 1+0-1 \\ \cot^2\theta - \text{cosec}^2\theta - 1 & 0+1+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

27. $\lim\limits_{x\to 1} \frac{x^3 - 1}{x-1} = \lim\limits_{x\to 1} \frac{x^3 - 1^3}{x-1} = 3(1)^{3-1} = 3.$

28. Given, $\int \frac{\sin x}{\cos^2 x} dx = \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} dx = \int \tan x \sec x dx = \sec x + c.$

29. Let S be the sample space and A be the event of taking 2 hundred rupee note.

Therefore, $n(S) = 12c_2 = 66$, $n(A) = 4c_2 = 6$ and $n(\overline{A}) = 66 - 6 = 60$

Therefore, odds in favour of A is 6: 60

That is, odds in favour of A is 1: 10, and P (A) $= \frac{1}{11}$

30.

$$\begin{aligned} \text{Take } u &= \sin x \quad \text{so that} \\ y &= e^u \\ \frac{dy}{dx} &= \frac{d(e^u)}{du} \times \frac{du}{dx} \\ &= e^u \times \cos x = \cos x e^{\sin x} \end{aligned}$$

PART - III

31. (i) The ordered pairs (c, c) should be included to R to make it reflexive.

$\therefore$ Minimum number of ordered pair is (c, c)

(ii) The ordered pairs (c,a) should be included to R to make it symmetric.

$\therefore$ Minimum number of ordered pair is (c, a).

(iii) The relation is transitive.

$\therefore$ Nothing needs to included.

(iv) The ordered pairs (c, c) and (c, a) should be included to R to make it equivalence.

$\therefore$ Minimum number of ordered pairs are (c, c) and (c, a).

32. Given $\left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)^2 = \frac{9}{2}$

$\Rightarrow$ $x+\frac{1}{x}+2x^{\frac{1}{2}}.\frac{1}{x^{\frac{1}{2}}} = \frac{9}{2} \Rightarrow x+\frac{1}{x}+2 = \frac{9}{2}$

$\Rightarrow$ $x+\frac{1}{x} = \frac{9}{2}-2 = \frac{9-4}{2} = \frac{5}{2}$ ...(1)

Consider $\left(x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)^2 = x+\frac{1}{x}-2x^{\frac{1}{2}}.\frac{1}{x^{\frac{1}{2}}}$

$= x+\frac{1}{x}-2$ [From (1)]

$\left(x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)^2 = \frac{5}{2}-2 = \frac{5-4}{2} = \frac{1}{2}$ [using 1]

$\therefore x^{\frac{1}{2}}-x^{-\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow$ $x^{\frac{1}{2}}-x^{-\frac{1}{2}} = \frac{1}{\sqrt{2}}$ since $x>1$.

33. Given $^nP_r = 720$ and $^nC_r = 120$

$\Rightarrow$ $\frac{n!}{(n-r)!} = 720$ ...(1)

$\frac{n!}{r!(n-r)!} = 120$ ...(2)

$\Rightarrow$ $\dfrac{\frac{n!}{(n-r)!}}{\frac{n!}{r!(n-r)!}} = \frac{720}{120}$ (60) [Dividing (1) by (2)]

$\Rightarrow$ $\frac{n!}{(n-r)!}\times\frac{r!(n-r)!}{n!} = 6$

$\Rightarrow$ $r! = 6$

$\Rightarrow$ $r! = 3\times2\times1 = 3!$

$\Rightarrow$ $r = 3.$

Substituting $r = 3$ in (1) we get,

$\frac{n!}{(n-r)!} = 720 \Rightarrow \frac{n!}{(n-3)!} = 720$

$\Rightarrow$ $\frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 720$

$\Rightarrow$ $n(n-1)(n-2) = 720$

$\Rightarrow$ $n(n-1)(n-2) = 10\times9\times8$

$\Rightarrow$ $n = 10.$

34. The sample space is S $= \{1,2,3,4,5,6,7,8,9,10\}, n(S) = 10$

Let A be the event of choosing an even number and

B be the event of choosing an integer multiple of three.

$$A = \{2, 4, 6, 8, 10\}, n(A) = 5,$$

$$B = \{3, 6, 9\}, n(B) = 3$$

$$P(\text{choosing an even integer}) = P(A) = \frac{n(A)}{n(S)} = \frac{5}{10} = \frac{1}{2}$$

$$P(\text{choosing an integer multiple of three}) = \frac{n(B)}{n(S)} = \frac{3}{10}$$

