12th STD

INSTANT SUPPLEMENTARY EXAM - JUNE 2023

Reg. No.

Part - III

Mathematics (with answers)

TIME ALLOWED : 3.00 Hours] [MAXIMUM MARKS : 90

Instructions :

(1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams

PART - I

Note : (i) **All** questions are **Compulsory.**

(ii) Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer. **20 × 1 = 20**

1. If $A^T . A^{-1}$ is symmetric, then A^2 is :

(a) A^{-1} (b) $(A^T)^2$ (c) A^T (d) $(A^{-1})^2$

2. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is :

(a) 1 (b) 2 (c) 4 (d) 3

3. If $|z - 2 + i| \le 2$, then the greatest value of $|z|$ is :

(a) $\sqrt{3} - 2$ (b) $\sqrt{3} + 2$

(c) $\sqrt{5} - 2$ (d) $\sqrt{5} + 2$

4. If $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|9 z_1 z_2 + 4 z_1 z_3 + z_2 z_3| = 12$, then the value of $|z_1 + z_2 + z_3|$ is :

(a) 1 (b) 2 (c) 3 (d) 4

5. A zero of $x^3 + 64$ is :

(a) 0 (b) 4 (c) $4i$ (d) -4

6. The number of positive zeros of the polynomial $\sum_{r=0}^{n} {}^nC_r (-1)^r x^r$ is :

(a) 0 (b) n (c) $< n$ (d) r

7. If $\cot^{-1} x = \frac{2\pi}{5}$ for some $x \in R$, then the value of $\tan^{-1} x$ is :

(a) $-\frac{\pi}{10}$ (b) $\frac{\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $-\frac{\pi}{5}$

8. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is :

(a) 1 (b) 3 (c) $\sqrt{10}$ (d) $\sqrt{11}$

9. If $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$, then the value of $[\vec{a}, \vec{b}, \vec{c}]$ is :

(a) $|\vec{a}|\ |\vec{b}|\ |\vec{c}|$ (b) $\frac{1}{3}|\vec{a}|\ |\vec{b}|\ |\vec{c}|$

(c) 1 (d) -1

10. If $\sin^{-1} x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$ then the value of x is :

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{5}}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{\sqrt{3}}{2}$

11. The number given by the Mean value theorem for the function $\frac{1}{x}$, $x \in [1, 9]$ is :

(a) 2 (b) 2.5 (c) 3 (d) 3.5

12. The curve $y = ax^4 + bx^2$ with $ab > 0$:

(a) has no horizontal tangent

(b) is concave up

(c) is concave down

(d) has no points of inflection

13. If $u(x, y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is :

(a) $e^{x^2+y^2}$ (b) $2xu$ (c) x^2u (d) y^2u

14. The value of $\int_{-1}^{2} |x|\, dx$ is :

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{7}{2}$

15. Linear approximation for $g(x) = \cos x$ at $x = \frac{\pi}{2}$ is :

(a) $x + \frac{\pi}{2}$ (b) $-x + \frac{\pi}{2}$

(c) $x - \frac{\pi}{2}$ (d) $-x - \frac{\pi}{2}$

16. The value of $\int_0^{\pi} \sin^4 x\, dx$ is :

(a) $\frac{3\pi}{10}$ (b) $\frac{3\pi}{8}$ (c) $\frac{3\pi}{4}$ (d) $\frac{3\pi}{2}$

17. The solution of the differential equation $\frac{dy}{dx} = 2xy$ is :

(a) $y = Ce^{x^2}$ (b) $y = 2x^2 + C$
(c) $y = Ce^{-x^2}$ (d) $y = x^2 + C$

18. The population P in any year t is such that the rate of increase in the population is proportional to the population then :

(a) $P = Ce^{kt}$ (b) $P = Ce^{-kt}$
(c) $P = Ckt$ (d) $P = C$

19. A random variable X has binomial distribution with $n = 25$ and $p = 0.8$ then standard deviation of X is :

(a) 6 (b) 4 (c) 3 (d) 2

20. If a compound statement involves 3 simple statements, then the number of rows in the truth table is :

(a) 9 (b) 8 (c) 6 (d) 3

PART - II

Note : Answer **any seven** questions. Question No. 30 is **Compulsory.** **7 × 2 = 14**

21. If $\text{adj}(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$ then find A^{-1}.

22. Find the principal argument Arg z, when $z = \frac{-2}{1+i\sqrt{3}}$.

23. Find the equation of the hyperbola with vertices $(0, \pm 4)$ and Foci $(0, \pm 6)$.

24. Find the distance from a point $(2, 5, -3)$ to the plane $\vec{r}\cdot(6\hat{i} - 3\hat{j} + 2\hat{k}) = 5$.

25. Prove that the function $f(x) = x^2 - 2x - 3$ is strictly increasing in $(2, \infty)$.

26. Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number.

27. Evaluate : $\int_0^{\infty} x^5 e^{-3x} dx$

28. Assume that a spherical rain drop evaporates at a rate proportional to its surface area. Form a differential equation involving the rate of change of the radius of the rain drop.

29. A pair, of Fair dice is rolled once. Find the probability mass function to get the number of four.

30. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two Boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$.

PART - III

Note : Answer **any seven** questions. Question No. **40** is **Compulsory.** **7 × 3 = 21**

31. Solve the system of linear equations $2x + 5y = -2$, $x + 2y = -3$ using matrix inversion method.

32. State and prove triangle inequality.

33. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$

34. With usual notations in any triangle ABC, prove by vector method $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

35. Find two positive numbers whose sum is 12 and their product is maximum.

36. If $U(x, y, z) = \log(x^3 + y^3 + z^3)$, find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$.

37. Prove that the point of intersection of the tangents at 't_1' and 't_2' on the parabola $y^2 = 4ax$ is $[at_1t_2, a(t_1 + t_2)]$.

38. Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000.

39. Establish the equivalence property connecting the bi-conditional with conditional $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$.

40. Show that the polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2$ has at least six imaginary roots.

PART - IV

Note : Answer **all** the questions. **7 × 5 = 35**

41. (a) Solve the system of linear equations by Cramer's Rule. $x_1 - x_2 = 3, 2x_1 + 3x_2 + 4x_3 = 17, x_2 + 2x_3 = 7$

OR

(b) Solve : $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$.

