11th STD.

INSTANT SUPPLEMENTARY EXAM - JUNE 2023

Register Number

PART - III

PHYSICS (With Answers)

TIME ALLOWED : 3.00 HOURS]   [MAXIMUM MARKS : 70

Instructions :

1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.
2) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams

PART - I

Note : (i) Answer **all** the questions. **(15 × 1 = 15)**
(ii) Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer.

1. Which of the following pairs of physical quantities have same dimension?
   (a) force and power (b) torque and energy
   (c) torque and power (d) force and torque

2. If the velocity of the particle is $\vec{v} = 2\hat{i} + t^2\hat{j} - 9\vec{k}$, then the magnitude of acceleration of the particle at t = 0.5 s is :
   (a) 1 ms^{-2} (b) 2 ms^{-2} (c) Zero (d) –1 ms^{-2}

3. An object of mass m begins to move on the plane inclined at an angle θ. The coefficient of static friction of inclined surface is μ_s. The maximum static frictional force experienced by the object is :
   (a) mg (b) $\mu_s$mg
   (c) $\mu_s$mgsinθ (d) $\mu_s$mgcosθ

4. The work done by the conservative force for a closed path is:
   (a) always negative (b) zero
   (c) always positive (d) not defined

5. The centre of mass of a system of particles does not depend upon:
   (a) position of particles
   (b) relative distance between particles
   (c) mass of particles
   (d) force acting on particle

6. If the masses of the Earth and Sun suddenly double, the gravitational force between them will:
   (a) remain the same (b) increase 2 times
   (c) increase 4 times (d) decrease 2 times

7. With an increase in temperature, the viscosity of liquid and gas, respectively will :
   (a) increase and increase (b) increase and decrease
   (c) decrease and increase (d) decrease and decrease

8. When a cycle tyre suddenly bursts, the air inside the tyre expands. This process is:
   (a) isothermal (b) adiabatic
   (c) isobaric (d) isochoric

9. The average translational kinetic energy of gas molecules depends on :
   (a) number of moles and T (b) only on T
   (c) P and T (d) P only

10. The damping force on an oscillator is directly proportional to the velocity. The unit of the constant of proportionality is :
    (a) kg m s^{-1} (b) kg m s^{-2}
    (c) kg s^{-1} (d) kg s

11. An organ pipe A closed at one end is allowed to vibrate in its first harmonic and another pipe B open at both ends is allowed to vibrate in its third harmonic. Both A and B are in resonance with a given tuning fork. The ratio of the length of A and B is :
    (a) $\frac{8}{3}$ (b) $\frac{3}{8}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

12. If the linear momentum of the object is increased by 0.1%, then the kinetic energy is increased by:
    (a) 0.1 % (b) 0.2 % (c) 0.4 % (d) 0.01 %

13. If an object is thrown vertically up with the initial speed 'u' from the ground, then the time taken by the object to return back to ground is :
    (a) $\frac{u^2}{2g}$ (b) $\frac{u^2}{g}$ (c) $\frac{u}{2g}$ (d) $\frac{2u}{g}$

14. Which of the following represents a wave?
    (a) $(x - vt)^3$ (b) $x(x + vt)$
    (c) $\frac{1}{(x+vt)}$ (d) $\sin(x + vt)$

15. The graph between volume and temperature in Charle's law is:
    (a) an ellipse (b) a circle
    (c) a straight line (d) a parabola

PART - II

Note: Answer **any six** questions. Question No **24** is **compulsory.** **(6 × 2 = 12)**

16. What is the principle of homogeneity of dimensions?
17. A metal cube of side 0.20 m is subjected to a shearing force of 4000 N. The top surface is displaced through 0.50 cm with respect to the bottom. Calculate the shear modulus of elasticity of the metal.
18. What is the meaning of 'Pseudo force'?
19. During a cyclic process, a heat engine absorbs 500 J of heat from a hot reservoir, does work and ejects an amount of heat 300 J into the surroundings (cold reservoir). Calculate the efficiency of the heat engine.
20. What is the difference between velocity and average velocity?

21. Why there is no lunar eclipse and solar eclipse every month?
22. Mention the four different types of oscillations.
23. State the law of equipartition of energy.
24. A fly wheel rotates with a uniform angular acceleration. If its angular velocity increases from 20π rad/s to 40π rad/s in 10 seconds, find the number of rotations in that period.