35. Let A, B and C be $\left(0, \frac{-3}{2}\right)$, $(1, -1)$ and $\left(2, -\frac{1}{2}\right)$ respectively.

The slope of AB is $\dfrac{-1+\frac{3}{2}}{1-0} = \dfrac{1}{2}$

The slope of BC is $\dfrac{-\frac{1}{2}+1}{2-1} = \dfrac{1}{2}$

Thus, the slope of AB is equal to slope of BC.

Hence, A, B and C are lying on the same line.

36. Area of the triangle = absolute value of $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$9 = \left|\frac{1}{2}\begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix}\right| = \left|\frac{1}{2}(-k)(-3-3)\right|$$

$\Rightarrow$ $9 = 3|k|$ and hence, $k = \pm 3$.

37. Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{r}\cdot\hat{i} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)\cdot\hat{i} = x$$

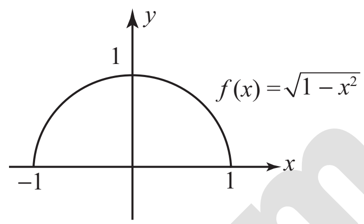
$$\vec{r}\cdot\hat{j} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)\cdot\hat{j} = y$$

$$\vec{r}\cdot\hat{k} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)\cdot\hat{k} = z$$

$$(\vec{r}\cdot\hat{i})\hat{i} + (\vec{r}\cdot\hat{j})\hat{j} + (\vec{r}\cdot\hat{k})\hat{k} = x\hat{i} + y\hat{j} + z\hat{k} = \vec{r}$$

Thus $\vec{r} = (\vec{r}\cdot\hat{i})\hat{i} + (\vec{r}\cdot\hat{j})\hat{j} + (\vec{r}\cdot\hat{k})\hat{k}$.

38. Let $f(x) = \sqrt{1-x^2}$



The domain of definition of f is the closed interval [–1,1].

(f is defined if $1 - x^2 \geq 0$). For any point $c \in (-1,1)$

$$\lim_{x\to c} f(x) = \lim_{x\to c}\sqrt{1-x^2} = \left[\lim_{x\to c}\left(1-x^2\right)\right]^{\frac{1}{2}}$$

$$= \left(1-c^2\right)^{\frac{1}{2}} = f(c).$$

$$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+}\left(1-x^2\right)^{\frac{1}{2}} = 0 = f(1)$$

$$\lim_{x\to -1^-} f(x) = \left[\lim_{x\to -1^-}\left(1-x^2\right)\right]^{\frac{1}{2}} = 0 = f(-1)$$

Thus f is continuous on [–1, 1]. One can also solve this problem using composite function theorem.

39. Given that $f'(x) = \dfrac{d}{dx}(f(x)) = 3x^2 - 4x + 5$

Integrating on both sides with respect to x, we get

$$\int f'(x)dx = \int (3x^2 - 4x + 5)\,dx$$

$$f(x) = x^3 - 2x^2 + 5x + c$$

To determine the constant of integration c, we have to apply the given information $f(1) = 3$

$$f(1) = 3 \Rightarrow 3 = (1)^3 - 2(1)^2 + 5(1) + c \Rightarrow c = -1$$

$$\text{Thus } f(x) = x^3 - 2x^2 + 5x - 1$$

40.

$$\tan 165° = \tan(120° + 45°)$$

$$= \frac{\tan 120° + \tan 45°}{1 - \tan 120° \tan 45°}$$

$$\text{But, } \tan 120° = \tan(90° + 30°) = -\cot 30°$$

$$= -\sqrt{3} \text{ and } \tan 45° = 1$$

$$\tan 165° = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

PART - IV

41. (a)

$$n(A \times A) = 16$$

$$\Rightarrow n(A) = 4.$$

Given $S = \{(a, b) \in A \times A : a < b\}$

$\therefore A = \{-1, 0, 1, 2\}$.

