LONDON SCHOOL

2022-23 ACHIEVERS

JAYASURIYA

100/100

ATCHAYA

99/100

YAZHLINI

99/100

+2 MATHS

(Volume -1)

PRACTICE BOOK

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CHAPTER 1

APPLICATIONS OF MATRICES AND DETERMINANTS

EXERCISE 1.1

2 MARKS

Example 1.2: If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is non-singular, find A^{-1}.

Example 1.4: If A is a non-singular matrix of odd order, prove that $|adj\ A|$ is positive.

Example 1.6: If $adj A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}.

Example1.7: If A is symmetric, prove that then $adj\ A$ is also symmetric.

Example 1.8: Verify the property $(A^T)^{-1} = (A^{-1})^T$, with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$.

Example 1.11: Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

1. Find the adjoint $(i)\begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$
2. Find the inverse $(i)\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$
3. If $adj\ (A) = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$ find A. **(EX 1.1 -8)**

3 MARKS

Example 1.3: Find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$. **(EG 1.3)**

Example 1.5: Find a matrix A if $adj A = \begin{bmatrix} 7 & 7 & -7 \\ -1 & 11 & 7 \\ 11 & 5 & 7 \end{bmatrix}$. **(EG 1.5)**

Example 1.9: Verify $(AB)^{-1} = B^{-1}A^{-1}$ with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$. **(EG 1.9)**

Example 1.10: If $= \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2 + xA + yI_2 = 0_2$. Hence, find A^{-1}. **(EG 1.10)**

Example 1.12: If $A = \frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a, b and c, and hence A^{-1}. **(EG1.12)**

1. Find the adjoint of the following:
   $(ii)\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$ $(iii)\frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$ **(EX 1.1 - 1)**
2. Find the inverse (if it exists) of the following:
   $(ii)\begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ $(iii)\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$ **(EX 1.1 -2)**

3. If $F(\alpha)=\begin{bmatrix}\cos\alpha & 0 & \sin\alpha\\ 0 & 1 & 0\\ \sin\alpha & 0 & \cos\alpha\end{bmatrix}$, show that $[F(\alpha)]^{-1}=F(-\alpha)$. **(EX 1.1 -3)**
4. If $A=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$, show that $A^2-3A-7I_2=0_2$. Hence find A^{-1}. **(EX 1.1 -4)**
5. If $A=\frac{1}{9}\begin{bmatrix}-8 & 1 & 4\\ 4 & 4 & 7\\ 1 & -8 & 4\end{bmatrix}$, prove that $A^{-1}=A^T$. **(EX 1.1 -5)**
6. If $A=\begin{bmatrix}8 & -4\\ -5 & 3\end{bmatrix}$, verify that $A(adj\ A)=(adj\ A)A=|A|I_2$. **(EX 1.1 -6)**
7. If $A=\begin{bmatrix}3 & 2\\ 7 & 5\end{bmatrix}$ and $B=\begin{bmatrix}-1 & -3\\ 5 & 2\end{bmatrix}$, verify that $(AB)^{-1}=B^{-1}A^{-1}$. **(EX 1.1 -7)**
8. If $adj\ (A)=\begin{bmatrix}0 & -2 & 0\\ 6 & 2 & -6\\ -3 & 0 & 6\end{bmatrix}$ find A^{-1}. **(EX 1.1 -9)**
9. $A=\begin{bmatrix}1 & \tan x\\ -\tan x & 1\end{bmatrix}$, show that $A^TA^{-1}=\begin{bmatrix}\cos 2x & -\sin 2x\\ \sin 2x & \cos 2x\end{bmatrix}$. **(EX 1.1 -11)**
10. Find the matrix A for which $A\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}=\begin{bmatrix}14 & 7\\ 7 & 7\end{bmatrix}$. **(EX 1.1 -12)**
11. Given $A=\begin{bmatrix}1 & -1\\ 2 & 0\end{bmatrix}$, $B=\begin{bmatrix}3 & -2\\ 1 & 1\end{bmatrix}$ and $C=\begin{bmatrix}1 & 1\\ 2 & 2\end{bmatrix}$ find a matrix X such that $AXB=C$. **(EX 1.1 -13)**
12. If $A=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$, show that $A^{-1}=\frac{1}{2}(A^2-3I)$. **(EX 1.1 -14)**
13. Decrypt the received encoded message $[2\quad -3][20\quad 4]$ with the encryption matrix $\begin{bmatrix}-1 & -1\\ 2 & 1\end{bmatrix}$ and the decryption matrix as its inverse, where the system of codes are described by the numbers $1-26$ to the letters $A-Z$ respectively, and the number 0 to a blank space. **(EX 1.1 -15)**

5 MARKS

Example 1.1 : If $A=\begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{bmatrix}$, verify that $A(adj\ A)=(adj\ A)A=|A|I_3$. **(EG 1.1)**

1. Find $adj(adj\ (A))$ if $(adj\ (A))=\begin{bmatrix}1 & 0 & 1\\ 0 & 2 & 0\\ -1 & 0 & 1\end{bmatrix}$. **(EX 1.1 -10)**

EXERCISE 1.2

2 MARKS

Example 1.16: Find the rank of the following matrices which are in row-echelon form :

$(i)\begin{bmatrix}2 & 0 & -7\\ 0 & 3 & 1\\ 0 & 0 & 1\end{bmatrix}$ $(ii)\begin{bmatrix}-2 & 2 & 1\\ 0 & 5 & 1\\ 0 & 0 & 0\end{bmatrix}$ $(iii)\begin{bmatrix}6 & 0 & -9\\ 0 & 2 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$ **(EG 1.16)**

1. Find the rank of the following matrices by minor method:

$(i)\begin{bmatrix}2 & -4\\ -1 & 2\end{bmatrix}$ $(ii)\begin{bmatrix}-1 & 3\\ 4 & -7\\ 3 & -4\end{bmatrix}$ $(iii)\begin{bmatrix}1 & -2 & -10\\ 3 & -6 & -31\end{bmatrix}$

$(iv)\begin{bmatrix}1 & -2 & 3\\2 & 4 & -6\\5 & 1 & -1\end{bmatrix}$ $(v)\begin{bmatrix}0 & 1 & 2 & 1\\0 & 2 & 4 & 3\\8 & 1 & 0 & 2\end{bmatrix}$ **(EX 1.2 - 1)**

2. Find the inverse of (i) $\begin{bmatrix}2 & -1\\5 & -2\end{bmatrix}$ by Gauss-Jordan method **(EX 1.2 - 3)**

3 MARKS

Example 1.13: Reduce the matrix $\begin{bmatrix}3 & -1 & 2\\-6 & 2 & 4\\-3 & 1 & 2\end{bmatrix}$ to a row-echelon form. **(EG 1.13)**

Example 1.14: Reduce the matrix $\begin{bmatrix}0 & 3 & 1 & 6\\-1 & 0 & 2 & 5\\4 & 2 & 0 & 0\end{bmatrix}$ to row-echelon form. **(EG 1.14)**

Example 1.15: Find the rank of each of the following matrices: $(i)\begin{bmatrix}3 & 2 & 5\\1 & 1 & 2\\3 & 3 & 6\end{bmatrix}$ $(ii)\begin{bmatrix}4 & 3 & 1 & -2\\-3 & -1 & -2 & 4\\6 & 7 & -1 & 2\end{bmatrix}$

Example 1.17: Find the rank of the matrix $\begin{bmatrix}1 & 2 & 3\\2 & 1 & 4\\3 & 0 & 5\end{bmatrix}$ by reducing it to a row-echelon form. **(EG 1.17)**

Example 1.18: Find the rank of the matrix $\begin{bmatrix}2 & -2 & 4 & 3\\-3 & 4 & -2 & -1\\6 & 2 & -1 & 7\end{bmatrix}$ by reducing it to an echelon form.

Example 1.19: Show that the matrix $\begin{bmatrix}3 & 1 & 4\\2 & 0 & -1\\5 & 2 & 1\end{bmatrix}$ is non-singular and reduce it to the identity matrix by elementary row transformations. **(EG 1.19)**

Example 1.20: Find the inverse of the non-singular matrix $A = \begin{bmatrix}0 & 5\\-1 & 6\end{bmatrix}$ by Gauss-Jordan method.

Example 1.21: Find the inverse of $A = \begin{bmatrix}2 & 1 & 1\\3 & 2 & 1\\2 & 1 & 2\end{bmatrix}$ by Gauss-Jordan method. **(EG 1.21)**

EXERCISE 1.2

1. Find the rank of the following matrices by row reduction method:

$(i)\begin{bmatrix}1 & 1 & 1 & 3\\2 & -1 & 3 & 4\\5 & -1 & 7 & 11\end{bmatrix}$ $(ii)\begin{bmatrix}1 & 2 & -1\\3 & -1 & 2\\1 & -2 & 3\\1 & -1 & 1\end{bmatrix}$ $(iii)\begin{bmatrix}3 & -8 & 5 & 2\\2 & -5 & 1 & 4\\-1 & 2 & 3 & -2\end{bmatrix}$ **(EX 1.2 - 2)**

5 MARKS

1. Find the inverse of each of the following by Gauss-Jordan method:

$(ii)\begin{bmatrix}1 & -1 & 0\\1 & 0 & -1\\6 & -2 & -3\end{bmatrix}$ $(iii)\begin{bmatrix}1 & 2 & 3\\2 & 5 & 3\\1 & 0 & 8\end{bmatrix}$ **(EX 1.2 - 3)**

EXERCISE 1.3

2 MARKS

Example 1.22: Solve, using matrix inversion method: $5x + 2y = 3, 3x + 2y = 5$. **(EG 1.22)**

1. Solve the following system of linear equations by matrix inversion method:
   $(i)\ 2x + 5y = -2, x + 2y = -3$ $(ii)\ 2x - y = 8, 3x + 2y = -2$ **(EX 1.3 - 1)**

3 MARKS

1. A man is appointed in a job with a monthly salary of certain amount and a fixed amount of annual increment. If his salary was ₹19,800 per month at the end of the first month after 3 years of service and ₹23,400 per month at the end of the first month after 9 years of service, find h is starting salary and h is annual increment. **(EX 1.3 - 3)**
2. 4 men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method. **(EX 1.3 - 4)**

5 MARKS

Example 1.23: Solve the following system of equations, using matrix inversion method:
$2x_1 + 3x_2 + 3x_3 = 5, x_1 - 2x_2 + x_3 = -4, 3x_1 - x_2 - 2x_3 = 3.$ **(EG 1.23)**

Example 1.24: If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ find the products AB and BA and hence solve the system of equations $x - y + z = 4,\ \ x - 2y - 2z = 9, 2x + y + 3z = 1.$ **(EG 1.24)**

1. Solve the following system of linear equations by matrix inversion method:
   $(iii)\ 2x + 3y - z = 9,\ x + y + z = 9,\ 3x - y - z = -1$
   $(iv)\ x + y + z - 2 = 0, 6x - 4y + 5z - 31 = 0, 5x + 2y + 2z = 13$ **(EX 1.3 - 1)**
2. If $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ find the products AB and BA and hence solve the system of equations $x + y + 2z = 1, 3x + 2y + z = 7, 2x + y + 3z = 2.$ **(EX 1.3 - 2)**
3. The prices of three commodities A, B and C are ₹xy , and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C . Person Q purchases 2 units of C and sells 3 units of A and one unit of B. Person Rpurchases one unit of $A$and sells 3 unit of $B$and one unit of C . In the process, P, Q and R earn ₹15,000, ₹1,000 and ₹4,000 respectively. Find the prices per unit of A, B and C. (Use matrix inversion method to solve the problem.) **(EX 1.3 - 5)**

EXERCISE 1.4

2 MARKS

1. Solve the following systems of linear equations by Cramer's rule:
   $(i)\ 5x - 2y + 16 = 0, x + 3y - 7 = 0$ $(ii)\ \frac{3}{x} + 2y = 12, \frac{2}{x} + 3y = 13$**(EX 1.4 - 1)**

3 MARKS

1. In a competetive examination, one mark is awarded for every correct answer while 14 mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly? (Use Cramer's rule to solve the problem). **(EX 1.4 - 2)**
2. A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution? (Use Cramer's rule to solve the problem). **(EX 1.4 - 3)**

3. A fish tank can be filled in 10 minutes using both pumps A and B simultaneously. However, pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes. How long would it take each pump to fill the tank by itself? (Use Cramer's rule to solve the problem). **(EX 1.4 - 4)**

5 MARKS

Example 1.25: Solve, by Cramer's rule, the system of equations $x_1 - x_2 = 3, 2x_1 + 3x_2 + 4x_3 = 17, x_2 + 2x_3 = 7$. **(EG 1.25)**

Example 1.26: In a $T20$ match, Chennai Super Kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y = ax^2 + bx + c$ with respect to a xy −coordinate system in the vertical plane and the ball traversed through the points (10,8), (20,16), (30,18) can you conclude that Chennai Super Kings won the match? Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is (70,0)). **(EG 1.26)**

1. Solve the following systems of linear equations by Cramer's rule:
   (iii) $3x + 3y - z = 11, 2x - y + 2z = 9, 4x + 3y + 2z = 25$
   (iv) $\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0, \frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0, \frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$ **(EX 1.4 - 1)**

2. A family of 3people went out for dinner in a restaurant. The cost of two dosai, three idlies adn two vadais is Rs. 150,. The cost of the two dosai, two idlies and four vadais is Rs. 200. The cost of five dosai, four idlies and two vadais is Rs. 250. The family has Rs. 350 in hand and they are ate 3 dosai and six dillies and six vadais. Will they be able to manage to pay the bill within amount they had? **(Ex. 1.4 – 5)**

EXERCISE 1.5

5 MARKS

Example 1.27: Solve the following system of linear equations, by Gaussian elimination method $4x + 3y + 6z = 25, x + 5y + 7z = 13, 2x + 9y + z = 1$. **(EG 1.27)**

