ALPHA MATHS ACADAMY

JEE, CBSE AND BOARD EXAMINATION COACHING CENTER
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UNIT TEST – CHAPTER 1

STANDARD 12

TIME: 3.00 HOURS **MATHEMATICS** **MARKS: 90**

PART 1

CHOOSE THE CORRECT ANSWER **$20 \times 1 = 20$**

1. If $|adj\,(adjA)| = |A|^9$, then the order of the square matrix A is

   (a) 3 (b) 4 (c) 2 (d) 5

2. If $A\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, $B = adjA$ and $C = 3A$, then $\frac{|adjB|}{|C|} =$

   $(a)\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{4}$ (d) 1

3. If $A = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$, then $A =$

   (a) $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

4. If $A\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then $9I_2 - A =$

   (a) A^{-1} (b) $\frac{A^{-1}}{2}$ (c) $3A^{-1}$ (d) $2A^{-1}$

5. If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then x is

   (a) 15 (b) 12 (c) 14 (d) 11

6. If $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of a_{23} is

   (a) 0 (b) – 2 (c) – 3 (d) – 1

7. If A, B and C are invertible matrices of some order, then which one of the following is not true?

   (a) $adj\ A = |A|\ A^{-1}$ (b) $adj(AB) = (adj\ A)(adj\ B)$

   (c) $detA^{-1} = (detA)^{-1}$ (d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

8. If A^TA^{-1} is symmetric, then $A^2 =$

$(a)\ A^{-1}$ $(b)\ (A^T)^2$ $(c)\ A^T$ $(d)\ (A^{-1})^2$

9. If A is a non-singular matrix, such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1} =$

$(a) \begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ $(b) \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ $(c) \begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ $(d) \begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

10. If $A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$ and $A^T = A^{-1}$, then the value of x is

$(a)\ \frac{-4}{5}$ $(b)\ \frac{-3}{5}$ $(c)\ \frac{3}{5}$ $(d)\ \frac{4}{5}$

11 If $A = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$ and $A(adjA) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then $k =$

(a) 0 (b) $\sin\theta$ (c) $\cos\theta$ (d) 1

12. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

(a) 17 (b) 14 (c) 19 (d) 21

13. If $adjA = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and $adjB = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ then $adj\ (AB)$ is

$(a) \begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ $(b) \begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ $(c) \begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ $(d) \begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

14. If $x^a y^b = e^m, x^c y^d = e^n, \Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}, \Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}, \Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of x and y are respectively,

$(a)\ e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ $(b)\ \log(\Delta_1/\Delta_3), \log(\Delta_2/\Delta_3)$

$(c)\ \log(\Delta_2/\Delta_1), \log(\Delta_3/\Delta_1)$ $(d)\ e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

15. Which of the following is/are correct?

(i) Adjoint of a symmetric matrix is also a symmetric matrix.

(ii) Adjoint of a diagonal matrix is also a diagonal matrix.

(iii) If A is a square matrix of order n and λ is a scalar, then $adj(\lambda A) = \lambda^n adj(A)$

(iv) $A(adjA) = (adj\ A)A = |A|I$

(a) *only* (i) (b) (ii) *and* (iii) (c) (iii) *and* (iv) (d) $(i), (ii)$ *and* (iv)

16. If $\rho(A) = \rho([A|B])$, then the system $AX = B$ of linear equations is

(a) Consistent and has a unique solution (b) Consistent

(c) Consistent and has infinitely many solution (d) Inconsistent

17. If $0 \le \theta \le \pi$ and the system of equations $x + (\sin\theta)y - (\cos\theta)z = 0, (\cos\theta)x - y + z = 0,$

$(\sin\theta)x + y - z = 0$ has a non- trivial solution then θ is

$(a)\ \frac{2\pi}{3}$ $(b)\ \frac{3\pi}{4}$ $(c)\ \frac{5\pi}{6}$ $(d)\ \frac{\pi}{4}$

18. The augmented matrix of a system of linear equation is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda-7 & \mu+5 \end{bmatrix}$. The system

has infinitely many solutions if

$(a)\ \lambda = 7, \mu \ne -5$ $(b)\ \lambda = -7, \mu = 5$ $(c)\ \lambda \ne 7, \mu \ne -5$ $(d)\ \lambda = 7, \mu = -5$

19. Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$. If B is the inverse of A, then the

value of x is

(a) 2 (b) 4 (c) 3 (d) 1

20. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then $adj(adj\ A)$ is

$(a) \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ $(b) \begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$ $(c) \begin{bmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{bmatrix}$ $(d) \begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

PART 2

ANSWER ANY 7 OF THE FOLLOWING QUESTIONS $(30^{TH}$ *QUESTION IS COMPULSARY*$)$ $7 \times 2 = 14$

21. If A is a non-singular matrix of odd order, prove that $|Adj\ A|$ is positive.

22. Prove that $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ is orthogonal.

23. If $adj(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}.

24. Find the rank of the matrices which are in row-echelon form: $\begin{bmatrix} 6 & 0 & -9 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

25. Find the inverse of the non-singular matrix $A = \begin{pmatrix} 0 & 5 \\ -1 & 6 \end{pmatrix}$ by Gauss-Jordan method.