Part - III

Note: Answer **any six** questions. Question No **33** is **compulsory.** **(6 × 3 = 18)**

25. Write a note on triangulation method to measure larger distances.
26. Define "molar specific heat capacity". Give its unit.
27. Write the various types of potential energy.
28. State the laws of transverse vibrations in stretched strings.
29. A car takes a turn with velocity 50 ms^{-1} on the circular road of radius of curvature 10 m. Calculate the centrifugal force experienced by a person of mass 60kg inside the car.
30. What are the differences between sliding and slipping?
31. A train was moving at the rate of 54 km h^{-1}. When brakes were applied, it came to rest within a distance of 225 m. Calculate the retardation in the train.
32. What are the factors affecting the surface tension of a liquid?
33. Ten particles are moving at the speed of 2, 3, 4, 5, 5, 5, 6, 6, 7 and 9 ms^{-1}. Calculate root mean square speed (V_{rms}) and most probable speed (V_{mp}).

Part - IV

Note: Answer **all** the questions. **(5 × 5 = 25)**

34. (a) What is an error? Explain the systematic errors.

OR

(b) State and prove Bernoulli's theorem for a flow of incompressible, non-viscous, and streamlined flow of liquid.

35. (a) Discuss in detail the energy in simple harmonic motion.

OR

(b) State Newton's three laws and discuss their significance.

36. (a) State and explain work energy principle.

OR

(b) Derive an expression for escape speed.

37. (a) Explain in detain the working of a refrigerator.

OR

(b) Derive the kinematic equations of motion for constant acceleration.

38. (a) How will you determind the velocity of sound using resonance air column apparatus?

OR

(b) State and prove parallel axis theorem.

ANSWERS

Part - I

1. (b) torque and energy
2. (a) 1 ms^{-2}
3. (d) $\mu_s mg\cos\theta$
4. (b) zero
5. (d) force acting on particle
6. (c) increase 4 times
7. (c) decrease and increase
8. (b) adiabatic
9. (a) number of moles and T
10. (c) kg s^{-1}
11. (c) $\frac{1}{6}$
12. (b) 0.2 %
13. (d) $\frac{2u}{g}$
14. (d) $\sin(x + vt)$
15. (c) a straight line

Part - II

16. **Principle of homogeneity of dimensions :** The principle of homogeneity of dimensions states that the dimensions of all the terms in a physical expression should be the same. For example, in the physical expression $v^2 = u^2 + 2as$, the dimensions of v^2, u^2 and 2 as are the same and equal to $[L^2T^{-2}]$.

17. Here, L = 0.20 m, F = 4000 N, x = 0.50 cm = 0.005 m and Area A = L^2 = 0.04 m^2 Therefore,

$$\eta R = \frac{F}{A} \times \frac{L}{x} = \frac{4000}{0.04} \times \frac{0.20}{0.005} = 4 \times 10^6 \text{ Nm}^{-2}$$

18. A pseudo force is an apparent force that acts on all masses whose motion is described using a non inertial frame of reference such as a rotating reference frame.

19. The efficiency of heat engine is given by $\eta = 1 - \frac{Q_L}{Q_H}$

$$\eta = 1 - \frac{300}{500} = 1 - \frac{3}{5} \; ; \quad \eta = 1 - 0.6 = 0.4$$

The heat engine has 40% efficiency, implying that this heat engine converts only 40% of the input heat into work.

20.

Velocity	Average Velocity
Velocity is equal to rate of change of position vector with respect to time.	Average velocity is the ratio of the displacement vector to the corresponding time interal.
$\vec{v} = \lim\limits_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$	$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$

21. (i) If the orbits of the Moon and Earth lie on the same plane, during full Moon of every month, we can observe lunar eclipse. If this is so during new Moon we can observe solar eclipse.

(ii) But Moon's orbit is tilted 5° with respect to Earth's orbit. Due to this 5° tilt, only during certain periods of the year, the Sun, Earth and Moon align in straight line leading to either lunar eclipse or solar eclipse depending on the alignment.

22. Damped oscillations, Maintained oscillations, Forced oscillations, Resonance

23. According to kinetic theory, the average kinetic energy of system of molecules in thermal equilibrium at temperature T is uniformly distributed to all degrees of freedom (x or y or z directions of motion) so that each degree of freedom will get $\frac{1}{2}kT$ of energy. This is called law of equipartition of energy.