Example 1.28: The upward speed $v(t)$ of a rocket at time t is approximated by $v(t) = at^2 + bt + c, 0 \le t \le 100$,where a, b and c are constants. It has been found that the speed at times $t = 3, t = 6$ and $t = 9$ seconds are respectively 64, 133, and 208 miles per second respectively. Find the speed at time $t = 15$ seconds. (Use Gaussian elimination method.) **(EG 1.28)**

1. Solve the following systems of linear equations by Gaussian elimination method:
   (i) $2x - 2y + 3z = 2, x + 2y - z = 3, 3x - y + 2z = 1$.
   (ii) $2x + 4y + 6z = 22, 3x + 8y + 5z = 27, -x + y + 2z = 2$ **(EX 1.5 - 1)**
2. If $ax^2 + bx + c$is divided by $x + 3, x - 5$ and $x - 1$,the remainders are 21,61 and 9 respectively. Find a, b and c. (Use Gaussian elimination method.) **(EX 1.5 - 2)**
3. An amount of ₹65,000 is invested in three bonds at the rates of 6%, 8%and 10%per annum respectively. The total annual income is ₹4,800. The income from the third bond is ₹600 more

than that from the second bond. Determine the price of each bond. (Use Gaussian elimination method.) **(EX 1.5 - 3)**

4. A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6,8), (-2,-12)$ and $(3,8)$. He wants to meet his friend at $P(7,60)$. Will he meet his friend? (Use Gaussian elimination method.) **(EX 1.5 - 4)**

EXERCISE 1.6

5 MARKS

Example 1.29: Test for consistency of the following system of linear equations and if possible solve: $x + 2y - z = 3, 3x - y + 2z = 1, x - 2y + 3z = 3,$ $x - y + z + 1 = 0$. **(EG 1.29)**

Example 1.30: Test for consistency of the following system of linear equations and if possible solve: $4x - 2y + 6z = 8, x + y - 3z = -1, 15x - 3y + 9z = 21$. **(EG 1.30)**

Example 1.31: Test for consistency of the following system of linear equations and if possible solve: $x - y + z = -9, 2x - 2y + 2z = -18,$ $3x - 3y + 3z + 27 = 0$. **(EG 1.31)**

Example 1.32: Test the consistency of the following system of linear equations $x - y + z = -9, 2x - y + z = 4, 3x - y + z = 6, 4x - y + 2z = 7$. **(EG 1.32)**

Example 1.33: Find the condition on a, b and c so that the following system of linear equations has one parameter family of solutions: $x + y + z = a,$ $x + 2y + 3z = b, 3x + 5y + 7z = c$. **(EG 1.33)**

Example 1.34: Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7, x + y + \lambda z = \mu, x + 3y - 5z = 5$has (i) no solution (ii) a unique solution (iii) an infinite number of solutions. **(EG 1.34)**

1. Test for consistency and if possible, solve the following systems of equations by rank method.
   (i) $x - y + 2z = 2, 2x + y + 4z = 7, 4x - y + z = 4.$
   (ii) $3x + y + z = 2, x - 3y + 2z = 1, 7x - y + 4z = 5.$
   (iii) $2x + 2y + z = 5, x - y + z = 1, 3x + y + 2z = 4.$
   (iv) $2x - y + z = 2, 6x - 3y + 3z = 6, 4x - 2y + 2z = 4$. **(EX 1.6 - 1)**
2. Find the value of k for which the equations $kx - 2y + z = 1, x - 2ky + z = -2, x - 2y + kz = 1$ have (i) no solution (ii) unique solution (iii) infinitely many solution **(EX 1.6 - 2)**
3. Investigate the values of λ and μ the system of linear equations $2x + 3y + 5z = 9, 7x + 3y - 5z = 8, 2x + 3y + \lambda z = \mu$,have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. **(EX 1.6 - 3)**

EXERCISE 1.7

5 MARKS

Example 1.35: Solve the following system: $x + 2y + 3z = 0, 3x + 4y + 4z = 0,$ $7x + 10y + 12z = 0$. **(EG 1.35)**

Example 1.36: Solve the system: $x + 3y - 2z = 0, 2x - y + 4z = 0, x - 11y + 14z = 0$. **(EG 1.36)**

Example 1.37: Solve the system: $x + y - 2z = 0, 2x - 3y + z = 0, 3x - 7y + 10z = 0,$ $6x - 9y + 10z = 0$. **(EG 1.37)**

Example 1.38: Determine the values of λ for which the following system of equations $(3\lambda-8)x+3y+3z=0, 3x+(3\lambda-8)y+3z=0,$ $3x+3y+(3\lambda-8)z=0$. has a non-trivial solution. **(EG 1.38)**

Example 1.39: By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8+O_2 \rightarrow CO_2+H_2O$. (The above is the reaction that is taking place in the burning of organic compound called isoprene.) **(EG 1.39)**

Example 1.40: If the system of equations $px+by+cz=0,\ ax+qy+cz=0,$ $ax+by+rz=0$, has a non-trivial solution and $p\neq a, q\neq b, r\neq c$, prove that $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}=2$. **(EG 1.40)**

1. Solve the following system of homogenous equations.
   $(i) 3x+2y+7z=0, 4x-3y-2z=0, 5x+9y+23z=0.$
   $(ii) 2x+3y-z=0, x-y-2z=0, 3x+y+3z=0.$ **(EX 1.7 - 1)**
2. Determine the values of λ for which the following system of equations $x+y+3z=0, 4x+3y+\lambda z=0, 2x+y+2z=0$, has (i) a unique solution (ii) a non-trivial solution. **(EX 1.7 - 2)**
3. By using Gaussian elimination method, balance the chemical reaction equation: $C_2H_6+O_2 \rightarrow H_2O+CO_2$. **(EX 1.7 - 3)**

CHAPTER 2

COMPLEX NUMBERS

EXERCISE 2.1

2 MARKS

Example 2.1: Simplify the following
$(i)\ i^7 \quad (ii)\ i^{1729} \quad (iii) i^{-1924}+i^{2018} (iv) \sum_{n=1}^{102} i^n\ (v)\ i.i^2.i^3 \ldots\ldots i^{40}$. **(EG 2.1)**

Simplify the following:
1. $i^{1947}+i^{1950}$ **(EX 2.1 - 1)** 2. $i^{1948}-i^{1869}$ **(EX 2.1 - 2)** 3. $\sum_{n=1}^{12} i^n$ **(EX 2.1 - 3)**
4. $i^{59}+\frac{1}{i^{59}}$ **(EX 2.1 - 3)** 5. $i.i^2.i^3 \ldots\ldots i^{2000}$ **(EX 2.1 - 5)** 6. $\sum_{n=1}^{10} i^{n+50}$ **(EX 2.1 - 6)**

EXERCISE 2.2

2 MARKS

1. Evaluate the following if $z=5-2i$ and $w=-1+3i$
   $(i) z+w \quad (ii) z-iw \quad (iii) 2z+3w$
   $(iv)\ zw \quad (v) z^2+2zw+w^2 \quad (vi)(z+w)^2$. **(EX 2.2 - 1)**
2. Given the complex number $z=2+3i$,represent the complex numbers in Argand diagram.
   $(i)\ z, iz$ and $z+iz \quad (ii) z, -iz$ and $z-iz$. **(EX 2.2 - 2)**

3 MARKS

Example 2.2: Find the value of the real numbers $x$and y, if the complex number $(2+i)x+(1-i)y+2i-3$ and $x+(-1+2i)y+1+i$ are equal. **(EG 2.2)**

1. Find the values of the real numbers x and y, if the complex numbers $(3-i)x-(2-i)y+2i+5$ and $2x+(-1+2i)y+3+2i$ are equal. **(EX 2.2 - 3)**

EXERCISE 2.3

3 MARKS

1. If $z_1 = 1-3i, z_2 = -4i$ and $z_3 = 5$ show that
   $(i)\ (z_1+z_2)+z_3 = z_1+(z_2+z_3)$ $(ii)\ (z_1z_2)z_3 = z_1(z_2z_3)$ **(EX 2.3 - 1)**
2. If $z_1 = 3, z_2 = -7i$ and $z_3 = 5+4i$ show that
   $(i)\ z_1(z_2+z_3) = z_1z_2+z_1z_3$ $(ii)\ (z_1+z_2)z_3 = z_1z_3+z_2z_3$ **(EX 2.3 - 2)**
3. If $z_1 = 2+5i, z_2 = -3-4i$ and $z_3 = 1+i$ find the additive and multiplicative inverse of z_1, z_2 and z_3. **(EX 2.3 - 3)**

EXERCISE 2.4

2 MARKS

Example 2.3: Write $\frac{3+4i}{5-12i}$ in the $x+iy$ form, hence find its real and imaginary parts. **(EG 2.3)**

Example 2.4: Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$. **(EG 2.4)**

Example 2.7: Find z^{-1}, if $z = (2+3i)(1-i)$. **(EG 2.7)**

1. Write the following in the rectangular form:
   $(i)\ \overline{(5+9i)+(2-4i)}$ $(ii)\ \frac{10-5i}{6+2i}$ $(iii)\ 3\bar{i}+\frac{1}{2-i}$. **(EX 2.4 - 1)**
2. If $z = x+iy$, find the following in rectangular form.
   $(i)\ Re\left(\frac{1}{z}\right)$ $(ii)\ Re(i\bar{z})$ $(iii)\ Im(3z+4\bar{z}-4i)$ **(EX 2.4 - 2)**

3 MARKS

Example 2.6: If $z_1 = 3-2i$ and $z_2 = 6+4i$, find $\frac{z_1}{z_2}$. **(EG 2.6)**

Example 2.5: If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$, find the complex number z. **(EG 2.5)**

1. If $z_1 = 2-i$ and $z_2 = -4+3i$ find the inverse of z_1z_2 and $\frac{z_1}{z_2}$. **(EX 2.4 - 3)**
2. The complex numbers u, v and w are related by $\frac{1}{u} = \frac{1}{v}+\frac{1}{w}$. If $v = 3-4i$ and $w = 4+3i$, find u in rectangular form. **(EX 2.4 - 4)**
3. Prove the following properties:
   (i) z is real if and only if $z = \bar{z}$ $(ii)\ Re(z) = \frac{z+\bar{z}}{2}$ and $Im(z) = \frac{z-\bar{z}}{2i}$ **(EX 2.4 - 5)**
4. Find the least value of the positive integer n for which $\left(\sqrt{3}+i\right)^n$
   (i)real(ii) purely imaginary. **(EX 2.4 - 6)**

5 MARKS

Example 2.8: Show that $(i)\ (2+i\sqrt{3})^{10}+(2-i\sqrt{3})^{10}$ is real and $(ii)\ \left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary. **(EG 2.8)**

1. Show that $(i)\left(2+i\sqrt{3}\right)^{10} - (2-i\sqrt{3})^{10}$ is purely imaginary and $(ii)\ \left(\frac{19-7i}{9+i}\right)^{15} + \left(\frac{20-5i}{7-6i}\right)^{15}$ is real. **(EX 2.4 - 7)**

EXERCISE 2.5

2 MARKS

Example 2.9: If $z_1 = 3+4i, z_2 = 5-12i$ and $z_3 = 6+8i$ find $|z_1|, |z_2|, |z_3|$, $|z_1+z_2|, |z_2-z_3|$ and $|z_1+z_3|$. **(EG 2.9)**

Example 2.10: Find the following (i) $\left|\frac{2+i}{-1+2i}\right|$ (ii) $\left|\overline{(1+i)}(2+3i)(4i-3)\right|$ (iii) $\left|\frac{i(2+i)^3}{(1+i)^2}\right|$ **(EG 2.10)**

Example 2.11: Which one of the points $i, -2+i$ and 3 is farthest from the origin? **(EG 2.11)**

Example 2.17: Find the square root of $6-8i$. **(EG 2.17)**

1. Find the modulus of the following complex numbers
   (i) $\frac{2i}{3+4i}$ (ii) $\frac{2-i}{1+i}+\frac{1-2i}{1-i}$ (iii) $(1-i)^{10}$ (iv) $2i(3-4i)(4-3i)$. **(EX 2.5 - 1)**
2. Find the square roots of $(i) 4+3i$ $(ii) -6+8i$ $(iii) -5-12i$. **(EX 2.5 - 10)**

3 MARKS

Example 2.12: If z_1, z_2 and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = |z_1+z_2+z_3| = 1$, find the value of $\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|$. **(EG 2.12)**

Example 2.13: If $|z| = 2$ show that $3 \le |z+3+4i| \le 7$. **(EG 2.13)**

Example 2.14: Show that the points $1, \frac{-1}{2}+i\frac{\sqrt{3}}{2}$ and $\frac{-1}{2}-i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle. **(EG 2.14)**

Example 2.16: Show that the equation $z^2 = \bar{z}$ has four solutions. **(EG 2.16)**

1. For any two complex numbers z_1 and z_2, such that $|z_1| = |z_2| = 1$ and $z_1z_2 \ne -1$, then show that $\frac{z_1+z_2}{1+z_1z_2}$ is a real number. **(EX 2.5 - 2)**
2. Which one of the points $10-8i$, $11+6i$ is closest to $1+i$. **(EX 2.5 - 3)**
3. If $|z| = 3$, show that $7 \le |z+6-8i| \le 13$. **(EX 2.5 - 4)**
4. If $|z| = 1$, show that $2 \le |z^2-3| \le 4$. **(EX 2.5 - 5)**
5. If $\left|z-\frac{2}{z}\right| = 2$, show that the greatest and least value of $|z|$ are $\sqrt{3}+1$ and $\sqrt{3}-1$ respectively. **(EX 2.5 - 6)**
6. If the area of the triangle formed by the vertices z, iz and $z+iz$ is 50 square units, find the value of $|z|$. **(EX 2.5 - 8)**
7. Show that the equation $z^3+2\bar{z} = 0$ has five solutions. **(EX 2.5 - 9)**

5 MARKS

Example 2.15: Let z_1, z_2 and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1+z_2+z_3 \ne 0$, P.T. $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right| = r$. **(EG 2.15)**