42. (a) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$.

OR

(b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u.$$

43. (a) Evaluate : $\lim_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$

OR

(b) If $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}, \vec{b} = 3\hat{i} + 5\hat{j} + 2\hat{k}, \vec{c} = -\hat{i} - 2\hat{j} + 3\hat{k}$, verify that $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a}$

44. (a) Solve the equation $2x^3 + 11x^2 - 9x - 18 = 0$

OR

(b) Solve : $(1 + x^2)\frac{dy}{dx} = 1 + y^2$.

45. (a) Find the equation of the circle passing through the points (1, 0), (–1, 0) and (0, 1).

OR

(b) The mean and variance of a binomial variate X are 2 and 1.5 respectively. Find (i) P(X= 0), (ii) P (X = 1), (iii) P(X ≥ 1).

46. (a) Find the foot of the perpendicular drawn from the point (5, 4, 2) to the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$. Also find the equation of the perpendicular.

OR

(b) Find the area of the region bounded by the lines $5x - 2y = 15$, $x + y + 4 = 0$ and the x - axis using integration.

47. (a) A particle moves along a line according to the law $s(t) = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$.

(i) At what time the particle changes direction?

(ii) Find the total distance travelled by the particle in the first 4 seconds.

(iii) Find the particles acceleration each time the velocity is zero.

OR

(b) Let $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{10\}\right\}$ and * be the matrix multiplication. Examine the closure, associative, existence of identity, existence of inverse for the operation * on M.

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Answers

PART - I

1. (b) $(A^T)^2$
2. (a) 1
3. (d) $\sqrt{5} + 2$
4. (b) 2
5. (d) –4
6. (b) n
7. (c) $\frac{\pi}{10}$
8. (c) $\sqrt{10}$
9. (a) $|\vec{a}|\ |\vec{b}|\ |\vec{c}|$
10. (b) $\frac{1}{\sqrt{5}}$
11. (c) 3
12. (a) has no points of inflection
13. (b) $2xu$
14. (c) $\frac{5}{2}$
15. (b) $-x + \frac{\pi}{2}$
16. (b) $\frac{3\pi}{8}$
17. (a) $y = Ce^{x^2}$
18. (a) $P = Ce^{kt}$
19. (d) 2
20. (b) 8

PART - II

21. Given adj (A)= $\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$

We know that $A^{-1} = \pm \frac{1}{\sqrt{|\text{adj}A|}}(\text{adj } A)$ ...(1)

$|\text{adj } A| = 0 + 2\begin{vmatrix} 6 & -6 \\ -3 & 6 \end{vmatrix} + 0$

[Expanded along R_1]

$= 2(36-18) = 2(18) = 36$

$\therefore A^{-1} = \pm \frac{1}{\sqrt{36}}\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$

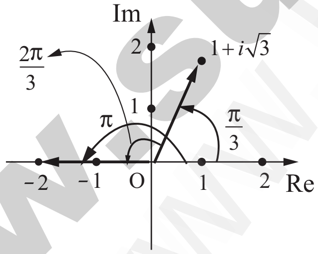
$= \pm \frac{1}{6}\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$

22. $\arg z = \arg \frac{-2}{1+i\sqrt{3}}$

$= \arg(-2) - \arg(1+i\sqrt{3})$

$\left(\because \arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2\right)$

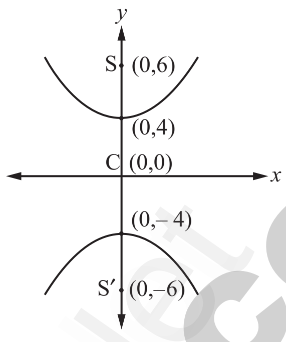
$= \left(\pi - \tan^{-1}\left(\frac{0}{2}\right)\right) - \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$



This implies that one of the values of arg z is $\frac{2\pi}{3}$

Since $\frac{2\pi}{3}$ lies between $-\pi$ and π, the principal argument Arg z is $\frac{2\pi}{3}$.

23. From the image, the midpoint of line joining foci is the centre C(0,0)



Transverse axis is y-axis

$AA' = 2a \Rightarrow 2a = 8$

$SS' = 2c = 12, c = 6$

$a = 4$

$b^2 = c^2 - a^2 = 36 - 16 = 20$

Hence the equation of the required hyperbola is

$\frac{y^2}{16} - \frac{x^2}{20} = 1$

24. Comparing the given equation of the plane with $\vec{r}\cdot\vec{n} = p$, we have $\vec{n} = 6\hat{i} - 3\hat{j} + 2\hat{k}$.

We know that the perpendicular distance from the given point with position vector $\vec{u}$ to the plane $\vec{r}\cdot\vec{n} = p$ is given by $\delta = \frac{|\vec{u}\cdot\vec{n} - p|}{|\vec{n}|}$. Therefore, substituting $\vec{u} = (2, 5, -3) = 2\hat{i} + 5\hat{j} - 3\hat{k}$ and $\vec{n} = 6\hat{i} - 3\hat{j} + 2\hat{k}$ in the formula, we get

$\delta = \frac{|\vec{u}\cdot\vec{n} - p|}{|\vec{n}|} = \frac{|(2\hat{i} + 5\hat{j} - 3\hat{k})\cdot(6\hat{i} - 3\hat{j} + 2\hat{k}) - 5|}{|6\hat{i} - 3\hat{j} + 2\hat{k}|}$

= 2 units.

25. Since $f(x) = x^2 - 2x - 3$, $f'(x) = 2x - 2 > 0$ $\forall x \in (2, \infty)$. Hence $f(x)$ is strictly increasing in $(2, \infty)$.

26. Let x be the number

Let $y = f(x) = x^{\frac{1}{n}}$

Then $\log y = \frac{1}{n}\log x$

Taking differential on both sides we get,

$\frac{1}{y}dy = \frac{1}{n}\times\frac{1}{x}dx$

i.e. $\frac{\Delta y}{y} \simeq \frac{dy}{y} = \frac{1}{n}.\frac{dx}{x}$

$$\therefore \frac{\Delta y}{y} \times 100 \simeq \frac{1}{n}\left(\frac{dx}{x} \times 100\right)$$

$\simeq \frac{1}{n}$ times the percentage error in the number.