24. Initial Angular Velocity ω_0 = 20π rad s^{-1}
Final Angular velocity ω = 40π rad s^{-1}
Time taken, t = 10 s
No. of rotations / s = ?
We know, $\omega = \omega_o + \alpha t$

$$\alpha = \frac{\omega - \omega_o}{t} = \frac{40\pi - 20\pi}{10} = \frac{20\pi}{10} = 2\pi$$

$\alpha = 2\pi$ rad s^{-1}

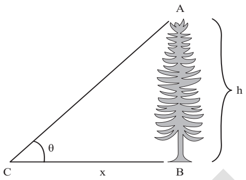
$$\theta = \omega_o t + \frac{1}{2}\alpha t^2 = 20\,\pi \times 10 + \frac{1}{2} \times 2\pi \times 10 \times 10$$

$= 200\,\pi + 100\pi = 300\,\pi$ rad

No. of rotations / sec. $= \dfrac{\theta}{2\pi} = \dfrac{300\,\pi}{2\pi} = 150$ rotations.

Part - III

25. **Triangulation method for the height of an accessible object:** Let AB = h be the height of the tree or tower to be measured. Let C be the point of observation at distance x from B. Place a range finder at C and measure the angle of elevation, $\angle ACB = \theta$ as shown in Figure.



Triangulation method

From right-angled triangle ABC,

$\tan\theta = \dfrac{AB}{BC} = \dfrac{h}{x}$ (or) height h = $x \tan\theta$.

Knowing the distance x, the height h can be determined.

26. Molar specific heat capacity is defined as heat energy required to increase the temperature of one mole of substance by 1K or 1°C.

$$C = \frac{1}{\mu}\left(\frac{\Delta Q}{\Delta T}\right)$$

Here C is known as molar specific heat capacity of a substance and μ is number of moles in the substance.

Unit of molar specific heat capacity = J/(kg k) or J/(kg°c)

27. **Various types of potential energies :** Each type is associated with a particular force. For example,
(i) The energy possessed by the body due to gravitational force gives rise to **gravitational potential energy**. U = mgh, U - Gravitational potential energy, m - Mass of the object, g - Acceleration due to gravity, h - Height from the ground.
(ii) The energy due to spring force and other similar forces give rise to **elastic potential energy.**

$$U = \frac{1}{2}kx^2$$

U - Elastic potential energy; k - String constant, x - Displacement
(iii) The energy due to electrostatic force on charges gives rise to **electrostatic potential energy.**

$$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r}$$

U - Electrostatic potential energy
ε_0 - absolute permittivity ; q_1, q_2 - electric charges.

28. There are three laws of transverse vibrations of stretched strings which are given as follows:
(i) **The law of length :** For a given wire with tension T (which is fixed) and mass per unit length μ (fixed) the frequency varies inversely with the vibrating length. Therefore,

$$f \propto \frac{1}{l} \Rightarrow f = \frac{C}{l}$$

$\Rightarrow l \times f = C$, where C is a constant.
(ii) **The law of tension :** For a given vibrating length l (fixed) and mass per unit length μ (fixed) the frequency varies directly with the square root of the tension T,

$f \propto \sqrt{T} \;\Rightarrow f = A\sqrt{T}$, where A is a constant
(iii) **The law of mass :** For a given vibrating length l (fixed) and tension T (fixed) the frequency varies inversely with the square root of the mass per unit length μ,

$f \propto \dfrac{1}{\sqrt{\mu}} \Rightarrow f = \dfrac{B}{\sqrt{\mu}}$, where B is a constant.

29. Velocity v = 50 ms^{-1}
Radius of curvature r = 10 m
Mass m = 60 kg

$$F = \frac{mv^2}{r} = \frac{60 \times 50 \times 50}{10} = \frac{150000}{10}$$

$\therefore$ F = 15,000 N.

30.

Sliding	Slipping
Sliding is the case when $v_{CM} > R\omega$ (or $v_{TRANS} > v_{ROT}$).	Slipping is the case when $v_{CM} < R\omega$ (or $v_{TRANS} < v_{ROT}$).
The translation is more than the rotation.	The rotation is more than the translation.
Sliding is also referred as forward slipping.	Slipping is sometimes empahasised as backward slipping.

31. The final velocity of the particle v = 0
The initial velocity of the particle

$u = 54 \times \dfrac{5}{18}\,ms^{-1} = 15\,ms^{-1}$; S = 225 m

Retardation is always against the velocity of the particle.

$v^2 = u^2 - 2aS$; $0 = (15)^2 - 2a\,(225)$

$450\,a = 225$; $a = \dfrac{225}{450}\,ms^{-2} = 0.5\,ms^{-2}$

Hence, retardation = 0.5 ms^{-2}

32. (i) **The presence of any contamination or impurities** considerably affects the force of surface tension depending upon the degree of contamination.
(ii) **The presence of dissolved substances** can also affect the value of surface tension.

(iii) **Electrification** affects the surface tension. When a liquid is electrified, surface tension decreases.