1. If z_1, z_2 and z_3 are three complex numbers such that $|z_1| = 1, |z_2| = 2, |z_3| = 3$ and $|z_1+z_2+z_3| = 1$, Prove that $|9z_1z_2+4z_1z_3+z_2z_3| = 6$. **(EX 2.5 - 7)**

EXERCISE 2.6

2 MARKS

1. Obtain the Cartesian form of the locus of $z = x+iy$ in each of the following cases:
   $(i) [Re(iz)]^2 = 3$ $(ii) Im[(1-i)z+1] = 0$
   $(iii) |z+i| = |z-1|$ $(iv)\ \bar{z} = z^{-1}$. **(EX 2.6 - 3)**

3 MARKS

Example 2.18: Given the complex number $z = 3 + 2i$, represent the complex numbers z, iz and $z + iz$ in one Argand diagram. Show that these complex numbers form the vertices of an isosceles right triangle. **(EG 2.18)**

Example 2.19: Show that $|3z - 5 + i|$ represents a circle, and, find its centre and radius. **(EG 2.19)**

Example 2.20: Show that $|z + 2 - i| < 2$ represents interior points of a circle. Find its centre and radius. **(EG 2.20)**

Example 2.21: Obtain the Cartesian form of the locus of z in each of the following cases.
$(i)\ |z| = |z - i|$ $\quad (ii)\ |2z - 3 - i| = 3$. **(EG 2.21)**

1. Show that the following equations represent a circle, and, find its centre and radius.
   $(i) |z - 2 - i| = 3$ $\quad (ii)\ |2z + 2 - 4i| = 2$ $\quad (iii)\ |3z - 6 + 12i| = 8$. **(EX 2.6 - 4)**
2. Obtain the Cartesian equation for the locus of $z = x + iy$ in each of the following cases:
   $(i)\ |z - 4| = 16$ $\quad (ii) |z - 4|^2 - |z - 1|^2 = 16$. **(EX 2.6 - 5)**

5 MARKS

1. If $z = x + iy$ is a complex number such that $\left|\frac{z-4i}{z+4i}\right| = 1$. Show that the locus of z is real axis. **(EX 2.6 - 1)**
2. If $z = x + iy$ is a complex number such that $Im\left(\frac{2z+1}{iz+1}\right) = 0$. Show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$. **(EX 2.6 - 2)**

EXERCISE 2.7

2 MARKS

Example 2.24: Find the principal argument $Arg\ z$, when $z = \frac{-2}{1+i\sqrt{3}}$. **(EG 2.24)**

3 MARKS

Example 2.22: Find the modulus and principal argument of the following complex numbers. $(i)\ \sqrt{3} + i$ $\ (ii) - \sqrt{3} + i$ $\ (iii) - \sqrt{3} - i$ $\ (iv)\sqrt{3} - i$ **(EG 2.22)**

Example 2.23: Represent the complex number $(i) - 1 - i$ $(ii)\ 1 + i\sqrt{3}$ in polar form. **(EG 2.23)**

Example 2.25: Find the product $\frac{3}{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right).6\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$ in rectangular from. **(EG 2.25)**

Example 2.26: Find the quotient $\frac{2\left(cos\frac{9\pi}{4}+i\,sin\frac{9\pi}{4}\right)}{4\left(cos\left(\frac{-3\pi}{2}\right)+i\,sin\left(\frac{-3\pi}{2}\right)\right)}$ in rectangular form. **(EG 2.26)**

1. Write in polar form of the following complex numbers
   $(i)\ 2 + i2\sqrt{3}$ $\quad (ii)\ 3 - i\sqrt{3}$ $\quad (iii) - 2 - 2i$ $\quad (iv)\frac{1-i}{cos\frac{\pi}{3}+i\,sin\frac{\pi}{3}}$. **(EX 2.7 - 1)**
2. Find the rectangular form of the complex numbers
   $(i)\ \left(cos\frac{\pi}{6} + i\,sin\frac{\pi}{6}\right)\left(cos\frac{\pi}{12} + i\,sin\frac{\pi}{12}\right)$ $\quad (ii)\frac{cos\frac{\pi}{6}-i\,sin\frac{\pi}{6}}{2\left(cos\frac{\pi}{3}+i\,sin\frac{\pi}{3}\right)}$. **(EX 2.7 - 2)**
3. If $(x_1 + iy_1)(x_2 + iy_2)(x_3 + iy_3) \ldots\ldots (x_n + iy_n) = a + ib$, show that
   $(i)({x_1}^2 + {y_1}^2)({x_2}^2 + {y_2}^2)({x_3}^2 + {y_3}^2) \ldots\ldots ({x_n}^2 + {y_n}^2) = a^2 + b^2$
   $(ii) \sum_{r=1}^{n} \tan^{-1}\left(\frac{y_r}{x_r}\right) = \tan^{-1}\left(\frac{b}{a}\right) + 2k\pi, k \in Z$. **(EX 2.7 - 3)**

4. If $\frac{1+z}{1-z} = \cos 2\theta + i \sin 2\theta$, show that $z = i \tan \theta$. **(EX 2.7 - 4)**
5. If $\cos \alpha + \cos \beta + \cos \gamma = \sin \alpha + \sin \beta + \sin \gamma = 0$, then show that
   $(i)\ cos\ 3\alpha + cos\ 3\beta + cos\ 3\gamma = 3 \cos(\alpha + \beta + \gamma)$ and
   $(ii)\ sin\ 3\alpha + sin\ 3\beta + sin\ 3\gamma = 3\ sin(\alpha + \beta + \gamma)$. **(EX 2.7 - 5)**

5 MARKS

Example 2.27: If $z = x + iy$and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, then show that $x^2 + y^2 = 1$. **(EG 2.27)**

1. If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, then show that $x^2 + y^2 + 3x - 3y + 2 = 0$. **(EX 2.7 - 6)**

EXERCISE 2.8

2 MARKS

1. If $\omega \neq 1$is a cube root of unity, then show that$\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = -1$. **(EX 2.8 - 1)**
2. Show that $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5 = -\sqrt{3}$. **(EX 2.8 - 2)**
3. If $\omega \neq 1$ is a cube root of unity, show that
   $(i)(1 - \omega + \omega^2)^6(1 + \omega - \omega^2)^6 = 128$
   $(ii)(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8) \ldots\ldots (1 + \omega^{2^{11}}) = 1$. **(EX 2.8 - 8)**
4. Find the value of $\sum_{k=1}^{8}\left(cos\frac{2k\pi}{9} + i\ sin\frac{2k\pi}{9}\right)$. **(EX 2.8 - 7)**

3 MARKS

Example 2.28: If $z = (cos\ \theta + i\ sin\ \theta)$, show that $z^n + \frac{1}{z^n} = 2\cos n\theta$ and
$z^n - \frac{1}{z^n} = 2i \sin n\theta$. **(EG 2.28)**

Example 2.29: Simplify $\left(sin\frac{\pi}{6} + i\ cos\frac{\pi}{6}\right)^{18}$ **(EG 2.29)**

Example 2.30: Simplify $\left(\frac{1+\cos 2\theta+i \sin 2\theta}{1+cos\ 2\theta-i\ sin\ 2\theta}\right)^{30}$. **(EG 2.30)**

Example 2.31: Simplify $(i)\ (1 + i)^n\ (ii)\ (-\sqrt{3} + 3i)^{31}$. **(EG 2.31)**

Example 2.32: Find the cube roots of unity. **(EG 2.32)**

Example 2.33: Find the fourth roots of unity. **(EG 2.33)**

Example 2.34: Solve the equation $z^3 + 8i = 0$, where $z \in C$. **(EG 2.34)**

Example 2.35: Find all cube roots of $\sqrt{3} + i$. **(EG 2.35)**

Example 2.36: Suppose z_1, z_2 and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$.If $z_1 = 1 + i\sqrt{3}$, then find z_2 and z_3. **(EG 2.36)**

1. Find the value of $\left(\frac{1+\sin\frac{\pi}{10}+i\cos\frac{\pi}{10}}{1+sin\frac{\pi}{10}-i\ cos\frac{\pi}{10}}\right)^{10}$. **(EX 2.8 - 3)**
2. If $\omega \neq 1$ is a cube root of unity, show that the roots of the equation $(z - 1)^3 + 8 = 0$are $-1, 1 - 2\omega, 1 - 2\omega^2$. **(EX 2.8 - 6)**
3. If $z = 2 - 2i$, find the rotation of $z$by θ radians in the counter clockwise direction about the origin when
   $(i)\ \theta = \frac{\pi}{3}$ $(ii)\ \theta = \frac{2\pi}{3}$ $(iii)\ \theta = \frac{3\pi}{2}$. **(EX 2.8 - 9)**

5 MARKS

1. If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$ show that
   $(i)\ \frac{x}{y} + \frac{y}{x} = 2\cos(\alpha - \beta)$ $\quad (ii)\ xy - \frac{1}{xy} = 2i\sin(\alpha + \beta)$
   $(iii) \frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$ $\quad (iv) x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$. **(EX 2.8 - 4)**
2. Solve the equation $z^3 + 27 = 0$. **(EX 2.8 - 5)**

CHAPTER 3

THEORY OF EQUATIONS

EXERCISE 3.1

2 MARKS

Example 3.1: If α and β are the roots of the quadratic equation $17x^2 + 43x - 73 = 0$, construct a quadratic equation whose roots are $\alpha + 2$ and $\beta + 2$. **(EG 3.1)**

Example 3.3: If α, β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \frac{1}{\beta\gamma}$ in terms of the coefficients. **(EG 3.3)**

1. Construct a cubic equation with roots (i) 1, 2 and 3 (ii) 1,1 and -2 (iii) 2 , -2 and 4. **(EX 3.1 - 2)**
2. Formalate into a mathematical problem to find a number such that when its cube root is added to it, the result is 6 . **(EX 3.1 - 11)**
3. A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was cut away. **(EX 3.1 - 12)**

3 MARKS

Example 3.2: If α and β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$, construct a quadratic equation whose roots are α^2 and β^2. **(EG 3.2)**

Example 3.4: Find the sum of the squares of the roots of $ax^4 + bx^3 + cx^2 + dx + e = 0$. **(EG 3.4)**

Example 3.5: Find the condition that the roots of $x^3 + ax^2 + bx + c = 0$, are in the ratio $p:q:r$. **(EG 3.5)**

Example 3.6: Form the equation whose roots are the squares of the roots of the cubic equation $x^3 + ax^2 + bx + c = 0$. **(EG 3.6)**

Example 3.7: If p is real, discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$, in terms of p. **(EG 3.7)**

EXERCISE 3.1

1. If the sides of a cubic box are increased by 1, 2, 3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Find the volume of the cuboid. **(EX 3.1 - 1)**
2. If α , β and γ are the roots of the cubic equation $x^3 + 2x^2 + 3x + 4 = 0$, form a cubic equation whose roots are
   $(i)\ 2\alpha, 2\beta, 2\gamma$ $\ (ii)\ \frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ $\quad (iii) -\alpha, -\beta, -\gamma$ **(EX 3.1 - 3)**
3. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$, if the product of two roots is 1. **(EX 3.1 - 4)**
4. Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$. **(EX 3.1 - 5)**

5. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$,if it is given that two of its roots are in the ratio $3:2$. **(EX 3.1 - 6)**
6. If $\alpha, \beta$and γ are the roots of the polynomial equation $ax^3 + bx^2 + cx + d = 0$, find the value of $\sum \frac{\alpha}{\beta\gamma}$in terms of the coefficients. **(EX 3.1 - 7)**
7. If $\alpha, \beta, \gamma$and δ are the roots of the polynomial equation $2x^4 + 5x^3 - 7x^2 + 8 = 0$,find a quadratic equation with integer coefficients whose roots are $\alpha + \beta + \gamma + \delta$and $\alpha\beta\gamma\delta$. **(EX 3.1 - 8)**
8. If $p$and $q$are the roots of the equation $lx^2 + nx + n = 0$,show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$. **(EX 3.1 - 9)**
9. If the equations $x^2 + px + q = 0$,and $x^2 + p'x + q' = 0$,have a common root, show that it must be equal to $\frac{pq' - p'q}{q - q'}$ or $\frac{q - q'}{p' - p}$. **(EX 3.1 - 10)**

EXERCISE 3.2

2 MARKS

Example 3.8: Find the monic polynomial equation of minimum degree with real coefficients having $2 - \sqrt{3}i$as a root. **(EG 3.8)**
Example 3.9: Find a polynomial equation of minimum degree with rational coefficients, having $2 - \sqrt{3}$as a root. **(EG 3.9)**
Example 3.10: Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$as a root. **(EG 3.10)**
Example 3.11: Show that the equation $2x^2 - 6x + 7 = 0$ cannot be satisfied by any real values of x. **(EG 3.11)**
Example 3.12: If $x^2 + 2(k+2)x + 9k = 0$has equal roots, find k. **(EG 3.12)**
Example 3.14: Prove that a line cannot intersect a circle at more than two points. **(EG 3.14)**

1. Find a polynomial equation of minimum degree with rational coefficients, having $2 + \sqrt{3}i$as a root. **(EX 3.2 - 1)**
2. Find a polynomial equation of minimum degree with rational coefficients, having $2i + 3$ as a root. **(EX 3.2 - 3)**
3. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5} - \sqrt{3}$as a root. **(EX 3.2 - 4)**
4. Prove that a straight line and parabola cannot intersect at more than two points. **(EX 3.2 - 5)**

3 MARKS

Example 3.13: Show that, if $p, q, r$are rational, the roots of the equation $x^2 - 2px + p^2 - q^2 + 2qr - r^2 = 0$ are rational. **(EG 3.13)**

1. If $k$is real, discuss the nature of the roots of the polynomial equation $2x^2 + kx + k = 0$, in terms of k. **(EX 3.2 - 1)**