Hence, percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number.

27. $= \frac{5!}{3^6}\left[\because \int_0^\infty x^n e^{-ax}dx = \frac{n!}{a^{n+1}}, \text{Here } n = 5, a = 3\right]$

28. Let V be the volume of the spherical rain drop and A be its surface area; and r be its radius.

$$\therefore V = \frac{4}{3}\pi r^3 \text{ and}$$

$$A = 4\pi r^2$$

$$\text{Given } \frac{dV}{dt} = -kA$$

[∵ the rain drop evaporates]

$$\Rightarrow \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = -k(4\pi r^2)$$

$$\Rightarrow \frac{4}{3}\pi.\frac{d}{dt}(r^3) = -4k\pi r^2$$

$$\Rightarrow \frac{\cancel{4}}{\cancel{3}}.\cancel{\pi}.\cancel{3}\cancel{r^2}.\frac{dr}{dt} = -\cancel{4}k\cancel{\pi}\cancel{r^2}$$

$$\Rightarrow \frac{dr}{dt} = -k$$

Which is the required differential equation.

29. Let X be a random variable whose values x are the number of fours.

The sample space S is given in the table.

$$\begin{Bmatrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{Bmatrix}$$

It can also be written as

S = {(i, j)}, where i = 1, 2, 3, 6,....and j = 1, 2, 3, ...6

Therefore X takes on the values of 0, 1, and 2.

We observe that

(i) X = 0, if (i, j) for $i \neq 4, j \neq 4$,

(ii) X = 1, if (1, 4), (2, 4), (3, 4), (5, 4), (6, 4) (4, 1), (4, 2), (4, 3), (4, 5), (4, 6),

(iii) X = 2, if (4, 4),

Therefore,

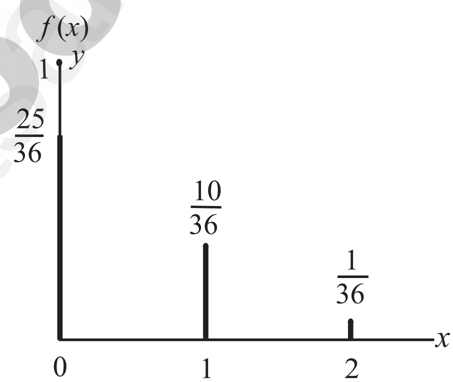
Values of the Random Variable X	0	1	2	Total
Number of elements in inverse images	25	10	1	36

The probabilities are

$$f(0) = P(X = 0) = \frac{25}{36},$$

$$f(1) = P(X = 1) = \frac{10}{36}$$

$$\text{and } f(2) = P(X = 2) = \frac{1}{36}$$



Probability mass function of $f(x)$

Clearly the function $f(x)$ satisfies the conditions

(i) $f(x) \geq 0, x = 0, 1, 2$ and

(ii) $\sum_x f(x) = \sum_{x=0}^{x=2} f(x) = 0 + f(1) + f(2) = 1$

$$= \frac{25}{36} + \frac{10}{36} + \frac{1}{36} = 1$$

The probability mass function is presented as

x	0	1	2
$f(x)$	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

(or)

$$f(x) = \begin{cases} \frac{25}{36} & \text{for } x = 0 \\ \frac{10}{36} & \text{for } x = 1 \\ \frac{1}{36} & \text{for } x = 2 \end{cases}$$

30. Then $A \vee B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0 \vee 1 & 1 \vee 1 \\ 1 \vee 0 & 1 \vee 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A \wedge B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \wedge \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \wedge 1 & 1 \wedge 1 \\ 1 \wedge 0 & 1 \wedge 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}.$$

PART - III

31. The matrix form of the system is

$$= \begin{pmatrix} 2 & 5 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$$

$\Rightarrow$ AX = B where

$$A = \begin{pmatrix} 2 & 5 \\ 1 & 2 \end{pmatrix}, B = \begin{pmatrix} -2 \\ -3 \end{pmatrix},$$

$$X = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow X = A^{-1} B$$

$$|A| = \begin{vmatrix} 2 & 5 \\ 1 & 2 \end{vmatrix} = 4 - 5 = -1 \neq 0.$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{ adj } A$$

$$= \frac{1}{-1} \begin{bmatrix} 2 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}$$

$$\therefore X = A^{-1} B = \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 - 15 \\ -2 + 6 \end{bmatrix} = \begin{bmatrix} -11 \\ 4 \end{bmatrix}$$

$\therefore x = -11, y = 4.$

32. Using $|z_1 + z_2|^2 = (z_1 + z_2)\overline{(z_1 + z_2)}$ $(\because |z|^2 = z\bar{z})$

$\Rightarrow$ $= (z_1 + z_2)(\bar{z}_1 + \bar{z}_2)$

$(\because \overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2)$

$= z_1\bar{z}_1 + (z_1\bar{z}_2 + z_1\bar{z}_2) + z_1\bar{z}_2$

$(\because \bar{\bar{z}} = z)$

$= |z_1|^2 + 2\,\text{Re}\,(\bar{z}_1 \bar{z}_2) + |z_2|^2$

$(\because \text{Re}\,(z) \leq |z|)$

$\leq |z_2|^2 + 2|z_1||z_2| + |z_2|^2$

$(\because |z_1 z_2) = |z_1||z_2|$ and $|z| = |z|)$

$\Rightarrow |z_1 + z_2|^2 \leq (|z_2| + |z_2|)^2$

$\Rightarrow |z_1 + z_2| \leq |z_2| + |z_2|$

33. $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$

$= \sin^{-1}\left(\sin\left(\frac{5\pi}{9} + \frac{\pi}{9}\right)\right)$

$(\because \sin A \cos B + \cos A \sin B = \sin (A + B))$

$= \sin^{-1}\left(\sin\left(\frac{6\pi}{9}\right)\right) = \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$

$= \sin^{-1}\left(\sin\left(\pi - \frac{\pi}{3}\right)\right)$ $[\because \frac{2\pi}{3} \notin \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]]$

$= \sin^{-1}\left(\sin\frac{\pi}{3}\right)$ $(\because \sin(\pi - \theta) = \sin\theta)$

$= \frac{\pi}{3}$ $[\because \frac{\pi}{3} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]]$

34. With usual notations in triangle ABC, we have

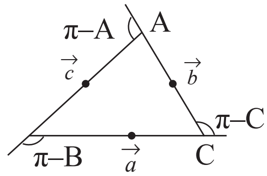
$\overrightarrow{BC} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$

and $\overrightarrow{AB} = \vec{c}$

Then $|\overrightarrow{BC}| = a,$

$|\overrightarrow{CA}| = b,$

and $|\overrightarrow{AB}| = c.$



Since in, ΔABC,

$\overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AB} = \vec{0}$, we have

$\overrightarrow{BC} \times (\overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AB}) = \vec{0}$.