(iv) **Temperature** plays a very crucial role in altering the surface tension of a liquid. Obviously, the surface tension decreases linearly with the rise of temperature.

33. **Solution :** The average speed

$$\bar{v} = \frac{2+3+4+5+5+5+6+6+7+9}{10} = 5.2 \text{ ms}^{-1}$$

To find the rms speed, first calculate the mean square speed $\overline{v^2}$

$$\overline{v^2} = \frac{2^2+3^2+4^2+5^2+5^2+5^2+6^2+6^2+7^2+9^2}{10}$$

$= 30.6 \text{ m}^2\text{s}^{-2}$

The rms speed

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{30.6} = 5.53 \text{ ms}^{-1}$$

The most probable speed is 5 ms^{-1} because three of the particles have that speed.

Part - IV

34.

(a) The uncertainty in a measurement is called an error. **Systematic errors :** They are reproducible inaccuracies that are consistently in the same direction.

It is classified as follows :

(i) **Instrumental errors :** It arises when an instrument is not calibrated properly at the time of manufacturing. It can be corrected by choosing accurate instruments.

(ii) **Imperfections in experimental technique or procedure:** It is due to the limitations in the experimental arrangement. To overcome this, necessary and proper correction is to be applied.

(iii) **Personal errors :** These errors are due to individuals performing the experiment, may be due to incorrect initial setting up of the experiment or carelessness of the individual making the observation due to improper precautions.

(iv) **Errors due to external causes :** The change in the external conditions during an experiment can cause error in measurement. For example, changes in temperature, humidity, or pressure during measurements may affect the result of the measurement.

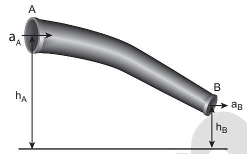
(v) **Least count error :** Least count is the smallest value that can be measured by the measuring instrument, and the error due to this measurement is least count error.

(OR)

(b) **Bernoulli's theorem:** According to Bernoulli's theorem, the sum of pressure energy, kinetic energy, and potential energy per unit mass of an incompressible, non-viscous fluid in a streamlined flow remains a constant.

Mathematically, $\frac{P}{\rho} + \frac{1}{2}v^2 + gh = \text{constant}$. This is known as Bernoulli's equation.

Proof:



Flow of liquid through a pipe AB

Let us consider a flow of liquid through a pipe AB as shown in Figure. Let V be the volume of the liquid when it enters A in a time t which is equal to the volume of the liquid leaving B in the same time. Let a_A, v_A and P_A be the area of cross section of the tube, velocity of the liquid and pressure exerted by the liquid at A respectively.

Let the force exerted by the liquid at A is $F_A = P_A a_A$

Distance travelled by the liquid in time t is $d = v_A t$

Therefore, the work done is $W = F_A d = P_A a_A v_A t$

But $a_A v_A t = a_A d = V$, volume of the liquid entering at A.

Thus, the work done is the pressure energy (at A), $W = F_A d = P_A V$

Pressure energy per unit volume at A

$$\frac{\text{Pressure energy}}{\text{volume}} = \frac{P_A V}{V} = P_A$$

Pressure energy per unit mass at A

$$\frac{\text{Pressure energy}}{\text{mass}} = \frac{P_A V}{m} = \frac{P_A}{\frac{m}{V}} = \frac{P_A}{\rho}$$

Since m is the mass of the liquid entering at A in a given time, therefore, pressure energy of the liquid at A is

$$E_{PA} = P_A V = P_A V \times \left(\frac{m}{m}\right) = m\frac{P_A}{\rho}$$

Potential energy of the liquid at A, $PE_A = mg\,h_A$,

Due to the flow of liquid, the kinetic energy of the liquid at A,

$$KE_A = \frac{1}{2} m\, v_A^2$$

Therefore, the total energy due to the flow of liquid at A,

$$E_A = E_{PA} + KE_A + PE_A \,;\; E_A = m\frac{P_A}{\rho} + \frac{1}{2} m\, v_A^2 + mg\,h_A$$

Similarly, let a_B, v_B, and P_B be the area of cross section of the tube, velocity of the liquid, and pressure exerted by the liquid at B. Calculating the total energy at E_B, we get

$$E_B = m\frac{P_B}{\rho} + \frac{1}{2} m v_B^2 + mg\,h_B$$

From the law of conservation of energy, $E_A = E_B$

$$m\frac{P_A}{\rho} + \frac{1}{2}mv_A^2 + mgh_A = m\frac{P_B}{\rho} + \frac{1}{2}mv_B^2 + mgh_B$$

$$\frac{P_A}{\rho} + \frac{1}{2}v_A^2 + gh_A = \frac{P_B}{\rho} + \frac{1}{2}v_B^2 + gh_B = \text{constant}$$