$A \times A = \{(-1, -1)\ (-1, 0)\ (-1, 1)\ (-1, 2) (0, -1), (0, 0)\ (0, 1)\ (0, 2)\ (1, -1)\ (1, 0) (1, 1)\ (1, 2)\ (2, -1)\ (2, 0)\ (2, 1)\ (2, 2)\}$

Now, $S = \{(-1, 0)\ (-1, 1)\ (-1, 2)\ (0, 1)\ (0, 2)\ (1, 2)\}$

$\therefore$ The remaining elements of S are $(-1, 0)\ (-1, 1)\ (0, 2)\ (1, 2)$

(OR)

(b)

$$\lim_{x \to 3} (x^2 - 6x + 5) = 3^2 - 6 \times 3 + 5 = -4$$

$$\lim_{x \to 3} (x^3 - 8x + 7) = 3^3 - 8 \times 3 + 7 = 10 \neq 0.$$

Therefore, $$\lim_{x \to 3} \frac{(x^2 - 6x + 5)}{x^3 - 8x + 7} = \frac{\lim_{x \to 3}(x^2 - 6x + 5)}{\lim_{x \to 3}(x^3 - 8x + 7)} = \frac{-4}{10} = -\frac{2}{5}$$

42. (a) Note that $x > 0$.

$\log_2 x + \log_4 x + \log_{16} x = \frac{7}{2}$ becomes

$\frac{1}{\log_x 2} + \frac{1}{\log_x 4} + \frac{1}{\log_x 16} = \frac{7}{2}$. (change of base rule)

Thus $\frac{1}{a} + \frac{1}{2a} + \frac{1}{4a} = \frac{7}{2}$ where $a = \log_x 2$.

That is $\frac{7}{4a} = \frac{7}{2}$

$$\Rightarrow a = \frac{1}{2}$$

$$\therefore \frac{1}{2} = \log_x 2$$

$$\Rightarrow x^{\frac{1}{2}} = 2$$

$$\Rightarrow \boxed{x = 4}$$

(OR)

(b) Let A, B, C be the given points and O be the point of reference or origin. Then

$$\overrightarrow{OA} = 2\hat{i}+4\hat{j}+3\hat{k}, \overrightarrow{OB} = 4\hat{i}+\hat{j}+9\hat{k} \text{ and } \overrightarrow{OC} = 10\hat{i}-\hat{j}+6\hat{k}$$

$$\overrightarrow{AB} = \overrightarrow{OB}-\overrightarrow{OA} = (4\hat{i}+\hat{j}+9\hat{k})-(2\hat{i}+4\hat{j}+3\hat{k}) = 2\hat{i}-3\hat{j}+6\hat{k}$$

$$AB = |\overrightarrow{AB}| = \sqrt{2^2+(-3)^2+6^2} = \sqrt{4+9+36} = 7$$

$$\overrightarrow{BC} = \overrightarrow{OC}-\overrightarrow{OB} = (10\hat{i}-\hat{j}+6\hat{k})-(4\hat{i}+\hat{j}+9\hat{k}) = 6\hat{i}-2\hat{j}-3\hat{k}$$

$$BC = |\overrightarrow{BC}| = \sqrt{6^2+(-2)^2+(-3)^2} = \sqrt{36+4+9} = 7$$

$$\overrightarrow{CA} = \overrightarrow{OA}-\overrightarrow{OC} = (2\hat{i}+4\hat{j}+3\hat{k})-(10\hat{i}-\hat{j}+6\hat{k}) = -8\hat{i}+5\hat{j}-3\hat{k}$$

$$CA = |\overrightarrow{CA}| = \sqrt{(-8)^2+5^2+(-3)^2} = \sqrt{64+25+9} = \sqrt{98}$$

$$BC^2 = 49,\ CA^2 = 98,\ AB^2 = 49.$$

Clearly $CA^2 = BC^2 + AB^2$.

Therefore, the given points form a right angled triangle.

43. (a) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$.