Simplification given,

$\overrightarrow{BC} \times \overrightarrow{CA} = \overrightarrow{AB} \times \overrightarrow{BC}$ ... (1)

Similarly, since $\overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AB} = \vec{0}$,

we have $\overrightarrow{CA} \times (\overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AB}) = \vec{0}$

On simplification, we obtain

$\overrightarrow{BC} \times \overrightarrow{CA} = \overrightarrow{CA} \times \overrightarrow{AB}$

Equations (1) and (2), we get

$$\vec{AB}\times\vec{BC} = \vec{CA}\times\vec{AB} = \vec{BC}\times\vec{CA}.$$

So, $|\vec{AB}\times\vec{BC}| = |\vec{CA}\times\vec{AB}| = |\vec{BC}\times\vec{CA}|$.

Then, we get

$$ca \sin(\pi - B) = bc\sin(\pi - A) = ab\sin(\pi - C).$$

That is,

$$ca\sin B = bc\sin A = ab\sin C.$$

Dividing by abc, leads to

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \text{ or } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Hence it is proved.

35. Let x, y be the two numbers then the sum $x + y = 12$ gives $\Rightarrow y = 12 - x$

Product of the numbers $P = xy = x(12 - x)$

$$p = 12x - x^2$$
$$p' = 12 - 2x$$
$$p'' = -2$$

Substituting $p' = 0$, we get

$$12 - 2x = 0$$
$$2x = 12 \text{ gives}$$
$$x = 6$$

Since $p'' = -2 < 0$, Product P is maximum at $x = 6$. Then $y = 12 - x$, gives $y = 12 - 6 = 6$.

The required two numbers are 6, 6.

36. Given $U(x, y, z) = \log(x^3 + y^3 + z^3)$

$$\frac{\partial U}{\partial x} = \frac{1}{x^3+y^3+z^3}(3x^2);$$

$$\frac{\partial U}{\partial y} = \frac{3y^2}{x^3+y^3+z^3} \text{ and}$$

$$\frac{\partial U}{\partial z} = \frac{3z^2}{x^3+y^3+z^3}$$

$$\therefore \frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z} = \frac{3x^2}{x^3+y^3+z^3} + \frac{3y^2}{x^3+y^3+z^3} + \frac{3z^2}{x^3+y^3+z^3}$$

$$= \frac{3(x^2+y^2+z^2)}{(x^3+y^3+z^3)}$$

37. The parametric equation of tangent at 't_1' to the parabola $y^2 = 4ax$ is $yt_1 = x + at_1^2$ ...(1)

Also, the parametric equation of tangent at 't_2' to the parabola $y^2 = 4ax$ is $yt_2 = x + at_2^2$ ...(2)

$$(1) \rightarrow yt_1 = x + at_1^2$$
$$\quad (-) \quad (-) \quad (-)$$
$$(2) \rightarrow yt_2 = x + at_2^2$$

$$(1) - (2) \quad y(t_1 - t_2) = a(t_1^2 - t_2^2)$$

$$\Rightarrow y(t_1 - t_2) = a(t_1 + t_2)(t_1 - t_2)$$

$$\Rightarrow y = a(t_1 + t_2)$$

Substituting $y = a(t_1 + t_2)$ in (1) we get,

$$a(t_1 + t_2)t_1 = x + at_1^2$$

$$\Rightarrow at_1^2 + at_1t_2 = x + at_1^2$$

$$\Rightarrow x = at_1t_2$$

Hence, the point of intersection of two tangents is $[at_1t_2, a(t_1 + t_2)]$

38. Let P be denote the population of a city.

Given that $\frac{dP}{dt} \propto P$

$$\Rightarrow kP = \frac{dP}{dt} \Rightarrow \frac{dP}{P} = kdt$$

$$\Rightarrow \int\frac{dP}{P} = k\int dt \Rightarrow \log P = kt + \log c$$

$$\Rightarrow \log\left(\frac{P}{c}\right) = kt$$

$$\Rightarrow \frac{P}{c} = e^{kt}$$

$$\Rightarrow P = c.e^{kt} \quad \text{... (1)}$$

Given when t = 0, P = 3,00,000

$$\therefore (1) \rightarrow 3{,}00{,}000 = ce^0 \Rightarrow c = 3{,}00{,}000$$

$$\therefore P = 3{,}00{,}000\, e^{kt} \quad \text{... (2)}$$

Again when t = 40, P = 4,00,000

$$\therefore (2) \Rightarrow 4{,}00{,}000 = 3{,}00{,}000\, e^{40k}$$

$$\Rightarrow \frac{4}{3} = e^{40k} \Rightarrow \log\left(\frac{4}{3}\right) = 40\text{ k}$$

$$\Rightarrow k = \frac{1}{40}\log\left(\frac{4}{3}\right)$$

$$\Rightarrow k = \log\left(\frac{4}{3}\right)^{\frac{1}{40}} \quad \text{... (3)}$$

$$\therefore (2) \text{ becomes, } P = 3{,}00{,}000\, e^{\log\left(\frac{4}{3}\right)^{\frac{1}{40}t}}$$

$$\Rightarrow P = 3{,}00{,}000\left(\frac{4}{3}\right)^{\frac{t}{40}}$$

39.

p	q	$p \to q$	$q \to p$	$p \leftrightarrow q$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

The entries in the columns corresponding to $p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$ are identical and hence they are equivalent.