Thus, the above equation can be written as

$$\frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g} + h = \text{constant.}$$

The above equation is the consequence of the conservation of energy which is true until there is no loss of energy due to friction. But in practice, some energy is lost due to friction. This arises due to the fact that in a fluid flow, the layers flowing with different velocities exert frictional forces on each other. This loss of energy is generally converted into heat energy. Therefore, Bernoulli's relation is strictly valid for fluids with zero viscosity or non-viscous liquids. Notice that when the liquid flows through a horizontal pipe, then $h = 0 \Rightarrow \frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g} =$ constant.

35.

(a) **Expression for Potential Energy** For the simple harmonic motion, the force and the displacement are related by Hooke's law

$$\vec{F} = -k\,\vec{r}$$

Since Force is a vector quantity, in three dimensions it has three components. In one dimensional case

$$F = -k\,x \qquad ....(1)$$

The potential energy U can be calculated from the following expression.

$$F = -\frac{dU}{dx} \qquad ....(2)$$

Comparing (1) and (2), we get $-\frac{dU}{dx} = -kx$

$$dU = k\,x\,dx$$

This work done by the force F during a small displacement dx stores as potential energy

$$U(x) = \int_0^x k\,x'dx' = \frac{1}{2}k(x')^2\Big|_0^x = \frac{1}{2}Kx^2 \qquad ...(3)$$

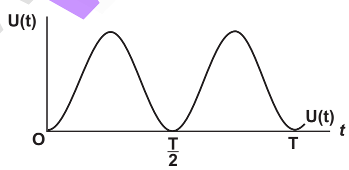
From equation $\omega = \sqrt{\frac{k}{m}}$, we can substitute the value of force constant $k = m\,\omega^2$ in equation (3)

$$U(x) = \frac{1}{2}m\omega^2x^2 \qquad ...(4)$$

where ω is the natural frequency of the oscillating system. For the particle executing simple harmonic motion from equation $y = A \sin \omega t$, we get $x = A \sin \omega t$

$$U(t) = \frac{1}{2}m\omega^2A^2\sin^2\omega t \qquad ...(5)$$

This variation of U is shown below.

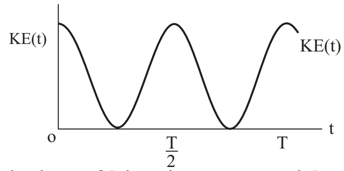


Variation of potential energy with time *t*

Expression for Kinetic Energy :

$$\text{Kinetic Energy} = \frac{1}{2}mv_x^2 = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2$$

Since the particle is executing simple harmonic motion, $x = A \sin \omega t$ Therefore, velocity is



Variation of kinetic energy with time t

$$v_x = \frac{dx}{dt} = A\omega\cos\omega t = A\omega\sqrt{1-\left(\frac{x}{A}\right)^2}$$

$$v_x = \omega\sqrt{A^2 - x^2} \qquad ...(6)$$

Hence,

$$KE = \frac{1}{2}mv_x^2 = \frac{1}{2}m\omega^2(A^2 - x^2) \qquad ....(7)$$

$$KE = \frac{1}{2}m\omega^2A^2\cos^2\omega t \qquad ...(8)$$

Expression for Total Energy : Total energy is the sum of kinetic energy and potential energy

$$E = KE + U \qquad ....(9)$$

$$E = \frac{1}{2}m\omega^2(A^2 - x^2) + \frac{1}{2}m\omega^2x^2$$

Hence canceling x^2 term,

$$E = \frac{1}{2}m\omega^2A^2 = \text{constant.} \qquad ...(10)$$

Alternatively, from equation (4) and equation (7), we get the total energy as

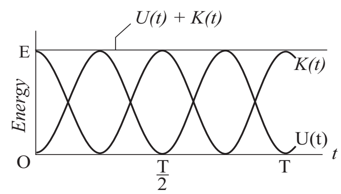
$$E = \frac{1}{2}m\omega^2A^2\sin^2\omega t + \frac{1}{2}m\omega^2A^2\cos^2\omega t$$

$$= \frac{1}{2}m\omega^2A^2(\sin^2\omega t + \cos^2\omega t)$$

From trigonometry identity, $(\sin^2\omega t + \cos^2\omega t) = 1$

$$E = \frac{1}{2}m\omega^2A^2 = \text{constant}$$

which gives the law of conservation of total energy.