Proof. Using the expressions for the "combination" we have,

$$ {}^nC_r + {}^nC_{r-1} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-(r-1))!}$$

$$= \frac{n!}{r!\times(n-r)!} + \frac{n!}{(r-1)!\times(n-r+1)!}$$

$$= \frac{n!}{r\cdot(r-1)!\times(n-r)!} + \frac{n!}{(r-1)!\times(n-r)!(n-r+1)}$$

$$= \frac{n!}{(r-1)!\times(n-r)!}\times\left(\frac{1}{r}+\frac{1}{(n-r+1)}\right) = \frac{n!}{(r-1)!\times(n-r)!}\times\frac{(n-r+1+r)}{r(n-r+1)}$$

$$= \frac{n!}{(r-1)!\times(n-r)!}\times\frac{(n+1)}{r(n-r+1)} = \frac{(n+1)!}{r!\times(n+1-r)!} = {}^{n+1}C_r$$

(OR)

(b) Given $\sin y = x\sin(a+y)$ ... (1)

Differentiating with respect to 'x' we get,

$$\cos y\frac{dy}{dx} = x\cos(a+y)\left(\frac{dy}{dx}\right) + \sin(a+y)(1) \quad \text{[product rule]}$$

$$\Rightarrow \cos y\frac{dy}{dx} = x\cos(a+y)\frac{dy}{dx} + \sin(a+y)$$

$$\Rightarrow \frac{dy}{dx}(\cos y - x\cos(a+y)) = \sin(a+y)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sin(a+y)}{\cos y - x\cos(a+y)} = \frac{\sin(a+y)}{\cos y - \frac{\sin y}{\sin(a+y)}\cdot\cos(a+y)}$$

[from (1)]

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin(a+y)\cos y - \sin y\cos(a+y)} = \frac{\sin^2(a+y)}{\sin(a+\not{y}-\not{y})}$$

[$\because$ sin (A+B) = sin A cos B – cos A sin B]

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

Hence proved.

44. (a) Napier's formula $\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\frac{C}{2}$

Proof: $\frac{a-b}{a+b}\cot\frac{C}{2} = \frac{2R\sin A - 2R\sin B}{2R\sin A + 2R\sin B}\cot\frac{C}{2}$

$$= \cot\left(\frac{A+B}{2}\right)\tan\left(\frac{A-B}{2}\right)\cot\frac{C}{2}$$

$$= \tan\frac{C}{2}\cdot\tan\left(\frac{A-B}{2}\right)\cdot\cot\frac{C}{2} = \tan\left(\frac{A-B}{2}\right).$$

(OR)

(b) Let $I = \int \frac{\sin x + \cos x}{\sin x - \cos x}\,dx$

Putting $\sin x - \cos x = u$, then $(\cos x + \sin x)\,dx = du$

Thus, $I = \int \frac{du}{u} = \log|u| + c = \log|\sin x - \cos x| + c$

Therefore, $\int \frac{\sin x + \cos x}{\sin x - \cos x}\,dx = \log|\sin x - \cos x| + c$

45. (a) LHS $= \left(x^3+6\right)^{\frac{1}{3}} - \left(x^3+3\right)^{\frac{1}{3}}$

$$= x^{\not{3}\times\frac{1}{\not{3}}}\left(1+\frac{6}{x^3}\right)^{\frac{1}{3}} - x^{\not{3}\times\frac{1}{\not{3}}}\left(1+\frac{3}{x^3}\right)^{\frac{1}{3}}$$

Since x large, $\frac{1}{x^3}$ is very small $\therefore \left|\frac{3}{x^3}\right| < 1$.

$$= x\left[1+\frac{1}{3}\left(\frac{6}{x^3}\right)+\frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)}{2!}\left(\frac{6}{x^3}\right)^2+\ldots\right] - x\left[1+\frac{1}{3}\left(\frac{3}{x^3}\right)+\frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)\left(\frac{3}{x^3}\right)}{2!}+\ldots\right] \text{(app)}$$

$$= x\left(1+\frac{2}{x^3}+\ldots\right) - x\left(1+\frac{1}{x^3}+\ldots\right) \text{(app)}$$

$$= \cancel{x} + \frac{2}{x^2} - \cancel{x} - \frac{1}{x^2} \text{ (app)}$$

$$= \frac{2}{x^2} - \frac{1}{x^2} = \frac{1}{x^2} \text{ (app)} = \text{RHS.}$$

Hence proved.

(OR)

(b) Let A $= \begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix}$ ... (1)

Putting a = 0 in (1) we get,

$$A = \begin{vmatrix} b+c & -c & -b \\ b-c & c & b \\ c-b & c & b \end{vmatrix} = 0$$

Taking c from c_2 and b from c_3, we have

$$A = cb\begin{vmatrix} b+c & -1 & -1 \\ b-c & 1 & 1 \\ c-b & 1 & 1 \end{vmatrix} = 0 \qquad [\because c_2 \equiv c_3]$$

$(a - 0)$ = a is a factor of A.