EXERCISE 3.3

Example 3.16: Solve the equation $x^4 - 9x^2 + 20 = 0$. **(EG 3.16)**

1. Solve the equation :$x^4 - 14x^2 + 45 = 0$. **(EX 3.3 - 7)**

3 MARKS

Example 3.17: Solve the equation $x^3 - 3x^2 - 33x + 35 = 0$. **(EG 3.17)**
Example 3.18: Solve the equation $2x^3 + 11x^2 - 9x - 18 = 0$. **(EG 3.18)**
Example 3.19: Obtain the condition that the roots of $x^3 + px^2 + qx + r = 0$,are in $A.P$. **(EG 3.19)**
Example 3.20: Find the condition that the roots of $ax^3 + bx^2 + cx + d = 0$, are in geometric progression. Assume $a, b, c, d \neq 0$ **(EG 3.20)**
Example 3.21: If the roots of $x^3 + px^2 + qx + r = 0$,are in $H.P$., prove that $9pqr = 27r^3 + 2p$. **(EG 3.21)**

1. Solve the cubic equation :$2x^3 - x^2 - 18x + 9 = 0$if sum of two of its roots vanishes. **(EX 3.3 - 1)**
2. Solve the equation $9x^3 - 36x^2 + 44x - 16 = 0$ if the roots form an arithmetic progression. **(EX 3.3 - 2)**
3. Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ if its roots form a geometric progression. **(EX 3.3 - 3)**
4. Determine $k$and solve the equation $2x^3 - 6x^2 + 3x + k = 0$if one of its roots is twice the sum of the other two roots. **(EX 3.3 - 4)**
5. Solve the cubic equations :(i) $2x^3 - 9x^2 + 10x = 3$, $(ii) 8x^3 - 2x^2 - 7x + 3 = 0$. **(EX 3.3 - 6)**

5 MARKS

Example 3.15: If $2 + i$and $3 - \sqrt{2}$are roots of the equation $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$, find all roots. **(EG3.15)**

Example 3.22: It is known that the roots of the equation $x^3 - 6x^2 - 4x + 24 = 0$ are in arithmetic progression. Find its roots. **(EG 3.22)**

1. Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$and $\sqrt{3}$ are two of its zeros. **(EX 3.3 - 5)**

EXERCISE 3.4

5 MARKS

Example 3.23: Solve the equation $(x - 2)(x - 7)(x - 3)(x + 2) + 19 = 0$. **(EG 3.23)**
Example 3.24: Solve the equation $(2x - 3)(6x - 1)(3x - 2)(x - 12) - 7 = 0$. **(EG 3.24)**

1. Solve :(i) $(x - 5)(x - 7)(x + 6)(x + 4) = 504$,
   (ii) $(x - 4)(x - 7)(x - 2)(x + 1) = 16$. **(EX 3.4 - 1)**
2. Solve :$(2x - 1)(x + 3)(x - 2)(2x + 3) + 20 = 0$. **(EX 3.4 - 2)**

EXERCISE 3.5

3 MARKS

Example 3.25: Solve the equation $x^3 - 5x^2 - 4x + 20 = 0$. **(EG 3.25)**
Example 3.26: Find the roots of $2x^3 + 3x^2 + 2x + 3$. **(EG 3.26)**

Example 3.27: Solve the equation $7x^3 - 43x^2 - 43x + 7 = 0$ **(EG 3.27)**
Example 3.28: Solve the following equation: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$. **(EG 3.28)**
Example 3.29: Find solution, if any, of the equation $2\cos^2 x - 9\cos x + 4 = 0$. **(EG 3.29)**

1. Solve the following equations
   (i) $\sin^2 x - 5\sin x + 4 = 0$ $(ii) 12x^3 + 8x = 29x^2 - 4$ **(EX 3.5 - 1)**
2. Examine for the rational roots of
   (i) $2x^3 - x^2 - 1 = 0$ (ii) $x^8 - 3x + 1 = 0$. **(EX 3.5 - 2)**
3. Solve : $8x^{\frac{3}{2n}} - 8x^{\frac{-3}{2n}} = 63$. **(EX 3.5 - 3)**
4. Solve : $2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}} = \frac{b}{a} + \frac{6a}{b}$. **(EX 3.5 - 4)**
5. Find all real numbers satisfying $4^x - 3(2^{x+2}) + 2^5 = 0$. **(EX 3.5 - 6)**

5 MARKS

1. Solve the equations
   (i) $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$, $(ii) x^4 + 3x^3 - 3x - 1 = 0$. **(EX 3.5 - 5)**
2. Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution. **(EX 3.5 - 7)**

EXERCISE 3.5

2 MARKS

Example 3.30: Show that the polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2$ has at least six imaginary roots. **(EG 3.30)**
Example 3.31: Discuss the nature of the roots of the following polynomials:
(i) $x^{2018} + 1947x^{1950} + 15x^8 + 26x^6 + 2019$
$(ii) x^5 - 19x^4 + 2x^3 + 5x^2 + 11$. **(EG 3.31)**

EXERCISE 3.6

1. Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$. **(EX 3.6 - 1)**
2. Discuss the maximum possible number of positive and negative roots of the polynomial equations $x^2 - 5x + 6$ and $x^2 - 5x + 16$. Also draw rough sketch of the graphs. **(EX 3.6 - 2)**
3. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions. **(EX 3.6 - 3)**
4. Determine the number of positive and negative roots of the equation $x^9 - 5x^8 - 14x^7 = 0$. **(EX 3.6 - 4)**
5. Find the exact number of real roots and imaginary of the equation $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$. **(EX 3.6 - 5)**

CHAPTER 4

INVERSE TRIGONOMETRIC FUNCTIONS

EXERCISE 4.1

2 MARKS

Example 4.1: Find the principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$(in radians and degrees). **(EG 4.1)**

Example 4.2: Find the principal value of $\sin^{-1}(2)$, if it exists. **(EG 4.2)**

Example 4.3: Find the principal value of

$(i)\ \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ $(ii)\ \sin^{-1}\left(\sin\left(\frac{-\pi}{3}\right)\right)$ $(iii)\ \sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)$. **(EG 4.3)**

1. Find all the values of x such that $(i) -10\pi \le x \le 10\pi$ and $\sin x = 0$ $(ii)\ -8\pi \le x \le 8\pi$ and $\sin x = -1$ **(EX 4.1 - 1)**
2. Find the period and amplitude of $(i)\ y = \sin 7x$ $(ii)\ y = -\sin\left(\frac{1}{3}x\right)$ $(iii) y = 4\sin(-2x)$. **(EX 4.1 - 2)**
3. Sketch the graph of $y = sin\left(\frac{1}{3}x\right)$ for $0 \le x < 6\pi$. **(EX 4.1 - 3)**
4. For what value of x does $\sin x = \sin^{-1} x$? **(EX 4.1 - 5)**

3 MARKS

Example 4.4: Find the domain of $\sin^{-1}(2-3x^2)$ **(EG 4.4)**

1. Find the value of $(i)\ \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$ $(ii)\ sin^{-1}\left(sin\left(\frac{5\pi}{4}\right)\right)$. **(EX 4.1 - 4)**
2. Find the domain of the following $(i)\ f(x) = \sin^{-1}\left(\frac{x^2+1}{2x}\right)$ $(ii)\ g(x) = 2\sin^{-1}(2x-1) - \frac{\pi}{4}$. **(EX 4.1 - 6)**
3. Find the value of $\sin^{-1}\left(sin\frac{5\pi}{9}cos\frac{\pi}{9} + cos\frac{5\pi}{9}sin\frac{\pi}{9}\right)$. **(EX 4.1 - 7)**

EXERCISE 4.2

2 MARKS

Example 4.5: Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$. **(EG 4.5)**

Example 4.6: Find $(i)\ \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ $(ii)\ cos^{-1}\left(\cos\left(\frac{-\pi}{3}\right)\right)$ $(iii)\ cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$ **(EG 4.6)**

EXERCISE 4.2

1. Find all values of x such that $(i) -6\pi \le x \le 6\pi$ and $\cos x = 0$ $(ii)\ -5\pi \le x \le 5\pi$ and $cos\, x = 1$. **(EX 4.2 - 1)**
2. State the reason for $cos^{-1}\left(\cos\left(\frac{-\pi}{6}\right)\right) \neq \frac{-\pi}{6}$ **(EX 4.2 - 2)**

3. Find the principal value of $\cos^{-1}\left(\frac{1}{2}\right)$. **(EX 4.2 - 4)**
4. Find the value of $(i)\ 2\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(\frac{1}{2}\right)$ $\quad (ii)\ cos^{-1}\left(\frac{1}{2}\right)+sin^{-1}(-1)$**(EX 4.2 - 5)**
5. Find the domain of $(ii)\ g(x)=\sin^{-1}x+\cos^{-1}x$. **(EX 4.2 - 6)**
6. Find the value of $(i)\ \cos\left(cos^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{4}{5}\right)\right)$ **(EX 4.2 - 8)**

3 MARKS

Example 4.7: Find the domain of $cos^{-1}\left(\frac{2+\sin x}{3}\right)$ **(EG 4.7)**

1. Is $cos^{-1}(-x)=\pi-cos^{-1}(x)$true? Justify your answer. **(EX 4.2 - 3)**
2. Find the value of $(iii)\ cos^{-1}\left(cos\frac{\pi}{7}cos\frac{\pi}{17}-sin\frac{\pi}{7}sin\frac{\pi}{17}\right)$. **(EX 4.2 - 5)**
3. Find the domain of $(i)\ f(x)=\sin^{-1}\left(\frac{|x|-2}{3}\right)+cos^{-1}\left(\frac{1-|x|}{4}\right)$ **(EX 4.2 - 6)**
4. For what value of x , the inequality $\frac{\pi}{2}<\cos^{-1}(3x-1)<\pi$holds? **(EX 4.2 - 7)**
5. Find the value of $(ii)\ cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right)+cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$ **(EX 4.2 - 8)**

EXERCISE 4.3

2 MARKS

Example 4.8: Find the principal value of $\tan^{-1}(\sqrt{3})$ **(EG 4.8)**
Example 4.9: Find $(i)\ tan^{-1}(-\sqrt{3})\ \ (ii)\ tan^{-1}\left(\tan\left(\frac{3\pi}{5}\right)\right)\ \ (iii)\ \tan(\tan^{-1}(2019))$ **(EG 4.9)**

EXERCISE 4.3

1. Find the value of $(i)\ tan^{-1}\left(\tan\left(\frac{5\pi}{4}\right)\right)$ $\quad (ii)\ tan^{-1}\left(\tan\left(\frac{-\pi}{6}\right)\right)$. **(EX 4.3 - 2)**
2. Find the value of
   $(i)\ tan\left(tan^{-1}\left(\frac{7\pi}{4}\right)\right)$ $\quad (ii)\ tan(tan^{-1}(1947))$ $\quad (iii)\ tan(tan^{-1}(-0.2021))$ **(EX 4.3 - 3)**

3 MARKS

Example 4.10: Find the value of $tan^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+sin^{-1}\left(\frac{-1}{2}\right)$ **(EG 4.10)**
Example 4.11: Prove that $\tan(\sin^{-1}x)=\frac{x}{\sqrt{1-x^2}},-1<x<1$. **(EG 4.11)**

1. Find the domain of the following functions:
   $(i)\ tan^{-1}(\sqrt{9-x^2})$ $\quad (ii)\frac{1}{2}tan^{-1}(1-x^2)-\frac{\pi}{4}$. **(EX 4.3 - 1)**
2. Find the value of $(i)\tan\left(\cos^{-1}\left(\frac{1}{2}\right)-\sin^{-1}\left(\frac{-1}{2}\right)\right)$ **(EX 4.3 - 4)**

5 MARKS

1. Find the value of
   $(ii)\ sin\left(tan^{-1}\left(\frac{1}{2}\right) - cos^{-1}\left(\frac{4}{5}\right)\right)$. $(iii)\ cos\left(sin^{-1}\left(\frac{4}{5}\right) - tan^{-1}\left(\frac{3}{4}\right)\right)$ **(EX 4.3 - 4)**

EXERCISE 4.4

2 MARKS

Example 4.12: Find the principal value of $(i)\ \text{cosec}^{-1}(-1)$ $(ii)\ \sec^{-1}(-2)$. **(EG 4.12)**

Example 4.15: Show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \sec^{-1} x\,, |x| > 1$. **(EG 4.15)**

EXERCISE 4.4

1. Find the principal value of
   $(i)\ sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ $(ii)\ \cot^{-1}(\sqrt{3})$ $(iii)\ \text{cosec}^{-1}(-\sqrt{2})$ **(EX 4.4 - 1)**

3 MARKS

Example 4.13: Find the value of $sec^{-1}\left(\frac{-2\sqrt{3}}{3}\right)$. **(EG 4.13)**

Example 4.14: If $\cot^{-1}\left(\frac{1}{7}\right) = \theta$, find the value of $\cos\theta$. **(EG 4.14)**

1. Find the value of
   $(i)\ \tan^{-1}(\sqrt{3}) - sec^{-1}(-2)$ $(ii)\ \sin^{-1}(-1) + cos^{-1}\left(\frac{1}{2}\right) + \cot^{-1}(2)$
   $(iii)\ cot^{-1}(1) + \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) - sec^{-1}(-\sqrt{2})$ **(EX 4.4 - 2)**

EXERCISE 4.5

2 MARKS

Example 4.16: Prove that $\frac{\pi}{2} \leq \sin^{-1} x + 2\cos^{-1} x \leq \frac{3\pi}{2}$. **(EG 4.16)**

Example 4.17: Simplify $(i)\ \cos^{-1}\left(\cos\left(\frac{13\pi}{3}\right)\right)$ $(ii)\ tan^{-1}\left(tan\left(\frac{3\pi}{4}\right)\right)$
$(iii)\ \sec^{-1}\left(\sec\left(\frac{5\pi}{3}\right)\right)$ $(iv)\ \sin^{-1}(\sin 10)$ **(EG 4.17)**

1. Find the value, if it exists. If not, give the reason for non-existence.
   $(i)\ \sin^{-1}(\cos\pi)$ $(ii)\ \tan^{-1}\left(\sin\left(\frac{-5\pi}{2}\right)\right)$ $(iii)\ \sin^{-1}(\sin 5)$ **(EX 4.5 - 1)**
2. Find the value of $(i)\ \sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$ **(EX 4.5 - 3)**