Thus the amplitude of simple harmonic oscillator, can be expressed in terms of total energy.

$$A = \sqrt{\frac{2E}{m\omega^2}} = \sqrt{\frac{2E}{k}}.$$

(OR)

(b) (i) **Newton's First Law :** Every object continues to be in the state of rest or of uniform motion (constant velocity) unless there is external force acting on it. This inability of objects to move on its own or change its state of motion is called inertia. Inertia means resistance to change its state.

(ii) **Newton's Second Law :** The force acting on an object is equal to the rate of change of its momentum

$$\vec{F} = \frac{d\vec{p}}{dt}$$

(iii) **Newton's Third Law :** Newton's third law states that for every action there is an equal and opposite reaction.

Significance: Newton's laws are vector laws. The equation $\vec{F} = m\vec{a}$ is a vector equation and essentially it is equivalent to three scalar equations. The acceleration along the x direction depends only on the component of force acting along the x-direction. $F_y = ma_y$ The acceleration along the y direction depends only on the component of force acting along the y-direction. $F_z = ma_z$ The acceleration along the z direction depends only on the component of force acting along the z-direction. The acceleration experienced by the body at time t depends on the force which acts on the body at that instant of time. It does not depend on the force which acted on the body before the time t. This can be expressed as

$$\vec{F}(t) = m\vec{a}(t)$$

In general, the direction of a force may be different from the direction of motion. Though in some cases, the object may move in the same direction as the direction of the force, it is not always true.

36.

(a) The work done by the force on the body changes the kinetic energy of the body. This is called work-kinetic energy theorem. Consider a body of mass m at rest on a frictionless horizontal surface. The work (W) done by the constant force (F) for a displacement (s) in the same direction is,

$$W = Fs \quad ...(1)$$

The constant force is given by the equation,

$$F = ma \quad ...(2)$$

The third equation of motion can be written as,

$$v^2 = u^2 + 2as$$

$$a = \left(\frac{v^2 - u^2}{2s}\right) \quad ...(3)$$

Substituting for a in equation (2),

$$F = m\left(\frac{v^2 - u^2}{2s}\right) \quad ...(4)$$

Substituting equation (4) in (1),

$$W = m\left(\frac{v^2}{2s}s\right) - m\left(\frac{u^2}{2s}s\right); \; W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \quad ...(5)$$

The expression for kinetic energy:

(i) The term $\left(\frac{1}{2}mv^2\right)$ in t equation (5) is the kinetic energy of the body of mass (m) moving with velocity (v). $KE = \frac{1}{2}mv^2$ ...(6)

(ii) Kinetic energy of the body is always positive. From equations (5) and (6) $\Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ ...(7)

Thus, $W = \Delta KE$

(iii) The expression on the right hand side (RHS) of equation (7) is the change in kinetic energy (ΔKE) of the body.

(OR)

(b) Consider an object of mass M on the surface of the Earth. When it is thrown up with an initial speed v_i, the initial total energy of the object is $E_i = \frac{1}{2}Mv_i^2 - \frac{GMM_E}{R_E}$ ...(1)

where, M_E is the mass of the Earth and R_E the radius of the Earth. The term $-\frac{GMM_E}{R_E}$ is the potential energy of the mass M. When the object reaches a height far away from Earth and hence treated as approaching infinity, the gravitational potential energy becomes zero [$U(\infty) = 0$] and the kinetic energy becomes zero as well. Therefore the final total energy of the object becomes zero. This is for minimum energy and for minimum speed to escape. Otherwise Kinetic energy can be nonzero.

$$E_f = 0$$

According to the law of energy conservation,

$$E_i = E_f \quad ...(2)$$

Substituting (1) in (2) we get,

$$\frac{1}{2}Mv_i^2 - \frac{GMM_E}{R_E} = 0 \; ; \; \frac{1}{2}Mv_i^2 = \frac{GMM_E}{R_E} \quad ...(3)$$

Consider the escape speed, the minimum speed required by an object to escape Earth's gravitational field, hence replace v_i *with* v_e. *i.e,*

$$\frac{1}{2}Mv_e^2 = \frac{GMM_E}{R_E} \; ; \; v_e^2 = \frac{GMM_E}{R_E}\cdot\frac{2}{M}$$

$$v_e^2 = \frac{2GM_E}{R_E}$$

$$\text{Using } g = \frac{GM_E}{R_E^2} \; ; \; v_e^2 = 2gR_E \; ; \; v_e = \sqrt{2gR_E} \quad ...(4)$$

From equation (4) the escape speed depends on two factors: acceleration due to gravity and radius of the Earth. It is completely independent of the mass of the object. By substituting the values of g (9.8 ms^{-2}) and R_e = 6400 km, the escape speed of the Earth is v_e = 11.2 kms^{-1}.

