ALPHA MATHS ACADAMY

JEE, CBSE AND BOARD EXAMINATION COACHING CENTER

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UNIT TEST – CHAPTER 2

STANDARD 12

TIME: 3.00 HOURS **MATHEMATICS** **MARKS: 90**

PART 1

CHOOSE THE CORRECT ANSWER $\mathbf{20 \times 1 = 20}$

1. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is

   $a)$ 0 $b)$ 1 $c)$ -1 $d)$ i

2. The conjugate of a complex number is $\frac{1}{i-2}$. Then the complex number is

   $a)$ $\frac{1}{i+2}$ $b)$ $\frac{-1}{i+2}$ $c)$ $\frac{-1}{i-2}$ $d)$ $\frac{1}{i-2}$

3. If $z = \frac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$, then $|z|$ is equal to

   $a)$ 0 $b)$ 1 $c)$ 2 $d)$ 3

4. If z is a non zero complex number, such that $2iz^2 = \bar{z}$ then $|z|$ is

   $a)$ $\frac{1}{2}$ $b)$ 1 $c)$ 2 $d)$ 3

5. If $|z - 2 + i| \leq 2$, then the greatest value of $|z|$ is

   $a)$ $\sqrt{3} - 2$ $b)$ $\sqrt{3} + 2$ $c)$ $\sqrt{5} - 2$ $d)$ $\sqrt{5} + 2$

6. If $\left|z - \frac{3}{z}\right| = 2$, then the least value of $|z|$ is

   $a)$ 1 $b)$ 2 $c)$ 3 $d)$ 5

7. If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

   $a)$ z $b)$ $\bar{z}$ $c)$ $\frac{1}{z}$ $d)$ 1

8. The solution of the equation $|z| - z = 1 + 2i$ is

   $a)$ $\frac{3}{2} - 2i$ $b)$ $-\frac{3}{2} + 2i$ $c)$ $2 - \frac{3}{2}i$ $d)$ $2 + \frac{3}{2}i$

9. If $|z_1| = 1, |z_2| = 2, |z_3| = 3$ and $|9z_1 z_2 + 4z_1 z_3 + z_2 z_3| = 12$, then the value of $|z_1 + z_2 + z_3|$ is

   $a)$ 1 $b)$ 2 $c)$ 3 $d)$ 4

10. If z is a complex number such that $z \in \mathbb{C}\backslash\mathbb{R}$ and $z + \frac{1}{z} \in \mathbb{R}$, then $|z|$ is

    $a)$ 0 $b)$ 1 $c)$ 2 $d)$ 3

11. z_1 , z_2 and z_3 are complex numbers such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$ then ${z_1}^2 + {z_2}^2 + {z_3}^2$ is

$a)$ 3 $b)$ 2 $c)$ 1 $d)$ 0

12. If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is

$a)$ $\frac{1}{2}$ $b)$ 1 $c)$ 2 $d)$ 3

13. If $z = x + iy$ is a complex number such that $|z + 2| = |z - 2|$, then the locus of z is

$a)$ real axis $b)$ imaginary axis $c)$ ellipse $d)$ circle

14. The principal argument of $\frac{3}{-1+i}$ is

$a)$ $\frac{-5\pi}{6}$ $b)$ $\frac{-2\pi}{3}$ $c)$ $\frac{-3\pi}{4}$ $d)$ $\frac{-\pi}{2}$

15. If $(1+i)(1+2i)(1+3i)\cdots(1+ni) = x + iy$, then $2.5.10\ldots(1+n^2)$ is

$a)$ 1 $b)$ i $c)$ $x^2 + y^2$ $d)$ $1 + n^2$

16. The principal argument of the complex number $\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is

$a)$ $\frac{2\pi}{3}$ $b)$ $\frac{\pi}{6}$ $c)$ $\frac{5\pi}{6}$ $d)$ $\frac{\pi}{2}$

17. If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{2020} + \beta^{2020}$ is

$a)$ -2 $b)$ -1 $c)$ 1 $d)$ 2

18. The product of all four values of $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is

$a)$ -2 $b)$ -1 $c)$ 1 $d)$ 2

19. If $\omega \neq 1$ is a cubic root of unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to

$a)$ 1 $b)$ -1 $c)$ $\sqrt{3}i$ $d)$ $-\sqrt{3}i$

20. If $\omega = cis\frac{2\pi}{3}$, then the number of distinct roots of $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$

$a)$ 1 $b)$ 2 $c)$ 3 $d)$ 4

PART 2

ANSWER ANY 7 OF THE FOLLOWING QUESTIONS $(30^{TH}$ ***QUESTION IS COMPULSARY***$)$ $\quad$ $\mathbf{7 \times 2 = 14}$

21. Simplify $i^1\, i^2\, i^3 \ldots\ldots i^{2000}$.

22. If $z = x + iy$, find the rectangular form $Re\left(\frac{1}{z}\right)$.

23. Prove that z is real $iff\ z = \bar{z}$.

24. Find the modulus of the complex numbers $(1 - i)^{10}$.

25. Obtain the Cartesian equation for the locus of $z = x + iy$ in each of the following cases $|z - 4| = 16$.

26. Write in polar form of the complex numbers $3 - i\sqrt{3}$.

27. Simplify $\left[\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right]^{18}$.

28. If $\omega \neq 1$ is a cube root of unity, show that $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = -1$.

29. If $z = 2 - 2i$, find the rotation of z $by\theta$ radians in the counter clockwise Direction about the origin when $\theta = \frac{3\pi}{2}$.