3 MARKS

Example 4.18: Find the value of $(i)\ \sin\left(\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right)$ $(ii)\ \cos\left(\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right)$
$(iii)\ \tan\left(\frac{1}{2}sin^{-1}\left(\frac{2a}{1+a^2}\right) + \frac{1}{2}cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right)$. **(EG 4.18)**

Example 4.19: Prove that $\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}$ for $|x| < 1$. **(EG 4.19)**

Example 4.24: Solve $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$ for $x>0$. **(EG 4.24)**

Example 4.25: Solve $\sin^{-1}x>\cos^{-1}x$. **(EG 4.25)**

Example 4.26: Show that $\cot(\sin^{-1}x)=\frac{\sqrt{1-x^2}}{x}$, $-1\le x\le 1$ and $x\neq 0$. **(EG 4.26)**

Example 4.27: Solve $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$, if $6x^2<1$. **(EG 4.27)**

Example 4.28: Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right)+tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$. **(EG 4.28)**

Example 4.29: Solve $\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right)=\sin\left(\cot^{-1}\left(\frac{3}{4}\right)\right)$. **(EG 4.29)**

1. Find the value of the expression in terms of x, with the help of a reference triangle.
   $(i)\sin(\cos^{-1}(1-x))$ $(iii)\cos(\tan^{-1}(3x-1))$ $(iii)\tan\left(\sin^{-1}\left(x+\frac{1}{2}\right)\right)$ **(EX 4.5 - 2)**
2. Prove that $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$. **(EX 4.5 - 5)**
3. Simplify: $\tan^{-1}\frac{x}{y}-\tan^{-1}\frac{x-y}{x+y}$. **(EX 4.5 - 8)**
4. Solve: $(i)\sin^{-1}\frac{5}{x}+\sin^{-1}\frac{12}{x}=\frac{\pi}{2}$
   $(ii)2\tan^{-1}x=\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)-\cos^{-1}\left(\frac{1-b^2}{1+b^2}\right), a>0, b>0.$
   $(iii)2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec}x)$ $(iv)\cot^{-1}x-\cot^{-1}(x+2)=\frac{\pi}{12}, x>0.$ **(EX 4.5 - 9)**

5 MARKS

Example 4.20: Evaluate $\sin\left[sin^{-1}\left(\frac{3}{5}\right)+sec^{-1}\left(\frac{5}{4}\right)\right]$. **(EG 4.20)**

Example 4.21: Prove that $(i)\tan^{-1}\left(\frac{1}{2}\right)+tan^{-1}\left(\frac{1}{3}\right)=\frac{\pi}{4}$

$(ii)\ 2\tan^{-1}\left(\frac{1}{2}\right)+tan^{-1}\left(\frac{1}{7}\right)=tan^{-1}\left(\frac{31}{17}\right)$ **(EG 4.21)**

Example 4.22: If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ and $0<x,y,z<1$, show that $x^2+y^2+z^2+2xyz=1$. **(EG 4.22)**

Example 4.23: If $a_1,a_2,a_3,\ldots,a_n$ is an arithmetic progression with common difference d,

P.T. $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+..+\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right]=\frac{a_n-a_1}{1+a_1a_n}$. **(EG 4.23)**

1. Find the value of $(ii)\cot\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{4}{5}\right)$ $(iii)\ tan\left(sin^{-1}\frac{3}{5}+cot^{-1}\frac{3}{2}\right)$ **(EX 4.5 - 3)**
2. Prove that $(i)\tan^{-1}\frac{2}{11}+tan^{-1}\frac{7}{24}=tan^{-1}\frac{1}{2}$ $(ii)\ sin^{-1}\frac{3}{5}+cos^{-1}\frac{12}{13}=sin^{-1}\frac{16}{65}$ **(EX 4.5 - 4)**
3. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$, show that $x+y+z=xyz$. **(EX 4.5 - 6)**
4. Prove that $\tan^{-1}x+\tan^{-1}\frac{2x}{1-x^2}=\tan^{-1}\frac{3x-x^3}{1-3x^2}, |x|<\frac{1}{\sqrt{3}}$. **(EX 4.5 - 7)**
5. Find the number of solution of the equation $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$. **(EX 4.5 - 10)**

CHAPTER 5

TWO DIMENSIONAL ANALYTICAL GEOMETRY-II

EXERCISE 5.1

2 MARKS

Example 5.1: Find the general equation of a circle with centre $(-3,-4)$ and radius 3 units. **(EG 5.1)**
Example 5.3: Determine whether $x+y-1=0$ is the equation of a diameter of the circle $x^2+y^2-6x+4y+c=0$, for all possible values of c. **(EG 5.3)**
Example 5.4: Find the general equation of the circle whose diameter is the line segment joining the points $(-4,-2)$ and $(1,1)$. **(EG 5.4)**
Example 5.5: Examine the position of the point $(2,3)$ with respect to the circle $x^2+y^2-6x-8y+12=0$. **(EG 5.5)**
Example 5.11: Find the equations of the tangent and normal to the circle $x^2+y^2=25$ at $P(-3,4)$. **(EG 5.11)**
Example 5.12: If $y=4x+c$ is a tangent to the circle $x^2+y^2=9$, find c. **(EG 5.12)**

1. Obtain the equation of the circles with radius 5 cm and touching x −axis at the origin in general form. **(EX 5.1 - 1)**
2. Find the equation of the circle with centre $(2,-1)$ and passing through the point $(3,6)$ in standard form. **(EX 5.1 - 2)**
3. Find the equation of circles that touch both the axes and pass through $(-4,-2)$ in general form. **(EX 5.1 - 3)**
4. Obtain the equation of the circle for which $(3,4)$ and $(2,-7)$ are the ends of a diameter. **(EX 5.1 - 5)**
5. Find the equation of the circle through the points $(1,0),(-1,0)$ and $(0,1)$. **(EX 5.1 - 6)**
6. If $y=2\sqrt{2}x+c$ is a tangent to the circle $x^2+y^2=16$, find the value of c. **(EX 5.1 - 8)**
7. Find the equation of the tangent and normal to the circle $x^2+y^2-6x+6y-8=0$ at $(2,2)$. **(EX 5.1 - 9)**
8. Determine whether the points $(-2,1),(0,0)$ and $(-4,-3)$ lie outside, on or inside the circle $x^2+y^2-5x+2y-5=0$. **(EX 5.1 - 10)**
9. Find centre and radius of the following circles. $(i)\ x^2+(y+2)^2=0$. **(EX 5.1 - 11)**

3 MARKS

Example 5.2: Find the equation of the circle described on the chord $3x+y+5=0$ of the circle $x^2+y^2=16$ as diameter. **(EG 5.2)**
Example 5.6: The line $3x+4y-12=0$ meets the coordinate axes at A and B. Find the equation of the circle drawn on AB as diameter. **(EG 5.6)**
Example 5.7: A line $3x+4y+10=0$ cuts a chord of length 6 units on a circle with centre of the circle $(2,1)$. Find the equation of the circle in general form. **(EG 5.7)**
Example 5.8: A circle of radius 3 units touches both the axes. Find the equations of all possible circles formed in the general form. **(EG 5.8)**
Example 5.9: Find the centre and radius of the circle $3x^2+(a+1)y^2+6x-9y+a+4=0$. **(EG 5.9)**
Example 5.13: A road bridge over an irrigation canal have two semi circular vents each with a span of $20m$ and the supporting pillars of width $2m$. to write the equations that model the arches. **(EG 5.13)**

1. Find the equation of the circle with centre $(2,3)$ and passing through the intersection of the lines $3x - 2y - 1 = 0$ and $4x + y - 27 = 0$. **(EX 5.1 - 4)**
2. A circle of area 9π square units has two of its diameters along the lines $x + y = 5$ and $x - y = 1$. Find the equation of the circle. **(EX 5.1 - 7)**
3. Find centre and radius of the following circles.
   $(i)\ x^2 + (y+2)^2 = 0.$ $(ii) x^2 + y^2 + 6x - 4y + 4 = 0.$
   $(iii)\ x^2 + y^2 - x + 2y - 3 = 0.$ $(iv) 2x^2 + 2y^2 - 6x + 4y + 2 = 0.$ **(EX 5.1 - 11)**
4. If the equation $3x^2 + (3-p)xy + qy^2 - 2px = 8pq$ represents a circle, Find p and q . Also determine the centre and radius of the circle. **(EX 5.1 - 12)**

5 MARKS

Example 5.10: Find the equation of the circle passing through the points $(1,1), (2,-1)$ and $(3,2)$. **(EG 5.10)**

EXERCISE 5.2

2 MARKS

Example 5.14: Find the length of Latus rectum of the parabola $y^2 = 4ax$. **(EG 5.14)**

Example 5.15: Find the length of Latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. **(EG 5.15)**

Example 5.14: Find the equation of the parabola with focus $(-\sqrt{2}, 0)$ and directrix $x = \sqrt{2}$. **(EG 5.14)**

Example 5.15: Find the equation of the parabola whose vertex is $(5,-2)$ and focus $(2,-2)$. **(EG 5.15)**

Example 5.16: Find the equation of the parabola with vertex $(-1,-2)$, axis parallel to $y-$ axis and passing through $(3,6)$. **(EG 5.16)**

1. Find the equation of the parabola in each of the cases given below:
   (i) focus $(4,0)$ and directrix $x = -4$.
   (ii) passes through $(2,-3)$ and symmetric about y $-$axis.
   (iii) vertex$(1,-2)$ and focus $(4,-2)$.
   (iv) end points of latus rectum$(4,-8)$ and $(4,8)$. **(EX 5.2 - 1)**

3 MARKS

Example 5.18: Find the equation of the ellipse with foci $(\pm 2,0)$ vertices $(\pm 3,0)$. **(EG 5.18)**

Example 5.22: Find the equation of the hyperbola with vertices $(0,\pm 4)$ and foci $(0,\pm 6)$. **(EG 5.22)**

Example 5.25: The orbit of Halley's Comet $(Fig. 5.51)$ is an ellipse 36.18 .astronomical units long and by 9.12 astronomical units wide. Find its eccentricity. **(EG 5.25)**

Example 5.23: Find the vertices, foci for the hyperbola $9x^2 - 16y^2 = 144$. **(EG 5.23)**

1. Find the equation of the ellipse in each of the cases given below:
   (i) foci $(\pm 3,0)$, $e = \frac{1}{2}$.
   (ii) foci $(0,\pm 4)$ and end points of major axis are $(0,\pm 5)$.
   (iii) length of latus rectum 8, eccentricity $= \frac{3}{5}$ and major axis on x $-$axis.

(iv) length of latus rectum 4, distance between foci $4\sqrt{2}$ and major axis as $y -$ axis. **(EX 5.2 - 2)**

2. Find the equation of the hyperbola in each of the cases given below:
   (i) foci $(\pm 2, 0)$, eccentricity $= \frac{3}{2}$.
   (ii) Centre $(2,1)$, one of the foci $(8,1)$ and corresponding directrix $x = 4$.
   (iii) passing through $(5, -2)$ and length of the transverse axis along x axis and of length 8 units. **(EX 5.2 - 3)**
3. Find the vertex, focus, equation of directrix and length of the latus rectum of the following:
   $(i)\ y^2 = 16x$ $(ii)\ x^2 = 24y$ $(iii) y^2 = -8x$ **(EX 5.2 - 4)**
4. Identify the type of conic and find centre, foci, vertices, and directrices of each of the following:
   $(i)\ \frac{x^2}{25} + \frac{y^2}{9} = 1$ $(ii)\ \frac{x^2}{3} + \frac{y^2}{10} = 1$ $(iii)\ \frac{x^2}{25} - \frac{y^2}{144} = 1$
   $(iv)\ \frac{y^2}{16} - \frac{x^2}{9} = 1$ **(EX 5.2 - 5)**
5. Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$. **(EX 5.2 - 6)**
6. Show that the absolute value of difference of the focal distances of any point P on the hyperbola is the length of its transverse axis. **(EX 5.2 - 7)**

5 MARKS

Example 5.17: Find the vertex, focus, directrix, and length of the latus rectum of the parabola $x^2 - 4x - 5y - 1 = 0$. **(EG 5.17)**

Example 5.19: Find the equation of the ellipse whose eccentricity is $\frac{1}{2}$, one of the foci is $(2,3)$ and a directrix is $x = 7$. Also find the length of the major and minor axes of the ellipse. **(EG 5.19)**

Example 5.20: Find the foci, vertices and length of major and minor axis of the conic $4x^2 + 36y^2 + 40x - 288y + 532 = 0$. **(EG 5.20)**

Example 5.21: For the ellipse $4x^2 + y^2 + 24x - 2y + 21 = 0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2. **(EG 5.21)**

Example 5.24: Find the centre, foci, and eccentricity of the hyperbola $11x^2 - 25y^2 - 44x + 50y - 256 = 0$. **(EG 5.24)**

1. Find the vertex, focus, equation of directrix and length of the latus rectum of the following:
   $(iv)\ x^2 - 2x + 8 + 17 = 0y$ $(v)\ y^2 - 4y - 8x + 12 = 0$ **(EX 5.2 - 4)**
2. Identify the type of conic and find centre, foci, vertices, and directrices of each of the following:
   $(i) \frac{(x-3)^2}{225} + \frac{(y-4)^2}{289} = 1$ $(ii)\ \frac{(x+1)^2}{100} + \frac{(y-2)^2}{64} = 1$ $(iii)\ \frac{(x+3)^2}{225} - \frac{(y-4)^2}{64} = 1$
   $(iv)\ \frac{(y-2)^2}{25} - \frac{(x+1)^2}{16} = 1$ $(v)\ 18x^2 + 12y^2 - 144x + 48y + 120 = 0$
   $(vi)\ 9x^2 - y^2 - 36x - 6y + 18 = 0$ **(EX 5.2 - 8)**

EXERCISE 5.3

2 MARKS

Example 5.26: Identify the type of the conic for the following equations:
$(1) 16y^2 = -4x^2 + 64$ $(2)\ x^2 + y^2 = -4x - y - 4$
$(3)\ x^2 - 2y = x + 3$ $(4)\ 4x^2 - 9y^2 - 16x + 18y - 29 = 0$ **(EG 5.26)**

Identify the type of conic section for each of the equations.

