ST. ANNE'S ACADEMY

(MATHS & PHYSICS TUITION CENTRE)

I FLOOR, JAFRO DENTAL CLINIC, HOLY CROSS COLLEGE ROAD, PUNNAI NAGAR, NAGERCOIL – 629004

Unit Test (Ch 2)

CLASS – XII - MATHEMATICS

Time Allowed : 2 Hrs Marks : 60

I. Answer ALL questions. **6x2 = 12**

1) Find the inverse (if it exists) of the following:

$$\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$$

2) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it to a row-echelon form.

3) State Rouché - Capelli theorem.

4) If A is a non-singular matrix of odd order, prove that $|\text{adj } A|$ is positive.

5) If $\text{adj}(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}.

6) If $A^T A^{-1}$ is symmetric, then show that, $A^2 = (A^T)^2$

II. Answer ALL questions. **6x3 = 18**

7) Verify the property $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$.

8) Find the inverse of the non-singular matrix $A = \begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$, by Gauss-Jordan method.

9) Four men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

10) Solve, by Cramer's rule, the system of equations
$x_1 - x_2 = 3, 2x_1 + 3x_2 + 4x_3 = 17, x_2 + 2x_3 = 7$.

11) Investigate for what values of λ and μ the system of linear equations
$x + 2y + z = 7,\ x + y + \lambda z = \mu,\ x + 3y - 5z = 5$
has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

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12) If $A=\begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB=I_2$, then, show that $B=\left(\cos^2\frac{\theta}{2}\right)A^T$

III. Answer ALL questions. **6x5 =30**

13) If $A=\frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a, b and c, and hence A^{-1}.

14) If $A=\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x-y+z=4, x-2y-2z=9, 2x+y+3z=1$.

15) The upward speed $v(t)$ of a rocket at time t is approximated by $v(t)=at^2+bt+c,\ 0\le t\le 100$ where $a, b,$ and c are constants. It has been found that the speed at times $t=3, t=6$, and $t=9$ second are, 64, 133, and 208 miles per second respectively. Find the speed at time $t=15$ seconds. (Use Gaussian elimination method.)

16) Find the value of k for which the equations $kx-2y+z=1,\ x-2ky+z=-2,\ x-2y+kz=1$ have (i) no solution (ii) unique solution (iii) infinitely many solution

17) By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8+O_2\rightarrow CO_2+H_2O$.

18) If the system of equations $px+by+cz=0,\ ax+qy+cz=0,\ ax+by+rz=0$ has a non-trivial solution and $p\ne a, q\ne b, r\ne c$, prove that $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}=2$.

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Unit Test (Ch 2)

CLASS – XII - MATHEMATICS

Time Allowed : 2 Hrs | Marks : 60

I. Answer ALL questions. **6x2 = 12**

1) Write $\frac{3+4i}{5-12i}$ in the $x+iy$ form, hence find its real and imaginary parts.

2) Simplify $\left(\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}\right)^{18}$.

3) Prove that, if $z=r(\cos\theta+i\sin\theta)$, then $z^{-1}=\frac{1}{r}(\cos\theta-i\sin\theta)$.

4) If $z=2-2i$, find the rotation of z by θ radians in the counter clockwise direction about the origin when $\theta=\frac{2\pi}{3}$

5) Simplify: $\sum_{n=1}^{10} i^{n+50}$

6) Find the square root of $6-8i$.

II. Answer ALL questions. **6x3 = 18**

7) For any two complex numbers z_1 and z_2, such that $|z_1|=|z_2|=1$ and $z_1z_2\neq-1$, then show that $\frac{z_1+z_2}{1+z_1z_2}$ is a real number.

8) If $2\cos\alpha=x+\frac{1}{x}$ and $2\cos\beta=y+\frac{1}{y}$, show that $x^m y^n+\frac{1}{x^m y^n}=2\cos(m\alpha+n\beta)$.

9) Show that the equation $z^5+2\overline{z}=0$ has five solutions.

10) If $z=x+iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right)=0$, show that the locus of z is $2x^2+2y^2+x-2y=0$.

11) Show that $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5=-\sqrt{3}$.

12) Simplify $\left(\frac{1+\cos2\theta+i\sin2\theta}{1+\cos2\theta-i\sin2\theta}\right)^{30}$.

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III. Answer ALL questions. **6x5 =30**

13) Suppose z_1, z_2, and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$, then find z_2 and z_3.

14) Show that (i) $\left(\frac{19-7i}{9+i}\right)^{12} + \left(\frac{20-5i}{7-6i}\right)^{12}$ is real.

(ii) A complex number z is purely imaginary if and only if $z = -\bar{z}$

15) State and prove Triangle inequality property of modulus of complex numbers.

16) Find all cube roots of $\sqrt{3} + i$.

17) If $\frac{1+z}{1-z} = \cos 2\theta + i \sin 2\theta$, show that $z = i \tan\theta$.

18) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$.

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Unit Test (Chapter 3)
CLASS – XII - MATHEMATICS

Time Allowed : 2 Hrs | Marks : 60

I. Answer ALL questions. **6x2 = 12**

1) If α and β are the roots of the quadratic equation $2x^2-7x+13=0$, construct a quadratic equation whose roots are α^2 and β^2.

2) Solve the equation $3x^3-16x^2+23x-6=0$ if the product of two roots is 1.

3) Find the monic polynomial equation of minimum degree with real coefficients having $2-\sqrt{3}\,i$ as a root.

4) Prove that a straight line and parabola cannot intersect at more than two points.

5) Solve the equation $x^4-9x^2+20=0$.

6) Find solution, if any, of the equation $2\cos^2 x-9\cos x+4=0$.

II. Answer ALL questions. **6x3 = 18**

7) Find the sum of the squares of the roots of $ax^4+bx^3+cx^2+dx+e=0$, $a\neq 0$

8) Show that the equation $x^9-5x^5+4x^4+2x^2+1=0$ has atleast 6 imaginary solutions.

9) Solve : $8x^{\frac{3}{2n}}-8x^{\frac{-3}{2n}}=63$

10) Solve the equations $x^4+3x^3-3x-1=0$

11) If the roots of $x^3+px^2+qx+r=0$ are in H.P., prove that $9pqr=27r^2+2q^3$. Assume $p, q, r\neq 0$

12) Determine k and solve the equation $2x^3-6x^2+3x+k=0$ if one of its roots is twice the sum of the other two roots.

III. Answer ALL questions. **6x5 =30**

13) (i) If the equations $x^2+px+q=0$ and $x^2+p'x+q'=0$ have a common root, show that it must be equal to $\dfrac{pq'-p'q}{q-q'}$ or $\dfrac{q-q'}{p'-p}$.

(ii) A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was left standing.

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14) (i) If α, β, and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \frac{1}{\beta\gamma}$ in terms of the coefficients.

(ii) If p is real, discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$, in terms of p.

15) Solve the following equation: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

16) If $2+i$ and $3-\sqrt{2}$ are roots of the equation $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$, find all roots.

17) Find the condition that the roots of $ax^3 + bx^2 + cx + d = 0$ are in geometric progression. Assume $a, b, c, d \neq 0$

18) Solve the equation $(2x-3)(6x-1)(3x-2)(x-2) - 5 = ($