40. Clearly there are 2 sign changes for the given polynomial P(x) and hence number of positive roots of P(x) cannot be more than two. Further, as $P(-x) = -9x^9 - 2x^5 - x^4 - 7x^2 + 2$, there is one sign change for P($-x$)and hence the number of negative roots cannot be more than one. Clearly 0 is not a root. So maximum number of real roots is 3 and hence there are atleast six imaginary roots.

PART - IV

41. (a) First we evaluate the determinants

$$\Delta = \begin{vmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{vmatrix} = 6 \neq 0,$$

$$\Delta_1 = \begin{vmatrix} 3 & -1 & 0 \\ 17 & 3 & 4 \\ 7 & 1 & 2 \end{vmatrix} = 12,$$

$$\Delta_2 = \begin{vmatrix} 1 & 3 & 0 \\ 2 & 17 & 4 \\ 0 & 7 & 2 \end{vmatrix} = -6,$$

$$\Delta_3 = \begin{vmatrix} 1 & -1 & 3 \\ 2 & 3 & 17 \\ 0 & 1 & 7 \end{vmatrix} = 24$$

By Cramer's rule, we get $x_1 = \dfrac{\Delta_1}{\Delta} = \dfrac{12}{6}$, $x_2 = \dfrac{\Delta_2}{\Delta} = \dfrac{-6}{6} = -1$, $x_3 = \dfrac{24}{6} = 4$.

So, the solution is ($x_1 = 2, x_2 = -1, x_3 = 4$).

OR

(b) Now, $\tan^{-1}\left(\dfrac{x-1}{x-2}\right) + \tan^{-1}\left(\dfrac{x+1}{x+2}\right)$

$$= \tan^{-1}\left|\frac{\dfrac{x-1}{x-2} + \dfrac{x+1}{x+2}}{1 - \dfrac{x-1}{x-2}\left(\dfrac{x+1}{x+2}\right)}\right| = \frac{\pi}{4}$$

Thus, $\dfrac{\dfrac{x-1}{x-2} + \dfrac{x+1}{x+2}}{1 - \dfrac{x-1}{x-2}\left(\dfrac{x+1}{x+2}\right)} = 1,$

Which on simplification gives $2x^2 - 4 = -3$

Thus, $x^2 = \dfrac{1}{2}$ gives $x = \pm \dfrac{1}{\sqrt{2}}$.

42.(a) Given $z = x + iy$ and $\arg\left(\dfrac{z-i}{z+2}\right) = \dfrac{\pi}{4}$

$\Rightarrow \arg(z - i) - \arg(z + 2) = \dfrac{\pi}{4}$

$\Rightarrow \arg(x + iy - i) - \arg(x + iy + 2) = \dfrac{\pi}{4}$

$\Rightarrow \arg(x + i(y - 1)) - \arg((x + 2) + iy) = \dfrac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\dfrac{y-1}{x}\right) - \tan^{-1}\left(\dfrac{y}{x+2}\right) = \dfrac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\dfrac{\dfrac{y-1}{x} - \dfrac{y}{x+2}}{1 + \dfrac{y-1}{x} \cdot \dfrac{y}{x+2}}\right) = \dfrac{\pi}{4}$

$\left[\because \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\dfrac{x-y}{1+xy}\right)\right]$

$\Rightarrow \dfrac{\left(\dfrac{(x+2)(y-1) - xy}{x(x+2)}\right)}{\left(\dfrac{x(x+2) + y(y-1)}{x(x+2)}\right)} = \tan\dfrac{\pi}{4} = 1$

$\Rightarrow \dfrac{(x+2)(y-1) - xy}{x(x+2) + y(y-1)} = 1$

$\Rightarrow -x + 2y - 2 = x^2 + 2x + y^2 - y$

$\Rightarrow x^2 + 2x + y^2 - y + x - 2y + 2 = 0$

$\Rightarrow x^2 + y^2 + 3x - 3y + 2 = 0$

Hence proved.

OR

(b) Note that the function u is not homogeneous. So we cannot apply Euler's Theorem for u. However, note that $f(x, y) = \dfrac{x+y}{\sqrt{x} + \sqrt{y}}$ = sin u is homogeneous; because

$$f(tx, ty) = \frac{tx + ty}{\sqrt{tx} + \sqrt{ty}} = t^{\frac{1}{2}} f(x, y), \forall x, y, t \geq 0.$$

Thus f is homogeneous with degree $\dfrac{1}{2}$, and so by Euler's Theorem we have

$$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y} = \frac{1}{2}f(x,y)$$

Now substituting $f = \sin u$ in the above equation, we obtain

$$x\frac{\partial(\sin u)}{\partial x}+y\frac{\partial(\sin u)}{\partial y} = \frac{1}{2}\sin u$$

$$x\cos u\frac{\partial u}{\partial x}+y\cos u\frac{\partial u}{\partial y} = \frac{1}{2}\sin u$$

Dividing both sides by $\cos u$ we obtain

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u.$$

Note: Solving this problem by direct calculation will be possible; but will involve lengthy calculations.

43. (a) This is an indeterminate of the form 1^{∞}

Let $g(x) = (\sin x)^{\tan x}$

Taking logarithm, we get

$\log(g(x)) = \log(\sin x)^{\tan x}$

$$= \tan x\,.\log(\sin x) = \frac{\log(\sin x)}{\cot x}$$

Now, $$\lim_{x\to\frac{\pi}{2}}\log(g(x)) = \lim_{x\to\frac{\pi}{2}}\frac{\log(\sin x)}{\cot x} = \left(\frac{0}{0}\text{ form}\right)$$

$$= \frac{\lim\limits_{x\to\frac{\pi}{2}}\frac{1}{\sin x}\times\cos x}{-\text{cosec}^2 x} \quad \text{[By L' Hôpital rule]}$$

$$= \lim_{x\to\frac{\pi}{2}}-\frac{\cos x}{\sin x\cdot\frac{1}{\sin^2 x}} = \lim_{x\to\frac{\pi}{2}}-\cos x\sin x = 0$$