1. $2x^2 - y^2 = 7$ **(EX 5.3 - 1)**
2. $3x^2 + 3y^2 - 4x + 3y + 10 = 0$ **(EX 5.3 - 2)**

3. $3x^2 + 2y^2 = 14$ **(EX 5.3 - 3)**
4. $x^2 + y^2 + x - y = 0$ **(EX 5.3 - 4)**
5. $11x^2 - 25y^2 - 44x + 50y - 256 = 0$ **(EX 5.3 - 5)**
6. $y^2 + 4x + 3y + 4 = 0$ **(EX 5.3 - 6)**

EXERCISE 5.4

3 MARKS

Example 5.27: Find the equations of tangent and normal to the parabola $x^2 + 6x + 4y + 5 = 0$ at $(1, -3)$. **(EG 5.27)**

Example 5.28: Find the equations of tangent and normal to the ellipse $x^2 + 4y^2 = 32$ when $\theta = \frac{\pi}{4}$. **(EG 5.28)**

1. Find the equation of the tangent to the parabola $y^2 = 16x$ perpendicular to $2x + 2y + 3 = 0$. **(EX 5.4 - 4)**
2. Find the equation of the tangent at $t = 2$to the parabola $y^2 = 8x$. (Hint: use parametric form) **(EX 5.4 - 5)**
3. Prove that the point of intersection of the tangents at t_1 and t_2 on the parabola$y^2 = 4ax$ is $[at_1t_2, a(t_1 + t_2)]$. **(EX 5.4 - 7)**
4. If the normal at the point t_1 on the parabola $y^2 = 4ax$ meets the parabola again at the point t_2 then prove that $t_2 = -\left(t_1 + \frac{2}{t_1}\right)$. **(EX 5.4 - 8)**

5 MARKS

5. Find the equations of the two tangents that can be drawn from $(5,2)$ to the ellipse $2x^2 + 7y^2 = 14$. **(EX 5.4 - 1)**
6. Find the equations of tangents to the hyperbola $\frac{x^2}{16} - \frac{y^2}{64} = 1$ which are parallel to$10x - 3y + 9 = 0$. **(EX 5.4 - 2)**
7. Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact. **(EX 5.4 - 3)**
8. Find the equations of the tangent and normal to hyperbola $12x^2 - 9y^2 = 108$ at $\theta = \frac{\pi}{3}$. (Hint: use parametric form) **(EX 5.4 - 6)**

EXERCISE 5.5

3 MARKS

Example 5.31: The maximum and minimum distances of the Earth from the Sun respectively are $152 \times 10^6 km$ and $94.5 \times 10^6 km$. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus. **(EG 5.31)**

Example 5.32: A concrete bridge is designed as a parabolic arch. The road over bridge is $40m$ long and the maximum height of the arch is $15m$. Write the equation of the parabolic arch. **(EG 5.32)**

Example 5.33: The parabolic communication antenna has a focus at $2m$ distance from the vertex of the antenna. Find the width of the antenna $3m$ from the vertex **(EG 5.33)**

Example 5.34: The equation $y = \frac{1}{32}x^2$ models cross sections of parabolic mirrors that are used for solar energy. There is a heating tube located at the focus of each parabola; how high is this tube located above the vertex of the parabola? **(EG 5.34)**

Example 5.35: A search light has a parabolic reflector (has a cross section that forms a bowl). The parabolic bowl is $40cm$ wide from rim to rim and $30cm$ deep. The bulb is located at the focus .

(a) What is the equation of the parabola used for reflector?

(b) How far from the vertex is the bulb to be placed so that the maximum distance covered? **(EG 5.35)**

Example 5.36: An equation of the elliptical part of an optical lens system is $\frac{x^2}{16} + \frac{y^2}{9} = 1$. The parabolic part of the system has a focus in common with the right focus of the ellipse .The vertex of the parabola is at the origin and the parabola opens to the right. Determine the equation of the parabola. **(EG 5.36)**

Example 5.37: A room $34m$ long is constructed to be a whispering gallery. The room has an elliptical ceiling, as shown in $Fig. 5.64$. If the maximum height of the ceiling is $8m$, determine where the foci are located. **(EG 5.37)**

Example 5.38: If the equation of the ellipse is $\frac{(x-11)^2}{484} + \frac{y^2}{64} = 1$ ($x$and y are measured in centimeters) where to the nearest centimeter, should the patient's kidney stone be placed so that the reflected sound hits the kidney stone? **(EG 5.38)**

5 MARKS

Example 5.30: A semielliptical archway over a one-way road has a height of $3m$ and a width of $12m$. The truck has a width of $3m$ and a height of $27.m$. Will the truck clear the opening of the archway? ($Fig. 5.6$) **(EG 5.30)**

Example 5.39: Two coast guard stations are located 600 km apart at points $A(0,0)$ and $B(0,600)$. A distress signal from a ship at P is received at slightly different times by two stations. It is determined that the ship is 200 km farther from station A than it is from station B . Determine the equation of hyperbola that passes through the location of the ship. **(EG 5.39)**

Example 5.40: Certain telescopes contain both parabolic mirror and a hyperbolic mirror. In the telescope shown in figure the parabola and hyperbola share focus F_1 which is $14m$above the vertex of the parabola. The hyperbola's second focus F_2 is $2m$ above the parabola's vertex. The vertex of the hyperbolic mirror is $1m$ below F_1. Position a coordinate system with the origin at the centre of the hyperbola and with the foci on the y −axis. Then find the equation of the hyperbola. **(EG 5.40)**

1. A bridge has a parabolic arch that is $10m$ high in the centre and $30m$ wide at the bottom. Find the height of the arch $6m$ from the centre, on either sides. **(EX 5.5 - 1)**
2. A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be $16m$, and the height at the edge of the road must be sufficient for a truck $4m$ high to clear if the highest point of the opening is to be $5m$ approximately . How wide must the opening be? **(EX 5.5 - 2)**
3. At a water fountain, water attains a maximum height of $4m$ at horizontal distance of 0.5 m from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75 m from the point of origin. **(EX 5.5 - 3)**

4. An engineer designs a satellite dish with a parabolic cross section. The dish is $5m$ wide at the opening, and the focus is placed $1.2\ m$ from the vertex (a) Position a coordinate system with the origin at the vertex and the x −axis on the parabola's axis of symmetry and find an equation of the parabola. (b) Find the depth of the satellite dish at the vertex. **(EX 5.5 - 4)**
5. Parabolic cable of a $60m$ portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every $6m$ along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex. **(EX 5.5 - 5)**
6. Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is $150\ m$ tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower. **(EX 5.5 - 6)**
7. A rod of length $1.2\ m$ moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is $0.3\ m$ from the end in contact with x −axis is an ellipse. Find the eccentricity. **(EX 5.5 - 7)**
8. Assume that water issuing from the end of a horizontal pipe, $7.5\ m$above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position $2.5\ m$below the line of the pipe, the flow of water has curved outward $3m$ beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground? **(EX 5.5 - 8)**
9. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of $4\ m$ when it is $6\ m$ away from the point of projection. Finally it reaches the ground $12\ m$ away from the starting point. Find the angle of projection. **(EX 5.5 - 9)**
10. Points A and B are $10\ km$ apart and it is determined from the sound of an explosion heard at those points at different times that the location of the explosion is $6\ km$ closer to A than B . Show that the location of the explosion is restricted to a particular curve and find an equation of it. **(EX 5.5 - 10)**

CHAPTER 6

APPLICATIONS OF VECTOR ALGEBRA

EXERCISE 6.1

2 MARKS

Example 6.11: Find the magnitude and the direction cosines of the torque about the point $(2,0,-1)$ of a force $2\vec{i} + \vec{j} - \vec{k}$ whose line of action passes through the origin. **(EG 6.11)**

1. A particle acted on by constant forces $8\vec{i} + 2\vec{j} - 6\vec{k}$ and $6\vec{i} + 2\vec{j} - 2\vec{k}$ is displaced from the point $(1,2,3)$ to the point $(5,4,1)$. Find the total work done by the forces. **(EX 6.1 – 11)**

3 MARKS

Example 6.1: (Cosine formulae): With usual notations, in any triangle ABC, prove the

following by vector method. $(i)\ a^2 = b^2 + c^2 - 2bc\cos A$
$(ii) b^2 = a^2 + c^2 - 2ac\cos B$ $(iii) c^2 = a^2 + b^2 - 2ab\cos C$. **(EG 6.1)**

Example 6.2: With usual notations, in any triangle ABC, prove the following by vector method. $(i)\ a = b\cos C + c\cos B$ $(ii)\ b = c\cos A + a\cos C$
$(iii)\ c = a\cos B + b\cos A$ **(EG 6.2)**

Example 6.4: With usual notations, in any triangle ABC, prove by vector method that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. **(EG 6.4)**

Example 6.6: (Apollonius's theorem): If D is the midpoint of the side BC of a triangle ABC, then show by vector method that
$|\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2 = 2\left(|\overrightarrow{AD}|^2 + |\overrightarrow{BD}|^2\right)$. **(EG 6.6)**

Example 6.8: In triangle ABC the points D, E, F are the midpoints of the sides BC, CA and AB respectively. Using vector method, show that the area of ΔDEF is equal to $\frac{1}{4}$ (area of ΔABC). **(EG 6.8)**

Example 6.10: A particle is acted upon by the forces $3\vec{\imath} - 2\vec{\jmath} + 2\vec{k}$ and $2\vec{\imath} + \vec{\jmath} - \vec{k}$ is displaced from the point $(1,3,-1)$ to the point $(4,-1,\lambda)$. If the work done by the forces is 16 units, find the value of λ. **(EG 6.10)**

Example 6.9: A particle acted upon by constant forces $2\vec{\imath} + 5\vec{\jmath} + 6\vec{k}$ and $-\vec{\imath} - 2\vec{\jmath} - \vec{k}$ is displaced from the point $(4,-3,-2)$ to the point $(6,1,-3)$. Find the total work done by the forces. **(EG 6.9)**

1. Prove by vector method that if a line is drawn from the centre of a circle to the midpoint of a chord, then the line is perpendicular to the chord. **(EX 6.1 - 1)**
2. Prove by vector method that the median to the base of an isosceles triangle is perpendicular to the base. **(EX 6.1 - 2)**
3. Prove by vector method that an angle in a semi-circle is a right angle. **(EX 6.1 - 3)**
4. Prove by vector method that the diagonals of a rhombus bisect each other at right angles. **(EX 6.1 - 4)**
5. Using vector method, prove that if the diagonals of a parallelogram are equal, then it is a rectangle. **(EX 6.1 - 5)**
6. Prove by vector method that the area of the quadrilateral $ABCD$ having diagonals AC and BD is $\frac{1}{2}|\overrightarrow{AC} \times \overrightarrow{BD}|$. **(EX 6.1 - 6)**
7. Prove by vector method that the parallelograms on the same base and between the same parallels are equal in area. **(EX 6.1 - 7)**
8. If G is the centroid of a ΔABC, prove that (area of ΔGAB) = (area of ΔGBC) = (area of ΔGCA) $= \frac{1}{3}$(area of ΔABC). **(EX 6.1 - 8)**
9. Forces of magnitudes $5\sqrt{2}$ and $10\sqrt{2}$ units acting in the directions $3\vec{\imath} + 4\vec{\jmath} + 5\vec{k}$ and $10\vec{\imath} + 6\vec{\jmath} - 8\vec{k}$ respectively, act on a particle which is displaced from the point with position vector $4\vec{\imath} - 3\vec{\jmath} - 2\vec{k}$ to the point with position vector $6\vec{\imath} + \vec{\jmath} - 3\vec{k}$. Find the work done by the forces. **(EX 6.1 - 12)**
10. Find the magnitude and direction cosines of the torque of a force represented by $3\vec{\imath} + 4\vec{\jmath} - 5\vec{k}$ about the point with position vector $2\vec{\imath} - 3\vec{\jmath} + 4\vec{k}$ acting through a point whose position vector is $4\vec{\imath} + 2\vec{\jmath} - 3\vec{k}$. **(EX 6.1 - 13)**
11. Find the torque of the resultant of the three forces represented by $-3\vec{\imath} + 6\vec{\jmath} - 3\vec{k}, 4\vec{\imath} - 10\vec{\jmath} + 12\vec{k}$ and $4\vec{\imath} + 7\vec{\jmath}$ acting at the point with position vector $8\vec{\imath} - 6\vec{\jmath} - 4\vec{k}$, about the point with position vector $18\vec{\imath} + 3\vec{\jmath} - 9\vec{k}$. **(EX 6.1 - 14)**

5 MARKS

Example 6.3: By vector method, prove that $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$. **(EG 6.3)**
Example 6.5: Prove by vector method that $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$. **(EG 6.5)**
Example 6.7: Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent. **(EG 6.7)**

1. Using vector method, prove that $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$. **(EX 6.1 - 9)**
2. Prove by vector method that $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$. **(EX 6.1 - 10)**