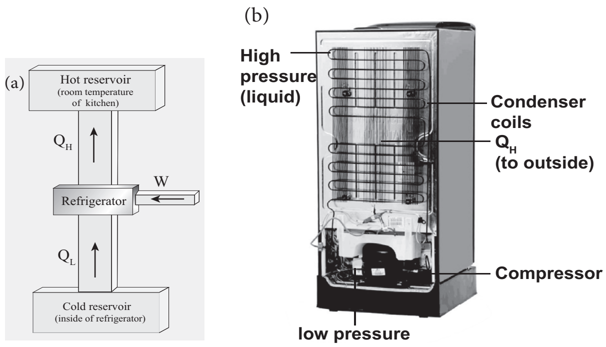
37.

(a) A refrigerator is a Carnot's engine working in the reverse order.

Working Principle: The working substance (gas) absorbs a quantity of heat Q_L from the cold body (sink) at a lower temperature T_L. A certain amount of work W is done on the working substance by the compressor and a quantity of heat Q_H is rejected to the hot body (source) i.e., the atmosphere at T_H. When you stand beneath of refrigerator, you can feel warmth air. From the first law of thermodynamics, we have

$$Q_L + W = Q_H \qquad \text{... (1)}$$

As a result the cold reservoir (refrigerator) further cools down and the surroundings (kitchen or atmosphere) gets hotter.



(a) Schematic diagram of a refrigerator (b) Actual refrigerator

Coefficient of performance (COP) (β): COP is a measure of the efficiency of a refrigerator. It is defined as the ratio of heat extracted from the cold body (sink) to the external work done by the compressor W.

$$\text{COP} = \beta = \frac{Q_L}{W} \qquad \text{... (2)}$$

From the equation (1)

$$\beta = \frac{Q_L}{Q_H - Q_L} \; ; \; \beta = \frac{1}{\frac{Q_H}{Q_L} - 1} \qquad \text{... (3)}$$

But we know that $\dfrac{Q_H}{Q_L} = \dfrac{T_H}{T_L}$

Substituting this equation into equation (3) we get

$$\beta = \frac{1}{\frac{T_H}{T_L} - 1} = \frac{T_L}{T_H - T_L}$$

Inferences: The greater the COP, the better is the condition of the refrigerator. A typical refrigerator has COP around 5 to 6. Lesser the difference in the temperatures of the cooling chamber and the atmosphere, higher is the COP of a refrigerator. In the refrigerator the heat is taken from cold object to hot object by doing external work. Without external work heat cannot flow from cold object to hot object. It is not a violation of second law of thermodynamics, because the heat is ejected to surrounding air and total entropy of (refrigerator + surrounding) is always increased.

(OR)

(b) Consider an object moving in a straight line with uniform or constant acceleration 'a'. Let 'u' be the initial velocity at time $t = 0$ and 'v' be the final velocity at time t.

Velocity - time relation: Acceleration, $a = \dfrac{dv}{dt}$ or $dv = a\,dt$

By integrating both sides, we get,

$$\int_u^v dv = \int_0^t a\,dt = a\int_0^t dt = a[t]_0^t$$

$$v - u = at$$

$$\boxed{v = u + at}$$

Displacement – time relation:

Velocity, $v = \dfrac{ds}{dt}$ or $ds = v\,dt$ and since $v = (u + at)$, we get, $ds = (u + at)\,dt$ Assume that initially at time t = 0, the particle started from the origin. Further assuming that acceleration is time-independent, we have

$$\int_0^s ds = \int_0^t u\,dt + \int_0^t at\,dt \qquad \text{(or)}$$

$$s = ut + \frac{1}{2}at^2 \qquad \text{...(1)}$$

Velocity – displacement relation : Acceleration $a = \dfrac{dv}{dt} = \dfrac{dv}{ds}\dfrac{ds}{dt} = \dfrac{dv}{ds}v$ [since $ds / dt = v$] where s is displacement traversed. This is rewritten as $a = \dfrac{1}{2}\dfrac{d}{ds}(v^2)$

or $ds = \dfrac{1}{2a}d(v^2)$

Integrating the above equation, using the fact when the velocity changes from u to v, displacement changes from 0 to s, we get,

$$\int_0^s ds = \int_u^v \frac{1}{2a}d(v^2)$$

$$\therefore s = \frac{1}{2a}(v^2 - u^2) \; ; \; \therefore v^2 = u^2 + 2as \qquad \text{...(2)}$$