But $$\lim_{x\to\frac{\pi}{2}}\log(g(x)) = \log\left(\lim_{x\to\frac{\pi}{2}}g(x)\right)$$

$$\therefore \log\left(\lim_{x\to\frac{\pi}{2}}g(x)\right) = 0$$

$$\Rightarrow e^{\log\left(\lim\limits_{x\to\frac{\pi}{2}}g(x)\right)} = e^0$$

$$\Rightarrow \lim_{x\to\frac{\pi}{2}}g(x) = 1 \Rightarrow \lim_{x\to\frac{\pi}{2}}(\sin x)^{\tan x} = 1$$

OR

(b) Given $\vec{a} = 2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b} = 3\hat{i}+5\hat{j}+2\hat{k}$ and $\vec{c} = -\hat{i}-2\hat{j}+3\hat{k}$

LHS = $(\vec{a}\times\vec{b})\times\vec{c}$

Consider $$(\vec{a}\times\vec{b}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & -1\\ 3 & 5 & 2\end{vmatrix}$$

$$= \hat{i}\begin{vmatrix}3 & -1\\ 5 & 2\end{vmatrix}-\hat{j}\begin{vmatrix}2 & -1\\ 3 & 2\end{vmatrix}+\hat{k}\begin{vmatrix}2 & 3\\ 3 & 5\end{vmatrix}$$

$$= \hat{i}(6+5)-\hat{j}(4+3)+\hat{k}(10-9)$$

$$= 11\hat{i}-7\hat{j}+\hat{k}$$

$$\therefore \text{ LHS} = (\vec{a}\times\vec{b})\times\vec{c} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 11 & -7 & 1\\ -1 & -2 & 3\end{vmatrix}$$

$$= \hat{i}\begin{vmatrix}-7 & 1\\ -2 & 3\end{vmatrix}-\hat{j}\begin{vmatrix}11 & 1\\ -1 & 3\end{vmatrix}+\hat{k}\begin{vmatrix}11 & -7\\ -1 & -2\end{vmatrix}$$

$$= \hat{i}(-21+2)-\hat{j}(33+1)+\hat{k}(-22-7)$$

$$= -19\hat{i}-34\hat{j}-29\hat{k} \quad \text{...(1)}$$

For RHS $\vec{a}\cdot\vec{c} = (2\hat{i}+3\hat{j}-\hat{k})\cdot(-\hat{i}-2\hat{j}+3\hat{k})$

$= -2-6-3 = -11$

$\vec{b}\cdot\vec{c} = (3\hat{i}+5\hat{j}+2\hat{k})\cdot(-\hat{i}-2\hat{j}+3\hat{k})$

$= -3-10+6 = -7$

$\therefore$ RHS $= (\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a}$

$= -11(3\hat{i}+5\hat{j}+2\hat{k})+7(2\hat{i}+3\hat{j}-\hat{k})$

$= -33\hat{i}-55\hat{j}-22\hat{k}+14\hat{i}+21\hat{j}-7\hat{k}$

$= -19\hat{i}-34\hat{j}-29\hat{k}$ ...(2)

From (1) & (2), LHS = RHS

Hence $(\vec{a}\times\vec{b})\times\vec{c} = (\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a}$

44. (a) We observe that the sum of the coefficients of the odd powers and that of the even powers are equal. Hence -1 is a root of the equation.

To find other roots, we divide $2x^3+11x^2-9x-18$ by $x+1$ and get $2x^2+9x-18$ as the quotient.

Solving this we get $\frac{3}{2}$ and -6 as roots.

Thus $-6, -1, \frac{3}{2}$, are the roots or solutions of the given equation.

OR

(b) Given that $(1 + x^2)\dfrac{dy}{dx} = 1 + y^2$ ... (1)

The given equation is written in the variables separable form

$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2} \quad \text{... (2)}$$

Integrating both sides of (2), we get $\tan^{-1} y = \tan^{-1} x + C$ ... (3)

$$\text{But } \tan^{-1} y - \tan^{-1} x = \tan^{-1}\left(\frac{y-x}{1+xy}\right) \quad \text{... (4)}$$

Using (4) in (3) leads to $\tan^{-1}\left(\dfrac{y-x}{1+xy}\right) = C$, which implies $\dfrac{y-x}{1+xy} = \tan C = a$ (say).

Thus, $y - x = a(1 + xy)$ gives the required solution.

45. (a) Let the equation of the circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \text{... (1)}$$

It passes through (1, 0)

$\Rightarrow$ $1^2 + 0 + 2g(1) + 2f(0) + c = 0$

$\Rightarrow$ $2g + c = -1$ ...(2)

It passes through (–1, 0)

$\Rightarrow$ $(-1)^2 + 0 + 2g(-1) + 2f(0) + c = 0$

$\Rightarrow$ $-2g + c = -1$ ...(3)

Also it passes through (0, 1)

$\Rightarrow$ $0 + 1^2 + 2g(0) + 2f(1) + c = 0$

$\Rightarrow$ $2f + c = -1$ ...(4)

(2) + (3) $\Rightarrow$ $2c = -2$

$\Rightarrow$ $c = -1$

Substituting $c = -1$ in (2), we get

$2g - 1 = -1$

$\Rightarrow$ $2g = 0$

$\Rightarrow$ $g = 0$

Substituting $c = -1$ in (4) we get,

$2f - 1 = -1$

$\Rightarrow$ $2f = 0$

$\Rightarrow$ $f = 0$

$\therefore$ (1) becomes,

$x^2 + y^2 + 0 + 0 - 1 = 0$

$\Rightarrow$ $x^2 + y^2 = 1$

OR

(b) To find the probabilities, the values of the parameters n and p must be known.