EXERCISE 6.2

2 MARKS

Example 6.12: If $\vec{a}=-3\vec{\imath}-\vec{\jmath}+5\vec{k},\ \vec{b}=\vec{\imath}-2\vec{\jmath}+\vec{k},\ \vec{c}=4\vec{\jmath}-5\vec{k}$, find $\vec{a}.(\vec{b}\times\vec{c})$. **(EG 6.12)**
Example 6.13: Find the volume of the parallelepiped whose coterminus edges are given by the vectors $2\vec{\imath}-3\vec{\jmath}+4\vec{k},\ \vec{\imath}+2\vec{\jmath}-\vec{k}$ and $3\vec{\imath}-\vec{\jmath}+2\vec{k}$. **(EG 6.13)**
Example 6.14: Show that the vectors $\vec{\imath}+2\vec{\jmath}-3\vec{k},\ 2\vec{\imath}-\vec{\jmath}+2\vec{k}$ and $3\vec{\imath}+\vec{\jmath}-\vec{k}$ are coplanar. **(EG 6.14)**
Example 6.15: If $2\vec{\imath}-\vec{\jmath}+3\vec{k},\ 3\vec{\imath}+2\vec{\jmath}+\vec{k},\vec{\imath}+m\vec{\jmath}+4\vec{k}$ are coplanar, find the value of m. **(EG 6.15)**
Example 6.16: Show that the four points $(6,-7,0),(16,-19,-4),(0,3,-6),(2,-5,10)$ Lie on a same plane. **(EG 6.16)**
Example 6.17: If the vectors $\vec{a},\vec{b},\vec{c}$ are coplanar, then prove that the vectors $\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}$ are also coplanar. **(EG 6.17)**
Example 6.18: If $\vec{a},\vec{b},\vec{c}$ are three vectors, prove that $[\vec{a}+\vec{c},\vec{a}+\vec{b},\vec{a}+\vec{b}+\vec{c}]=-[\vec{a},\vec{b},\vec{c}]$. **(EG 6.18)**

1. If $\vec{a}=\vec{\imath}-2\vec{\jmath}+3\vec{k},\ \vec{b}=2\vec{\imath}+\vec{\jmath}-2\vec{k},\ \vec{c}=3\vec{\imath}+2\vec{\jmath}+\vec{k}$, find $\vec{a}.(\vec{b}\times\vec{c})$. **(EX 6.2 - 1)**
2. Find the volume of the parallelepiped whose coterminous edges are represented by the vectors $-6\vec{\imath}+14\vec{\jmath}+10\vec{k},\ 14\vec{\imath}-10\vec{\jmath}-6\vec{k}$ and $2\vec{\imath}+4\vec{\jmath}-2\vec{k}$. **(EX 6.2 - 2)**
3. The volume of the parallelepiped whose coterminus edges are $7\vec{\imath}+\lambda\vec{\jmath}-3\vec{k},\ \vec{\imath}+2\vec{\jmath}-\vec{k}$ and $-3\vec{\imath}+7\vec{\jmath}+5\vec{k}$ is 90 cubic units. Find the value of λ. **(EX 6.2 - 3)**
4. Determine whether the three vectors $2\vec{\imath}+3\vec{\jmath}+\vec{k},\ \vec{\imath}-2\vec{\jmath}+2\vec{k}$ and $3\vec{\imath}+\vec{\jmath}+3\vec{k}$ are coplanar. **(EX 6.2 - 6)**
5. Let $\vec{a}=\vec{\imath}+\vec{\jmath}+\vec{k},\ \vec{b}=\vec{\imath}$ and $\vec{c}=c_1\vec{\imath}+c_2\vec{\jmath}+c_3\vec{k}$. If $c_1=1$ and $c_2=2$, find c_3 such that $\vec{a},\vec{b}$ and $\vec{c}$ are coplanar. **(EX 6.2 - 7)**
6. If $\vec{a}=\vec{\imath}-\vec{k},\ \vec{b}=x\vec{\imath}+\vec{\jmath}+(1-x)\vec{k},\ \vec{c}=y\vec{\imath}+x\vec{\jmath}+(1+x-y)\vec{k}$ show that $[\vec{a},\vec{b},\vec{c}]$ depends on neither x nor y. **(EX 6.2 - 8)**
7. If the vectors $\vec{a}=a\vec{\imath}+a\vec{\jmath}+a\vec{k},\ \vec{\imath}+\vec{k},\ c\vec{\imath}+c\vec{\jmath}+b\vec{k}$ are coplanar, prove that c is the geometric mean of a and b. **(EX 6.2 - 9)**
8. Let $\vec{a},\vec{b},\vec{c}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, show that $[\vec{a},\vec{b},\vec{c}]^2=\frac{1}{4}|\vec{a}|^2|\vec{b}|^2$. **(EX 6.2 - 10)**

3 MARKS

1. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors represented by concurrent edges of a parallelepiped of volume 4 cubic units, find the value of
$(\vec{a}+\vec{b}).(\vec{b}\times\vec{c})+(\vec{b}+\vec{c}).(\vec{c}\times\vec{a})+(\vec{c}+\vec{a}).(\vec{a}\times\vec{b})$. **(EX 6.2 - 4)**
2. Find the altitude of a parallelepiped determined by the vectors $\vec{a}=-2\vec{i}+5\vec{j}+3\vec{k},\ \vec{b}=\vec{i}+3\vec{j}-2\vec{k}$ and $\vec{c}=-3\vec{i}+\vec{j}+4\vec{k}$ if the base is taken as the parallelogram determined by $\vec{b}$ and $\vec{c}$. **(EX 6.2 - 5)**

EXERCISE 6.3

2 MARKS

Example 6.20: Prove that $\left(\vec{a}.(\vec{b}\times\vec{c})\right)\vec{a}=(\vec{a}\times\vec{b})\times(\vec{a}\times\vec{c})$. **(EG 6.20)**

1. If $\vec{a}=\vec{i}-2\vec{j}+3\vec{k},\ \vec{b}=2\vec{i}+\vec{j}-2\vec{k},\ \vec{c}=3\vec{i}+2\vec{j}+\vec{k}$, find $(i)\ (\vec{a}\times\vec{b})\times\vec{c}\ (ii)\ \vec{a}\times(\vec{b}\times\vec{c})$. **(EX 6.3 - 1)**
2. For any vector $\vec{a}$, prove that $\vec{i}\times(\vec{a}\times\vec{i})+\vec{j}\times(\vec{a}\times\vec{j})+\vec{k}\times(\vec{a}\times\vec{k})=2\vec{a}$. **(EX 6.3 - 2)**
3. Prove that $[\vec{a}-\vec{b},\vec{b}-\vec{c},\vec{c}-\vec{a}]=0$. **(EX 6.3 - 3)**
4. $\vec{a}=2\vec{i}+3\vec{j}-\vec{k},\ \vec{b}=-\vec{i}+2\vec{j}-4\vec{k},\ \vec{c}=\vec{i}+\vec{j}+\vec{k}$, then find the value of $(\vec{a}\times\vec{b}).(\vec{a}\times\vec{c})$. **(EX 6.3 - 5)**
5. If $\vec{a},\vec{b},\vec{c},\vec{d}$ are coplanar vectors, then show that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{0}$. **(EX 6.3 - 6)**
6. If $\vec{a}=\vec{i}+2\vec{j}+3\vec{k},\ \vec{b}=2\vec{i}-\vec{j}+\vec{k},\ \vec{c}=3\vec{i}+2\vec{j}+\vec{k}$, and $\vec{a}\times(\vec{b}\times\vec{c})=l\vec{a}+m\vec{b}+n\vec{c}$, find the values of l,m,n. **(EX 6.3 - 7)**
7. If $\vec{a},\vec{b},\vec{c}$ are three unit vectors such that $\vec{b}$ and $\vec{c}$ are non-parallel and $\vec{a}\times(\vec{b}\times\vec{c})=\frac{1}{2}\vec{b}$, find the angle between $\vec{a}$ and $\vec{c}$. **(EX 6.3 - 8)**

3 MARKS

Example 6.19: Prove that $[\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}]=[\vec{a},\vec{b},\vec{c}]^2$. **(EG 6.19)**

Example 6.21: For any four vectors $\vec{a},\vec{b},\vec{c},\vec{d}$ we have
$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{b},\vec{d}]\vec{c}-[\vec{a},\vec{b},\vec{c}]\vec{d}=[\vec{a},\vec{c},\vec{d}]\vec{b}-[\vec{b},\vec{c},\vec{d}]\vec{a}$ **(EG 6.21)**

Example 6.22: If $\vec{a}=-2\vec{i}+3\vec{j}-2\vec{k},\ \vec{b}=3\vec{i}-\vec{j}+3\vec{k},\ \vec{c}=2\vec{i}-5\vec{j}+\vec{k}$, find $(\vec{a}\times\vec{b})\times\vec{c}$ and $\vec{a}\times(\vec{b}\times\vec{c})$ State whether they are equal. **(EG 6.22)**

Example 6.23: If $\vec{a}=2\vec{i}-\vec{j},\ \vec{b}=\vec{i}-\vec{j}-4\vec{k},\ \vec{c}=3\vec{j}-\vec{k}$ and $\vec{d}=2\vec{i}+5\vec{j}+\vec{k}$, verify that
$(i)\ (\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{b},\vec{d}]\vec{c}-[\vec{a},\vec{b},\vec{c}]\vec{d}$
$(ii)\ (\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{c},\vec{d}]\vec{b}-[\vec{b},\vec{c},\vec{d}]\vec{a}$ **(EG 6.23)**

5 MARKS

1. If $\vec{a}=2\vec{i}+3\vec{j}-\vec{k},\ \vec{b}=3\vec{i}+5\vec{j}+2\vec{k},\ \vec{c}=-\vec{i}-2\vec{j}+3\vec{k}$, verify that
$(i)\ (\vec{a}\times\vec{b})\times\vec{c}=(\vec{a}.\vec{c})\vec{b}-(\vec{b}.\vec{c})\vec{a}\ \ (ii)\ \vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$. **(EX 6.3 - 4)**

EXERCISE 6.4

2 MARKS

Example 6.29: Find the angle between the lines $\vec{r} = (\vec{i} + 2\vec{j} + 4\vec{k}) + t(2\vec{i} + 2\vec{j} + \vec{k})$ and the straight line passing through the points $(5,1,4)$ and $(9,2,12)$. **(EG 6.29)**

Example 6.32: Show that the lines $\frac{x-1}{4} = \frac{2-y}{6} = \frac{z-4}{12}$ and $\frac{x-3}{-2} = \frac{y-3}{3} = \frac{5-z}{6}$ are parallel. **(EG 6.32)**

1. Find the angle between the following lines.
   $(i)\ \vec{r} = (4\vec{i} - \vec{j}) + t(\vec{i} + 2\vec{j} - 2\vec{k}),\ \vec{r} = (\vec{i} - 2\vec{j} + 4\vec{k}) + s(-\vec{i} - 2\vec{j} + 2\vec{k})$. **(EX 6.4 - 5)**
   $(ii)\ \frac{x+4}{3} = \frac{y-7}{4} = \frac{z+5}{5},\ \vec{r} = 4\vec{k} + t(2\vec{i} + \vec{j} + \vec{k})$
   $(iii) 2x = 3y = -z$ and $6x = -y = -4z$.
2. The vertices of ΔABC are $A(7,2,1)$, $B(6,0,3)$, and $C(4,2,4)$. Find $\angle ABC$. **(EX 6.4 - 6)**
3. If the straight line joining the points $(2,1,4)$ and $(a-1,4,-1)$ is parallel to the line joining the points $(0,2,b-1)$ and $(5,3,-2)$, find the values of a and b. **(EX 6.4 - 7)**
4. If the straight lines $\frac{x-5}{5m+2} = \frac{2-y}{5} = \frac{1-z}{-1}$ and $x = \frac{2y+1}{4m} = \frac{1-z}{-3}$ are perpendicular to each other, find the value of m . **(EX 6.4 - 8)**
5. Show that the points $(2,3,4), (-1,4,5)$ $-$and $(8,1,2)$ are collinear. **(EX 6.4 - 9)**

3 MARKS

Example 6.24: A straight line passes through the point $(1,2,-3)$ and parallel to $4\vec{i} + 5\vec{j} - 7\vec{k}$. Find (i) vector equation in parametric form (ii) vector equation in non-parametric form (iii) Cartesian equations of the straight line. **(EG 6.24)**

Example 6.25: The vector equation in parametric form of a line is $\vec{r} = (3\vec{i} - 2\vec{j} + 6\vec{k}) + t(2\vec{i} - \vec{j} + 3\vec{k})$. Find (i) the direction cosines of the straight line (ii) vector equation in non-parametric form of the line (iii) Cartesian equations of the line. **(EG 6.25)**

Example 6.26: Find the vector equation in parametric form and Cartesian equations of the line passing through $(-4,2,-3)$ and is parallel to the line $\frac{-x-2}{4} = \frac{y+3}{-2} = \frac{2z-6}{3}$. **(EG 6.26)**

Example 6.27: Find the vector equation in parametric form and Cartesian equations of a straight passing through the points $(-5,7,-4)$ and $(13,-5,2)$. Find the point where the straight line crosses the xy $-$plane. **(EG 6.27)**

Example 6.28: Find the angle between the straight line $\frac{x+3}{2} = \frac{y-1}{2} = -z$ with coordinate axes. **(EG 6.28)**

Example 6.30: Find the angle between the straight lines $\frac{x-4}{2} = \frac{y}{1} = \frac{z+1}{-2}$ and $\frac{x-1}{4} = \frac{y+1}{-4} = \frac{z-2}{2}$ and state whether they are parallel or perpendicular. **(EG 6.30)**

Example 6.31: Show that the straight line passing through the points $A(6,7,5)$ and $B(8,10,6)$ is perpendicular to the straight line passing through the points $C(10,2,-5)$ and $D(8,3,-4)$. **(EG 6.31)**

1. Find the non-parametric form of vector equation and Cartesian equations of the straight line passing through the point with position vector $4\vec{i} + 3\vec{j} - 7\vec{k}$ and parallel to the vector $2\vec{i} - 6\vec{j} + 7\vec{k}$. **(EX 6.4 - 1)**
2. Find the parametric form of vector equation and Cartesian equations of the straight line passing through the point $(-2,3,4)$ and parallel to the straight line $\frac{x-1}{-4} = \frac{y+3}{5} = \frac{8-z}{6}$. **(EX 6.4 - 2)**
3. Find the points where the straight line passes through $(6,7,4)$ and $(8,4,9)$ cuts the xz and yz planes. **(EX 6.4 - 3)**
4. Find the direction cosines of the straight line passing through the points $(5,6,7)$ and $(7,9,13)$. Also, find the parametric form of vector equation and Cartesian equations of the straight line passing through two given points. **(EX 6.4 - 4)**

EXERCISE 6.5

3 MARKS

Example 6.33: Find the point of intersection of the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$. **(EG 6.33)**