26. Solve the system of linear equations $2x + 5y = -2, x + 2y = -3$ by matrix inversion method .

27. Solve the systems of linear equations $\frac{3}{x} + 2y = 12$; $\frac{2}{x} + 3y = 13$ by Cramer's rule.

28. Find the rank of the matrix $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$ by minor method.

29. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ and $A^2 + xI = yA$, then the values of x and y are respectively.

30. If A is symmetric, prove that then $Adj\ A$ is also symmetric.

PART 3

ANSWER ANY 7 OF THE FOLLOWING QUESTIONS $(40^{TH}$ $QUESTION\ IS\ COMPULSARY)$ $\quad$ $7 \times 3 = 21$

31. Verify $(AB)^{-1} = B^{-1}A^{-1}$ with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$.

32. Find the inverse (if it exists) $\begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$.

33. If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$, Verify $(AB)^{-1} = B^{-1}A^{-1}$.

34. Decrypt the received encoded message $[2\ -3]\ [20\ \ 4]$ with the encryption matrix $\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$ and the decryption matrix as its inverse, where the system of codes are described by the numbers $1 - 26$ to the letters $A - Z$ respectively and the number 0 to the blank space.

35. Show that the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{bmatrix}$ is non-singular and reduce it to the identity matrix by elementary row transformation.

36. Find the inverse of $A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$ by Gauss-Jordan method.

37. Four man and 4 woman can finish a piece of work jointly in 3 days while 2 man and 5 woman can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

38. Solve $x + 2y + 3z = 0$, $3x + 4y + 4z = 0$, $7x + 10y + 12z = 0$.

39. A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution? (Use Cramer's rule to solve the problem)

40. If the system of equations $px + by + cz = 0, ax + qy + cz = 0, ax + by + rz = 0$ has a non-trivial solution and $p \neq a, q \neq b, r \neq c$ prove that $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$.

PART 4

ANSWER ALL THE FOLLOWING QUESTIONS **$7 \times 5 = 35$**

41. (a) If $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, Verify that $A(Adj\ A) = (Adj\ A)A = |A|I_3$ (or)

(b) Solve the systems of linear equations by Cramer's rule

$\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0\ , \frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0\ , \frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$

42. (a) If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, Find x, y such that $A^2 + xA + yI_2 = O_2$. Hence find A^{-1}. (or)

(b) The upward speed $v(t)$ of a rocket at time t is approximated by $v(t) = at^2 + bt + c, 0 \le t \le 100$ where a, b and c are constants. It has been found that the speed at times $t = 3, t = 6$ and $t = 9$ seconds are respectively 64 , 133 and 208 miles per second respectively. Find the speed at time $t = 15$ seconds. (Use Gaussian elimination method)

43. (a) Solve the system of equations using matrix inversion method

$2x_1 + 3x_2 + 3x_3 = 5\ ,\ x_1 - 2x_2 + x_3 = -4\ ,\ 3x_1 - x_2 - 2x_3 = 3.$ (or)

(b) A boy is walking along the path $\boldsymbol{y = ax^2 + bx + c}$ through the points $(\boldsymbol{-6}\ , \boldsymbol{8}), (\boldsymbol{-2}\ , \boldsymbol{-12})$ and $(\boldsymbol{3}\ , \boldsymbol{8})$. He wants to meet his friend at $\boldsymbol{P}(\boldsymbol{7}\ , \boldsymbol{60})$. Will he meet his friend? (Use Gaussian elimination method).

44. (a) If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, Find the products AB and BA and hence solve the system of equations $x - y + z = 4\ ,\ x - 2y - 2z = 9\ , 2x + y + 3z = 1.$ (or)

(b) Find the condition on $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{c}$ so that the following system of linear equations has one parameter family of solution $\boldsymbol{x + y + z = a\ , x + 2y + 3z = b, 3x + 5y + 7z = c}$

45. (a) Solve the system of linear equations by matrix inversion method

$2x + 3y - z = 9\ , x + y + z = 9\ , 3x - y - z = -1.$ (or)

(b) Find the value of $\boldsymbol{k}$ for which the equations k$\boldsymbol{x - 2y + z = 1\ , x - 2ky + z = -2\ ,\ x - 2y + kz = 1}$ has $(\boldsymbol{i})$ no solution $(\boldsymbol{ii})$ a unique solution $(\boldsymbol{iii})$ an infinitely many solution

46. (a) The prices of three commodities A, B and C are rupees x, y and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q purchase 2 unit of C and

sells 3 units of A and one unit of B. Person R purchases one unit of A and sells 3 units of B and 1 unit of C. In the process P, Q and R earn rupees 15,000 , rupees 1,000 and rupees 4,000 respectively. Find the prices per unit of A, B and C. (Use matrix inversion method). (or)

(b) By using Gaussian elimination method balances the chemical reaction equation

$\boldsymbol{C_5H_8 + O_2 \longrightarrow CO_2 + H_2O}$.

47. (a) Solve the Cramer's rule, the system of equations $x_1 - x_2 = 3$, $2x_1 + 3x_2 + 4x_3 = 17$, $x_2 + 2x_3 = 7$.

(or)

(b) Determine the values of λ for which the following system of equations $x + y + 3z = 0$, $4x + 3y + \lambda z = 0$, $2x + y + 2z = 0$ has (i) a unique solution (ii) a non- trivial solution.

*************** ***ALL THE BEST*** **************

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