ST. ANNE'S ACADEMY
(MATHS & PHYSICS TUITION CENTRE)
I FLOOR, JAFRO DENTAL CLINIC, HOLY CROSS COLLEGE ROAD, PUNNAI NAGAR, NAGERCOIL – 629004

Common Mid-Term Test 1 (2023 – 24)
CLASS – XII - MATHEMATICS

Time Allowed : 1.5 Hrs Maximum Marks : 45

PART – I

I. Answer ALL questions. **10x1 = 10**

1) The area of the triangle formed by the complex numbers $z, iz,$ and $z+iz$ in the Argand's diagram is

(1) $\frac{1}{2}|z|^2$ (2) $|z|^2$ (3) $\frac{3}{2}|z|^2$ (4) $2|z|^2$

2) According to the rational root theorem, which number is not possible rational zero of $4x^7+2x^4-10x^3-5$?

(1) -1 (2) $\frac{5}{4}$ (3) $\frac{4}{5}$ (4) 5

3) If A^TA^{-1} is symmetric, then $A^2 =$

(1) A^{-1} (2) $(A^T)^2$ (3) A^T (4) $(A^{-1})^2$

4) If $|\text{adj}(\text{adj } A)| = |A|^{16}$, then the order of the square matrix A is

(1) 3 (2) 4 (3) 2 (4) 5

5) The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

(1) 1 (2) 2 (3) 4 (4) 3

6) If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then x is

(1) 15 (2) 12 (3) 14 (4) 11

7) The principal argument of $\frac{3}{-1+i}$ is

(1) $\frac{-5\pi}{6}$ (2) $\frac{-2\pi}{3}$ (3) $\frac{-3\pi}{4}$ (4) $\frac{-\pi}{2}$

8) If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

(1) z (2) $\bar{z}$ (3) $\frac{1}{z}$ (4) 1

9) The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

(1) $cis\frac{2\pi}{3}$ (2) $cis\frac{4\pi}{3}$ (3) $-cis\frac{2\pi}{3}$ (4) $-cis\frac{4\pi}{3}$

10) The polynomial x^3-kx^2+9x has three real zeros if and only if, k satisfies

(1) $|k|\le 6$ (2) $k=0$ (3) $|k|>6$ (4) $|k|\ge 6$

PART – II

II. Answer any THREE questions. [Question 15 is compulsory] **3x2 = 6**

11) Solve the equation $x^4-9x^2+20=0$.

12) Find the square roots of $-5-12i$.

13) If $\text{adj}(A)=\begin{bmatrix}0 & -2 & 0\\ 6 & 2 & -6\\ -3 & 0 & 6\end{bmatrix}$, find A^{-1}.

14) If $z=2-2i$, find the rotation of z by θ radians in the counter clockwise direction about the origin when $\theta=\frac{\pi}{3}$

15) If A^TA^{-1} is symmetric, then show that, $A^2=(A^T)^2$

PART – III

III. Answer any THREE questions. [Question 20 is compulsory] **3x3 = 9**

16) Solve, by Cramer's rule, the system of equations
$x_1-x_2=3, 2x_1+3x_2+4x_3=17, x_2+2x_3=7.$

17) Simplify $\left(\frac{1+\cos 2\theta+i\sin 2\theta}{1+\cos 2\theta-i\sin 2\theta}\right)^{30}$.

18) If the roots of $x^3+px^2+qx+r=0$ are in H.P., prove that $9pqr=27r^2+2q^3$.
Assume $p, q, r\neq 0$

19) For any two complex numbers z_1 and z_2, such that $|z_1|=|z_2|=1$ and $z_1z_2\neq -1$, then show that $\frac{z_1+z_2}{1+z_1z_2}$ is a real number.

20) If p is real, discuss the nature of the roots of the equation $4x^2+4px+p+2=0$, in terms of p.

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PART – IV

IV. Answer ALL questions. **4x5 = 20**

21) a) Solve the following equation: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

OR

b) (i) If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, show that it must be equal to $\frac{pq' - p'q}{q - q'}$ or $\frac{q - q'}{p' - p}$.

(ii) A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was left standing.

22) a) Find all cube roots of $\sqrt{3} + i$.

OR

b) Show that (i) $\left(\frac{19 - 7i}{9 + i}\right)^{12} + \left(\frac{20 - 5i}{7 - 6i}\right)^{12}$ is real.

(ii) A complex number z is purely imaginary if and only if $Z = -\bar{Z}$

23) a) Find the value of k for which the equations $kx - 2y + z = 1$, $x - 2ky + z = -2$, $x - 2y + kz = 1$ have

(i) no solution (ii) unique solution (iii) infinitely many solution

OR

b) If $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x + y + 2z = 1, 3x + 2y + z = 7, 2x + y + 3z = 2$.

24) a) By using Gaussian elimination method, balance the chemical reaction equation:
$C_5H_8 + O_2 \rightarrow CO_2 + H_2O$.

OR

b) Suppose z_1, z_2, and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$, then find z_2 and z_3.

Solve the equation $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$