Given that Mean $= np = 2$ and variance $= npq = 1.5$

This gives $\frac{npq}{np} = \frac{1.5}{2} = \frac{3}{4}$

$$q = \frac{3}{4} \text{ and } p = 1 - q = 1 - \frac{3}{4} = \frac{1}{4}$$

$$np = 2, \text{ gives } n = \frac{2}{p} = 8.$$

$$\text{Therefore X} \sim B\left(8, \frac{1}{4}\right)$$

Therefore the binomial distribution is

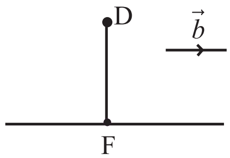
$$P(X = x) = f(x) = \binom{8}{x}\left(\frac{1}{4}\right)^x\left(\frac{3}{4}\right)^{8-x} \quad x = 0, 1, 2, \ldots 8$$

(i) $$P(X = 0) = f(0) = \binom{8}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{8-0} = \left(\frac{3}{4}\right)^8$$

(ii) $$P(X = 1) = f(1) = \binom{8}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^{8-1} = 2\left(\frac{3}{4}\right)^7$$

(iii) $$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0) = 1 - \left(\frac{3}{4}\right)^8$$

46. (a) Given equation of line is



$$\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1} \Rightarrow \text{A is } (-1, 3, 1) \text{ and}$$

$$\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$$

$$\text{Let } \frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1} = t$$

If F is the foot of the perpendicular from D to the line, then it is of the form $(2t - 1, 3t + 3, -t + 1)$ ...(1)

$\Rightarrow$ $$\overrightarrow{OF} = (2t-1)\hat{i} + (3t+3)\hat{j} + (-t+1)\hat{k}$$

Given point is D(5, 4, 2)

$\Rightarrow$ $$\overrightarrow{OD} = 5\hat{i} + 4\hat{j} + 2\hat{k}$$

$\therefore$ $\overrightarrow{DF} = \overrightarrow{OF} - \overrightarrow{OD}$

$$= (2t-1-5)\hat{i} + (3t+3-4)\hat{j} + (-t+1-2)\hat{k} \quad ...(2)$$

Since $\vec{b} \perp \overrightarrow{DF}$, we have

$$\vec{b} \cdot \overrightarrow{DF} = 0$$

$\Rightarrow$ $2(2t - 6) + 3(3t - 1) - 1(-t - 1) = 0$

$\Rightarrow$ $4t - 12 + 9t - 3 + t + 1 = 0$

$\Rightarrow$ $14t - 14 = 0$

$\Rightarrow$ $14t = 14$

$\Rightarrow \quad t = 1$

From (1), F is (2(1) – 1, 3(1) + 3, –1 + 1) = (1, 6, 0)

∴ The foot of the perpendicular is (1, 6, 0)

∴ Equation of the perpendicular DF is the equation of the line passing through two points (5, 4, 2) and (1, 6, 0). Its Cartesian equation is

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\Rightarrow \quad \frac{x-5}{1-5} = \frac{y-4}{6-4} = \frac{z-2}{0-2}$$

[∵ (x_1, y_1, z_1) is (5, 4, 2) (x_2, y_2, z_2) is (1, 6, 0)]

$$\Rightarrow \quad \frac{x-5}{-4} = \frac{y-4}{2} = \frac{z-2}{-2}$$

is the equation of required perpendicular.

OR

(b) The lines $5x - 2y = 15$, $x + y$ +4, intersect at (1, –5). The line $5x - 2y = 15$ meets the x-axis at(3, 0). The line $x + y + 4 = 0$ meets the x-axis at (–4, 0). The required area is shaded in the figure. It lies below the x-axis. It can be computed either by considering vertical strips or horizontal strips.

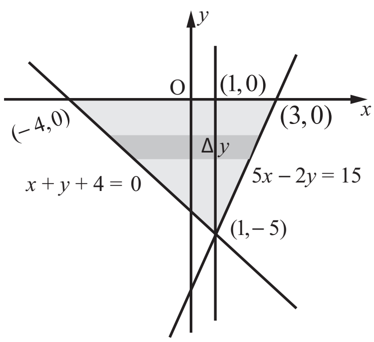
When we do by vertical strips, the region has to be divided into two sub-regions by the line x =1. Then, we get

$$A = \left|\int_{-4}^{1} y\,dx\right| + \left|\int_{1}^{3} y\,dx\right| = \left|\int_{-4}^{1} (-4-x)\,dx\right| + \left|\int_{1}^{3}\left(\frac{5x-15}{2}\right)dx\right|$$

$$= \left|\left(-4x - \frac{x^2}{2}\right)_{-4}^{1}\right| + \left|\left(\frac{5x^2}{4} - \frac{15x}{2}\right)_{1}^{3}\right|$$

$$= \left|\left(-\frac{9}{2}\right) - (8)\right| + \left|\left(-\frac{45}{4}\right) - \left(-\frac{25}{4}\right)\right| = \frac{25}{2} + 5 = \frac{35}{2}$$

When we do by horizontal strips, there is no need to subdivide the region. In this case, the area is bounded on the right by the line $5x - 2y = 15$ and on the left by $x + y + 4 = 0$. So, we get



$$A = \int_{-5}^{0} [x_R - x_L]\, dy = \int_{-5}^{0} \left[\frac{15+2y}{5} - (-4-y)\right] dy$$

$$= \int_{-5}^{0} \left[7 + \frac{7y}{5}\right] dy = \left[7y + \frac{7y^2}{10}\right]_{-5}^{0}$$

$$= 0 - \left[-35 + \frac{35}{2}\right] = \frac{35}{2}$$

Note : The region is triangular with base 7 units and height 5 units. Hence its area is $\frac{35}{2}$ without using integration.

47. (a) Given $s(t) = 2t^3 - 9t^2 + 12t - 4$, $t \geq 0$.

(i) On differentiating we get,

$$V(t) = 6t^2 - 18t + 12 \quad \text{... (1)}$$
$$= 6(t^2 - 3t + 2)$$
$$= 6(t-1)(t-2)$$

Now $V(t) = 0$

$\Rightarrow$ $6(t-1)(t-2) = 0$

$\Rightarrow$ $t = 1, 2$

The particle changes direction when V(t) changes its sign.

If $0 \leq t < 1$ then both $(t-1)$ and $(t-2) < 0$

$\Rightarrow$ $V(t) > 0$

If $1 < t < 2$ then $(t-1) > 0$ and $(t-2) < 0$

$\Rightarrow$ $V(t) < 0$

If $t > 2$ then both $(t-1)$ and $(t-2) > 0$

$\Rightarrow$ $V(t) > 0$

$\therefore$ The particle changes direction when $t = 1$ and $t = 2$ sec.