Example 6.35: Determine whether the pair of straight lines $\vec{r}=(2\vec{\imath}+6\vec{\jmath}+3\vec{k})+t(2\vec{\imath}+3\vec{\jmath}+4\vec{k}), \vec{r}=(2\vec{\jmath}-3\vec{k})+s(\vec{\imath}+2\vec{\jmath}+3\vec{k})$ are parallel. Find the shortest distance between them. **(EG 6.35)**

Example 6.36: Find the shortest distance between the two given straight lines $\vec{r}=(2\vec{\imath}+3\vec{\jmath}+4\vec{k})+t(-2\vec{\imath}+\vec{\jmath}-2\vec{k})$ and $\frac{x-3}{2}=\frac{y}{-1}=\frac{z+2}{2}$. **(EG 6.36)**

1. Find the parametric form of vector equation and Cartesian equations of a straight line passing through $(5,2,8)$ and is perpendicular to the straight lines $\vec{r}=(\vec{\imath}+\vec{\jmath}-\vec{k})+s(2\vec{\imath}-2\vec{\jmath}+\vec{k})$ and $\vec{r}=(2\vec{\imath}-\vec{\jmath}-3\vec{k})+t(\vec{\imath}+2\vec{\jmath}+2\vec{k})$. **(EX 6.5 - 1)**
2. Show that the lines $\vec{r}=(6\vec{\imath}+\vec{\jmath}+2\vec{k})+s(\vec{\imath}+2\vec{\jmath}-3\vec{k})$ and $\vec{r}=(3\vec{\imath}+2\vec{\jmath}-2\vec{k})+t(2\vec{\imath}+4\vec{\jmath}-5\vec{k})$ are skew lines and hence find the shortest distance between them. **(EX 6.5 - 2)**
3. If the two lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-m}{2}=z$ intersect at a point, find the value of m. **(EX 6.5 - 3)**
4. Show that the straight lines $x+1=2y=-12z$ and $x=y+2=6z-6$ are skew and hence find the shortest distance between them. **(EX 6.5 - 5)**

5 MARKS

Example 6.34: Find the equation of a straight line passing through the point of intersection of the straight lines $\vec{r}=(\vec{\imath}+3\vec{\jmath}-\vec{k})+t(2\vec{\imath}+3\vec{\jmath}+2\vec{k})$ and $\frac{x-2}{1}=\frac{y-4}{2}=\frac{z+3}{4}$, and perpendicular to both straight lines. **(EG 6.34)**

Example 6.37: Find the coordinates of the foot of the perpendicular drawn from the point $(-1,2,3)$ to the straight line $\vec{r}=(\vec{\imath}-4\vec{\jmath}+3\vec{k})+t(2\vec{\imath}+3\vec{\jmath}+\vec{k})$. Also, find the shortest distance from the point to the straight line. **(EG 6.37)**

5. Show that the lines $\frac{x-3}{3}=\frac{y-3}{-1}, z-1=0$ and $\frac{x-6}{2}=\frac{z-1}{3}, y-2=0$ intersect. Also find the point of intersection. **(EX 6.5 - 4)**
6. Find the parametric form of vector equation of the straight line passing through $(-1,2,1)$ and parallel to the straight line $\vec{r}=(2\vec{\imath}+3\vec{\jmath}-\vec{k})+t(\vec{\imath}-2\vec{\jmath}+\vec{k})$ and hence find the shortest distance between the lines. **(EX 6.5 - 6)**
7. Find the foot of the perpendicular drawn from the point $(5,4,2)$ to the line $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}$. Also, find the equation of the perpendicular. **(EX 6.5 - 7)**

EXERCISE 6.6

2 MARKS

Example 6.38: Find the vector and Cartesian form of the equations of a plane which is at a distance of 12 units from the origin and perpendicular to $6\vec{i}+2\vec{j}-3\vec{k}$. **(EG 6.38)**

Example 6.39: If the cartesian equation of a plane is $3x-4y-3z=-8$, find the vector equation of the plane in the standard form. **(EG 6.39)**

Example 6.40: Find the direction cosines and length of the perpendicular from the origin to the plane $\vec{r}.\left(3\vec{i}-4\vec{j}+12\vec{k}\right)=5$. **(EG 6.40)**

1. Find a parametric form of vector equation of a plane which is at a distance of 7 units from the origin having $3,-4,5$ as direction ratios of a normal to it. **(EX 6.6 - 1)**
2. Find the vector and Cartesian equations of the plane passing through the point with position vector $2\vec{i}+6\vec{j}+3\vec{k}$ and normal to the vector $\vec{i}+3\vec{j}+5\vec{k}$. **(EX 6.6 - 3)**

3 MARKS

Example 6.41: Find the vector and Cartesian equations of the plane passing through the point with position vector $4\vec{i}+2\vec{j}-3\vec{k}$ and normal to vector $2\vec{i}-\vec{j}+\vec{k}$. **(EG 6.41)**

Example 6.42: A variable plane moves in such a way that the sum of the reciprocals of its intercepts on the coordinate axes is a constant. Show that the plane passes through a fixed point. **(EG 6.42)**

1. Find the direction cosines of the normal to the plane $12x+3y-4z=65$. Also, find the non-parametric form of vector equation of a plane and the length of the perpendicular to the plane from the origin. **(EX 6.6 - 2)**
2. A plane passes through the point $(-1,1,2)$ and the normal to the plane of magnitude $3\sqrt{3}$ makes equal acute angles with the coordinate axes. Find the equation of the plane. **(EX 6.6 - 4)**
3. Find the intercepts cut off by the plane $\vec{r}.\left(6\vec{i}+4\vec{j}-3\vec{k}\right)=12$ on the coordinate axes. **(EX 6.6 - 5)**
4. If a plane meets the coordinate axes at A,B,C such that the centriod of the triangle ABC is the point (u,v,w), find the equation of the plane. **(EX 6.6 - 6)**

EXERCISE 6.7

5 MARKS

Example 6.43: Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point $(0,1,-5)$ and parallel to the straight lines $\vec{r}=\left(\vec{i}+2\vec{j}-4\vec{k}\right)+s\left(2\vec{i}+3\vec{j}+6\vec{k}\right)$ and $\vec{r}=\left(\vec{i}-3\vec{j}+5\vec{k}\right)+t\left(\vec{i}+\vec{j}-\vec{k}\right)$. **(EG 6.43)**

Example 6.44: Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points $(-1,2,0),(2,2,-1)$ and parallel to the straight line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$. **(EG 6.44)**

1. Find the non-parametric form of vector equation, and cartesian equation of the plane passing through the point $(2,3,6)$ and parallel to the straight lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-3}{1}$ and $\frac{x+3}{2}=\frac{y-3}{-5}=\frac{z+1}{-3}$. **(EX 6.7 – 1)**

2. Find the parametric form of vector equation, and Cartesian equations of the plane passing through the points $(2,2,1),(9,3,6)$ and perpendicular to the plane $2x+6y+6z=9$. **(EX 6.7 – 2)**
3. Find parametric form of vector equation and Cartesian equations of the plane passing through the points $(2,2,1),(1,-2,3)$ and parallel to the straight line passing through the points $(2,1,-3)$ and $(-1,5,-8)$. **(EX 6.7 – 3)**
4. Find the non-parametric form of vector equation of the plane passing through the point $(1,-2,4)$ and perpendicular to the plane $x+2y-3z=11$ and parallel to the line $\frac{x+7}{3}=\frac{y+3}{-1}=\frac{z}{1}$. **(EX 6.7 – 4)**
5. Find the parametric form of vector equation, and Cartesian equations of the plane containing the line $\vec{r}=(\vec{\imath}-\vec{\jmath}+3\vec{k})+t(2\vec{\imath}-\vec{\jmath}+4\vec{k})$ and perpendicular to plane $\vec{r}.(\vec{\imath}+2\vec{\jmath}+\vec{k})=8$. **(EX 6.7 – 5)**
6. Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the points $(3,6,-2),(-1,-2,6)$ and $(6,-4,-2)$. **(EX 6.7 – 6)**
7. Find the non-parametric form of vector equation, and Cartesian equations of the plane $\vec{r}=(6\vec{\imath}-\vec{\jmath}+\vec{k})+s(-\vec{\imath}+2\vec{\jmath}+\vec{k})+t(-5\vec{\imath}-4\vec{\jmath}-5\vec{k})$. **(EX 6.7 – 7)**

EXERCISE 6.7

2 MARKS

Example 6.45: Verify whether the line $\frac{x-3}{-4}=\frac{y-4}{-7}=\frac{z+3}{12}$ lies in the plane $5x-y+z=8$. **(EG 6.45)**

3 MARKS

Example 6.46: Show that the lines $\vec{r}=(-\vec{\imath}-3\vec{\jmath}-5\vec{k})+s(3\vec{\imath}+5\vec{\jmath}+7\vec{k})$ and $\vec{r}=(2\vec{\imath}+4\vec{\jmath}+6\vec{k})+t(\vec{\imath}+4\vec{\jmath}+7\vec{k})$ are coplanar. Also, find the non-parametric form of vector equation of the plane containing these lines. **(EG 6.46)**

1. Show that the straight lines $\vec{r}=(5\vec{\imath}+7\vec{\jmath}-3\vec{k})+s(4\vec{\imath}+4\vec{\jmath}-5\vec{k})$ and $\vec{r}=(8\vec{\imath}+4\vec{\jmath}+5\vec{k})+t(7\vec{\imath}+\vec{\jmath}+3\vec{k})$ are coplanar. Find the vector equation of the plane in which they lie. **(EX 6.8 – 1)**

5 MARKS

1. Show that the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{3}$ and $\frac{x-1}{-3}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar. Also, find the plane containing these lines. **(EX 6.8 – 2)**
2. Show that the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{m^2}$ and $\frac{x-3}{1}=\frac{y-2}{m^2}=\frac{z-1}{2}$ are coplanar, find the distinct real values of m. **(EX 6.8 – 3)**

3. If the straight lines $\frac{x-1}{2}=\frac{y+1}{\lambda}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{\lambda}$ are coplanar, find λ and equations of the planes containing these two lines. **(EX 6.8 – 4)**

EXERCISE 6.8

2 MARKS

Example 6.47: Find the acute angle between the planes $\vec{r}.(2\vec{\imath}+2\vec{\jmath}+2\vec{k})=11$ and $4x-2y+2z=15$. **(EG 6.47)**

Example 6.48: Find the angle between the straight line $\vec{r}=(2\vec{\imath}+3\vec{\jmath}+\vec{k})+t(\vec{\imath}-\vec{\jmath}+\vec{k})$ and the plane $2x-y+z=5$. **(EG 6.48)**

Example 6.49: Find the distance of a point $(2,5,-3)$ from the plane $\vec{r}.(6\vec{\imath}-3\vec{\jmath}+2\vec{k})=5$. **(EG 6.49)**

Example 6.51: Find the distance between the parallel planes $x+2y-2z+1=0$ and $2x+4y-4z+5=0$. **(EG 6.51)**

Example 6.52: Find the distance between the planes $\vec{r}.(2\vec{\imath}-\vec{\jmath}-2\vec{k})=6$ and $\vec{r}.(6\vec{\imath}-3\vec{\jmath}-6\vec{k})=27$. **(EG 6.52)**

1. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.(2\vec{\imath}-\overline{7}\vec{\jmath}+4\vec{k})=3$ and $3x-5y+4z+11=0$, and the point $(-2,1,3)$. **(EX 6.9 – 1)**
2. Find the angle between the line $\vec{r}=(2\vec{\imath}-\vec{\jmath}+\vec{k})+t(\vec{\imath}+2\vec{\jmath}-2\vec{k})$ and the plane $\vec{r}.(6\vec{\imath}+3\vec{\jmath}+2\vec{k})=8$. **(EX 6.9 – 3)**
3. Find the angle between the planes $\vec{r}.(\vec{\imath}+\vec{\jmath}-2\vec{k})=3$and $2x-2y+z=2$. **(EX 6.9 – 4)**
4. Find the equation of the plane which passes through the point $(3,4,-1)$ and is parallel to the plane $2x-3y+5z+7=0$. Also, find the distance between the two planes. **(EX 6.9 – 5)**
5. Find the length of the perpendicular from the point $(1,-2,3)$ to the plane $x-y+z=5$. **(EX 6.9 – 6)**
6. Find the point of intersection of the line $x-1=\frac{y}{2}=z+1$ with the plane $2x-y+2z=2$. Also, find the angle between the line and the plane. **(EX 6.9 – 7)**
7.

3 MARKS

Example 6.50: Find the distance of the point $(5,-5,-10)$ from the point of intersection of a straight line passing through the points $A(4,1,2)$ and $B(7,5,4)$ with the plane $x-y+z=5$. **(EG 6.50)**

Example 6.53: Find the equation of the plane passing through the intersection of the planes $\vec{r}.(\vec{\imath}+\vec{\jmath}+\vec{k})+1=0$ and $\vec{r}.(2\vec{\imath}-3\vec{\jmath}+5\vec{k})=2$ and the point $(-1,2,1)$. **(EG 6.53)**

Example 6.54: Find the equation of the plane passing through the intersection of the planes $2x+3y-z+7=0$ and $x+y-2z+5=0$ and is perpendicular to the plane $x+y-3z-5=0$. **(EG 6.54)**

Example 6.55: Find the image of the point whose position vector is $\vec{\imath}+2\vec{\jmath}+3\vec{k}$ in the plane $\vec{r}.(\vec{\imath}+2\vec{\jmath}+4\vec{k})=38$. **(EG 6.55)**

Example 6.56: Find the coordinates of the point where the straight line $\vec{r}=(2\vec{\imath}-\vec{\jmath}+2\vec{k})+t(3\vec{\imath}+4\vec{\jmath}+2\vec{k})$ intersects the plane

$x - y + z - 5 = 0$. **(EG 6.56)**

1. Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point $(4,3,2)$ to the plane $x + 2y + 3z = 2$. **(EX 6.9 – 8)**

5 MARKS

1. Find the equation of the plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z + 11 = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3,1,-1)$. **(EX 6.9 – 2)**