30. Find the square root of i.

PART 3

ANSWER ANY 7 OF THE FOLLOWING QUESTIONS $\left(40^{TH} QUESTION\ IS\ COMPULSARY\right)$ $7 \times 3 = 21$

31. Find the values of the real numbers x and,y if the complex numbers $(3-i)x - (2-i)y + 2i + 5$ and $2x + (-1+2i)y + 3 + 2i$ are equal .

32. If $(1+i)(1+2i)(1+3i) \ldots\ldots\ldots (1+ni) = (x+iy)$ .Show that $2.5.10 \ldots\ldots.. (1+n^2) = x^2 + y^2$.

33. If z_1 , z_2 and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = |z_1 + z_2 + z_3| = 1$, Find the value of $\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$.

34. If $|z| = 3$, show that $7 \leq |z + 6 - 8i| \leq 13$

35. Show that the equation $z^3 + 2\,\bar{z} = 0$ has five solutions

36. Obtain the Cartesian equation for the locus of $z = x + iy$ in $|z-4|^2 - |z-1|^2 = 16$

37. The principal argument $Arg\ z = \frac{-2}{1+i\sqrt{3}}$.

38. Simplify $\left[\frac{1+\cos 2\theta + i\sin 2\theta}{1+\cos 2\theta - i \sin 2\theta}\right]^{30}$.

39. Suppose z_1, z_2, and z_3, are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$.

If $z_1 = 1 + i\sqrt{3}$, then find z_2 and z_3.

40. If $\omega \neq 1$ is a cube root of unity, show that $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8) \ldots \left(1+\omega^{2^{11}}\right) = 1$

PART 4

ANSWER ALL THE FOLLOWING QUESTIONS $7 \times 5 = 35$

41. (a) Let z_1 , z_2 and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1 + z_2 + z_3 \neq 0$. Prove that $\left|\frac{z_1 z_2 + z_2 z_3 + z_3 z_1}{z_1 + z_2 + z_3}\right| = r$. $(\boldsymbol{or})$

(b) $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$, show that

(i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$ (ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$

42. (a) If $z = x + iy$ is a complex number such that $Im\left(\frac{2z+1}{iz+1}\right) = 0$ show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$. $(\boldsymbol{or})$

(b) If z = $(cos\theta + isin\theta)$, show that $z^n + \frac{1}{z^n}$ = 2cos nθ and $z^n - \frac{1}{z^n}$ = 2i sin nθ

43. (a) Find the fourth roots of unity. **(or)**

(b) If $\left|z - \frac{2}{z}\right| = 2$, show that the greatest and least value of $|z|$ are $\sqrt{3} + 1$ and $\sqrt{3} - 1$ respectively.

44. (a) If $2\,cos\alpha = x + \frac{1}{x}$ and $2\,cos\,\beta = y + \frac{1}{y}$, show that (i) $\frac{x}{y} + \frac{y}{x} = 2\,cos\,(\alpha - \beta)$

(ii) $xy - \frac{1}{xy} = 2i\,sin\,(\alpha + \beta)$ (ii) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\,sin\,(m\alpha - n\beta)$ **(or)**

(b) Show that $\left[\frac{\sqrt{3}}{2} + \frac{i}{2}\right]^5 + \left[\frac{\sqrt{3}}{2} - \frac{i}{2}\right]^5 = -\sqrt{3}$.

45. (a) Prove that the values of $\sqrt[4]{-1}$ are $\pm\frac{1}{\sqrt{2}}(1 \pm i)$ **(or)**

(b) The value of $\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}} - 1$

46. (a) If $z = x + iy$ and $arg\left[\frac{z-i}{z+2}\right] = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$ **(or)**

(b) P represents the variable complex number z. find the locus of P, if arg $\left[\frac{z-1}{z+3}\right] = \frac{\pi}{2}$.

47. (a) Find the value of $\sum_{k=1}^{8}\left(cos\frac{2k\pi}{9} + isin\frac{2k\pi}{9}\right)$ **(or)**

(b) Find the least positive integer n such that $\left(\frac{1+i}{1-i}\right)^n = 1$.

***************** *ALL THE BEST* *****************

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