(ii) Total distance travelled by the particle in the first 4 seconds is $|s(0) - s(1)| + |s(1) - s(2)| + |s(2) - s(4)|$

$s(0) = -4$

$s(1) = 2(1)^3 - 9(1)^2 + 12(1) - 4 = 2 - 9 + 12 - 4 = 1$

$s(2) = 2 \times 2^3 - 9 \times 2^2 + 12 \times 2 - 4 = 16 - 36 + 24 - 4 = 0$

$s(4) = 2(4)^3 - 9(4)^2 + 12(4) - 4$

$= 128 - 144 + 48 - 4 = 28$

$\therefore |s(0) - s(1)| + |s(1) - s(2)| + |s(2) - s(4)| = |-4-1| + |1-0| + |0-28|$

$= |-5| + |1| + |-28|$

$= 5 + 1 + 28 = 34$ m

(iii) Acceleration $A = 12t - 18$ $\left[\text{acceleration} = \frac{dv}{dt}\right]$

When $t = 1$,

Acceleration $= 12(1) - 18 = -6$ m/sec^2

When $t = 2$

Acceleration $= 12(2) - 18 = 6$ m/sec^2

OR

(b) Given $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in \mathbb{R} - \{0\}\right\}$ and * be the matrix multiplication.

Let A $= \begin{pmatrix} x & x \\ x & x \end{pmatrix}$ and B $= \begin{pmatrix} y & y \\ y & y \end{pmatrix} \in M$

where $x, y \in \mathbb{R} - \{0\}$.

$$A * B = \begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} y & y \\ y & y \end{pmatrix}$$

$$= \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} \in M \qquad [\because 2xy \in \mathbb{R} - \{0\}]$$

∴ M is closed under *.

Commutative property:

$$\text{We know } A * B = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} \quad ...(1)$$

Let $x, y \in \mathbb{R} - \{0\}$

$$\text{Now } B * A = \begin{pmatrix} y & y \\ y & y \end{pmatrix}\begin{pmatrix} x & x \\ x & x \end{pmatrix} = \begin{pmatrix} xy+xy & xy+xy \\ xy+xy & xy+xy \end{pmatrix}$$

$$= \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix}$$

From (1) & (2), A * B = B * A

∴ * has commutative property on M.

Associative property:

Let A $= \begin{pmatrix} x & x \\ x & x \end{pmatrix}$,

B $= \begin{pmatrix} y & y \\ y & y \end{pmatrix}$ and

C $= \begin{pmatrix} z & z \\ z & z \end{pmatrix}$

for $x, y, z \in \mathbb{R} - \{0\}$

$$(A * B) * C = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} * \begin{pmatrix} z & z \\ z & z \end{pmatrix}$$

$$= \begin{pmatrix} 2xyz+2xyz & 2xyz+2xyz \\ 2xyz+2xyz & 2xyz+2xyz \end{pmatrix}$$

$$= \begin{pmatrix} 4xyz & 4xyz \\ 4xyz & 4xyz \end{pmatrix} \quad ...(1)$$

$$\text{Now } A * (B * C) = A * \begin{pmatrix} 2yz & 2yz \\ 2yz & 2yz \end{pmatrix}$$

$$= \begin{pmatrix} x & x \\ x & x \end{pmatrix} * \begin{pmatrix} 2yz & 2yz \\ 2yz & 2yz \end{pmatrix}$$

$$= \begin{pmatrix} 4xyz & 4xyz \\ 4xyz & 4xyz \end{pmatrix} \quad ...(2)$$

From (1) & (2), (A * B) * C = A * (B * C)

∴ * has associative property on M.

(ii) **(1) Closure**

$$\text{Let A} = \begin{pmatrix} x & x \\ x & x \end{pmatrix} \text{ and}$$

$$\text{B} = \begin{pmatrix} y & y \\ y & y \end{pmatrix} : x, y \in \mathbb{R} - (0).$$

$$\text{Now, AB} = \begin{pmatrix} x & x \\ x & x \end{pmatrix} \begin{pmatrix} y & y \\ y & y \end{pmatrix}$$

$$= \begin{pmatrix} xy+xy & xy+xy \\ xy+xy & xy+xy \end{pmatrix} = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} \in \text{M}.$$

Since, $x, y \in \mathbb{R} - (0)$ gives xy also $y \in \mathbb{R} - (0)$

So, AB $y \in$ M $\Rightarrow$ A * B $\in$ M

∴ * is closed on M.

(2) Existence of Identity :

$$\text{Let A} = \begin{pmatrix} x & x \\ x & x \end{pmatrix} \text{ and}$$

$$\text{E} = \begin{pmatrix} e & e \\ e & e \end{pmatrix} \text{ be the identity,such that : } a, e \in \mathbb{R} - (0).$$

Hence M = (A, E)

$$\text{Now, A * E} = \text{E * A} = \text{A}$$

$$\begin{pmatrix} x & x \\ x & x \end{pmatrix} \begin{pmatrix} e & e \\ e & e \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$$

$$\begin{pmatrix} 2xe & 2xe \\ 2xe & 2xe \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$$

$$2xe = x$$

$$2e = 1$$

$$e = \frac{1}{2} \in \mathbb{R} - (0)$$

$$\therefore \text{E} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \text{ is the identity} \in \text{M}$$

∴ * has identity on M.

(3) **Existence of Inverse :**

$$\text{Let } A = \begin{pmatrix} x & x \\ x & x \end{pmatrix} \text{ and}$$

$$A^{-1} = \begin{pmatrix} x^{-1} & x^{-1} \\ x^{-1} & x^{-1} \end{pmatrix} \text{ be the inverse of A.}$$

Then * $A * A^{-1} = A^{-1} * A = E$

$$\begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} x^{-1} & x^{-1} \\ x^{-1} & x^{-1} \end{pmatrix} = \begin{pmatrix} e & e \\ e & e \end{pmatrix}$$

$$\begin{pmatrix} 2xx^{-1} & 2xx^{-1} \\ 2xx^{-1} & 2xx^{-1} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$2xx^{-1} = \frac{1}{2}$$

$$x^{-1} = \frac{1}{4x}, \in \mathbb{R} - (0)$$

$$e = \frac{1}{2} \in \mathbb{R} - (0)$$

$$\therefore A^{-1} = \begin{pmatrix} \frac{1}{4x} & \frac{1}{4x} \\ \frac{1}{4x} & \frac{1}{4x} \end{pmatrix} \text{ is the inverse of } A \in M$$

$\therefore$ * has inverse on M.

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