We can also derive the displacement s in terms of initial velocity u and final velocity v. From the equation we can write, $at = v - u$

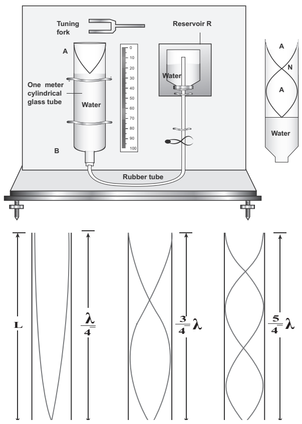
Substituting equation (1) , we get,

$$s = ut + \frac{1}{2}(v - u)\,t$$

$$s = \frac{(u + v)t}{2}$$

38.

(a) The resonance air column apparatus is one of the simplest techniques to measure the speed of sound in air at room temperature. It consists of a cylindrical glass tube of one meter length whose one end A is open and another end B is connected to the water reservoir R through a rubber tube as shown in Figure.



The resonance air column apparatus and first, second and third resonance

This cylindrical glass tube is mounted on a vertical stand with a scale attached to it. The tube is partially filled with water and the water level can be adjusted by raising or lowering the water in the reservoir R. The surface of the water will act as a closed end and other as the open end. The length of the air column is varied by changing the water level until a loud sound is produced in the air column. At this particular length the frequency of waves in the air column resonates with the frequency of the tuning fork (natural frequency of the tuning fork). At resonance, the frequency of sound waves produced is equal to the frequency of the tuning fork. This will occur only when the length of air column is proportional to $\left(\frac{1}{4}\right)^{th}$ of the wavelength of the sound waves produced.

Let the first resonance occur at length L_1, then

$$\frac{1}{4}\lambda = L_1$$

Including this end correction, the first resonance is

$$\frac{1}{4}\lambda = L_1 + e \qquad ...(1)$$

Now the length of the air column is increased to get the second resonance. Let L_2 be the length at which the second resonance occurs. Again taking end correction into account,

$$\frac{3}{4}\lambda = L_2 + e \qquad ...(2)$$

In order to avoid end correction, let us take the difference of equation (2) and equation (1), we get

$$\frac{3}{4}\lambda - \frac{1}{4}\lambda = (L_2 + e) - (L_1 + e)$$

$$\Rightarrow \frac{1}{2}\lambda = L_2 - L_1 = \Delta L$$

$$\Rightarrow \lambda = 2\Delta L$$

The speed of the sound in air at room temperature can be computed by using the formula

$$v = f\lambda = 2f\Delta L$$

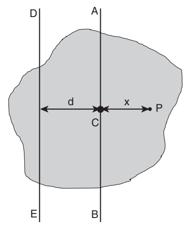
Further, to compute the end correction, we use equation (1) and (2), we get $e = \frac{L_2 - 3L_1}{2}$

(OR)

(b) **Statement:** The moment of inertia of a body about any axis is equal to the sum of its moment of inertia about a parallel axis through its center of mass and the product of the mass of the body and the square of the perpendicular distance between the two axes.

To Prove : Ic + Md²

Proof : Let us consider a rigid body as shown in figure.



If I_C is the moment of inertia of the body of mass M about an axis passing through the centre of mass, then the moment of inertia I about a parallel axis at a distance d from it is given by the relation,

$$I = I_C + Md^2$$

DE is another axis parallel to AB at a perpendicular distance d from AB. The moment of inertia of the body about DE is I. We attempt to get an expression for I in terms of I_C. The moment of inertia of the point mass about the axis DE is, $m(x + d)^2$. The moment of inertia I of the whole body about DE is the summation of the above expression.

$$I = \sum m(x+d)^2$$

$$I = \sum m(x^2 + d^2 + 2xd)$$

$$I = \sum (mx^2 + md^2 + 2dmx)$$

$$I = \sum mx^2 + \sum md^2 + 2d\sum mx$$

Here, $\sum mx^2$ is the moment of inertia of the body about the center of mass.

Hence, $I_C = \sum mx^2$

The term, $\Sigma Mx = 0$ because, x can take positive and negative values with respect to the axis AB. The summation (Σmx) will be zero. Thus, $I = I_C + \Sigma md^2 = I_C + (\Sigma m)d^2$

Here, Σm is the entire mass M of the object $(\Sigma m = M)$

$$I = I_C + \Sigma Md^2$$

Hence, the parallel axis theorem is proved.

◆◆◆