Putting b = 0 in (1) we get,

$$A = \begin{vmatrix} c & a-c & a \\ -c & c+a & -a \\ c & c-a & a \end{vmatrix}$$

Taking c from c_1 and a from c_3, we have

$$A = ca\begin{vmatrix} 1 & a-c & 1 \\ -1 & c+a & -1 \\ 1 & c-a & 1 \end{vmatrix} = 0 \qquad [\because c_1 \equiv c_3]$$

$(b - 0)$ = b is a factor of A.

Putting c = 0 in (1) we get,

$$A = \begin{vmatrix} b & a & a-b \\ b & a & b-a \\ -b & -a & a+b \end{vmatrix} = 0$$ [By taking b from c_1 and a from c_2 we have $c_1 \equiv c_3$]

$\therefore (c - 0)$ = c is a factor of A.

Since the leading diagonal of A is of degree 3, only 3 factors are available and there may exist a constant k.

$$\therefore \begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = k(abc)$$

Putting $a = 1$, $b = 1$ and $c = 1$ in the above equation, we get

$$\begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix} = k(1)(1)(1) \quad \Rightarrow \quad 8 = k$$

$$\therefore \begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$$

46. **(a)** Given $\theta + \phi = \alpha$ and $\tan\theta = k\tan\phi$

$$\tan\theta = k\tan\phi$$

$$\Rightarrow \frac{\tan\theta}{\tan\phi} = k$$

$$\Rightarrow \frac{\sin\theta\cos\phi}{\cos\theta\cdot\sin\phi} = \frac{k}{1}$$

(By Components and dividends)

$$\Rightarrow \frac{\sin\theta\,\cos\phi - \cos\theta\,\sin\phi}{\sin\theta\,\cos\phi + \cos\theta\,\sin\phi} = \frac{k-1}{k+1}$$

$$\Rightarrow \frac{\sin\,(\theta-\phi)}{\sin\,(\theta+\phi)} = \frac{k-1}{k+1}$$

$$\Rightarrow \frac{\sin\,(\theta-\phi)}{\sin\,\alpha} = \frac{k-1}{k+1}$$

$$\Rightarrow \sin\,(\theta-\phi) = \frac{k-1}{k+1}\sin\alpha.$$

(OR)

(b) Given $y = e^{\tan^{-1}x}$

$$y' = e^{\tan^{-1}x}\cdot\frac{d}{dx}(\tan^{-1}x)$$

$$\Rightarrow y' = e^{\tan^{-1}x}\,\frac{1}{1+x^2} \Rightarrow (1+x^2)\,y' = y \qquad \left[\because y = e^{\tan^{-1}x}\right]$$

Differentiating again with respect to 'x', we get

$$(1+x^2)\,y'' + y'(2x) = y'$$

$$\Rightarrow (1+x^2)\,y'' + 2x\,y' - y' = 0$$

$$\Rightarrow (1+x^2)\,y'' + (2x-1)\,y' = 0$$

Hence proved.

47. **(a)** Any point on the line $x + y = 5$ is $x = t$; $y = 5 - t$

The distance from $(t, 5 - t)$ to the line $4x + 3y - 12 = 0$ is given by 2 units.

$$\therefore \frac{4(t)+3(5-t)-12}{\sqrt{4^2+3^2}} = 2$$

$$\Rightarrow \frac{|t+3|}{5} = 2$$

$$\Rightarrow t + 3 = \pm 10 \Rightarrow t = -13;\ t = 7$$

$\therefore$ The points $(-13, 18)$ and $(7, -2)$.

(OR)

(b) Sample space S = {1, 2, 3, 4, 5, 6}

Let A be the event of die shows an odd number.

Let B be the event of getting 5

Then A = {1, 3, 5}, B = {5} and $A \cap B = \{5\}$

Therefore, $P(A) = \frac{3}{6}$ and $P(A \cap B) = \frac{1}{6}$

P (getting 5 / die shows an odd number) = P (B/A)

$$= \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{6}}{\frac{3}{6}}$$

$$P(B/A) = \frac{1}{3}$$