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UNIT-4- VECTOR CALCULUS

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UNIT-4-VECTOR CALCULUS

Vector calculus deals with variable vectors which are varying in magnitude or direction or both. A physical quantity, that is a function of the points in space is called a scalar function (denoted by ∅) or a vector function(denoted by $\underset{F}{\rightarrow}$) according as the quantity is a scalar or a vector.

- Temperature at any point and electric potential are examples of a scalar function.
- Velocity of a moving particle and gravitational force are examples of a vector function.

Vector Differential operator - ∇

∇ is an operator which can be operated on both scalar and vector function. It is known as Del.

$$\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

- When **∇** is operated on a scalar function ,It gives the gradient ∇∅.
- When **∇** is operated on a vector function,It gives the Divergence and Curl.

Gradient of a scalar point function.

Let ∅ (x, y, z) be a scalar point function defined in a certain region of space.

$$\nabla\emptyset = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)\emptyset \quad = \vec{i}\frac{\partial\emptyset}{\partial x} + \vec{j}\frac{\partial\emptyset}{\partial y} + \vec{k}\frac{\partial\emptyset}{\partial z}$$

is defined as the gradient of ∅ and shortly defined as Grad ∅.

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- **Find grad $\emptyset$ at the point $(1,-2,-1)$ when $\emptyset = 3x^2y - y^3z^2$.**

Solution:

Let $\emptyset = 3x^2y - y^3z^2$

$$\text{Grad}\emptyset = \vec{i}\frac{\partial\emptyset}{\partial x} + \vec{j}\frac{\partial\emptyset}{\partial y} + \vec{k}\frac{\partial\emptyset}{\partial z}$$

$$\frac{\partial\emptyset}{\partial x} = 6xy \ ; \ \frac{\partial\emptyset}{\partial y} = 3x^2 - 3y^2z^2 \ ; \frac{\partial\emptyset}{\partial z} = -2y^3z$$

$$\text{Grad}\ \emptyset = \vec{i}\ 6xy + \vec{j}(3x^2 - 3y^2z^2) + \vec{k}\ (-2y^3z)$$

$$\text{Grad}\emptyset_{(1,-2,-1)} = -12\,\vec{i} - 9\vec{j} - 16\vec{k}$$

- **If $\vec{r}$ is the position vector of the point (x,y,z), prove that $\nabla(r) = \left(\frac{1}{r}\right)\vec{r}$.**

Solution:

Let $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

$$|\vec{r}\,| = r = \sqrt{x^2 + y^2 + z^2}$$

$r^2 = x^2 + y^2 + z^2$

Partially differentiating, with respect to x, y, z we get,

$$2r\frac{\partial r}{\partial x} = 2x \ ; \ 2r\frac{\partial r}{\partial y} = 2y \ ; \ 2r\frac{\partial r}{\partial z} = 2z$$

$$\nabla(r) = \vec{i}\frac{\partial r}{\partial x} + \vec{j}\frac{\partial r}{\partial y} + \vec{k}\frac{\partial r}{\partial z}$$

$$= \vec{i}\frac{x}{r} + \vec{j}\frac{y}{r} + \vec{k}\frac{z}{r}$$

$$= \frac{x\vec{i} + y\vec{j} + z\vec{k}}{r} \quad \frac{\vec{}}{r} \quad = \quad r$$

- **Find the unit vector normal to the surface $xy^3z^2 = 4$ at $(-1,-1,2)$.**

Solution:

Unit vector normal to the surface is $\hat{n} = \dfrac{\vec{n}}{|\vec{n}|}$

where $\hat{n}$ is the unit in the direction of the normal.

$\vec{n} = \nabla\emptyset$ at the given point.

Let $\emptyset = xy^3z^2 - 4$

$$\vec{n} = \nabla\emptyset = \vec{i}(y^3z^2) + \vec{j}(3xy^2z^2) + \vec{k}(2xy^3z)$$

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$$\nabla\phi_{(-1,-1,2)} = -4\vec{i} - 12\vec{j} + 4\vec{k}$$

$$|\nabla\phi| = \sqrt{16+144+16} = \sqrt{176} = 4\sqrt{11}$$

$$\therefore \ |\vec{n}| = 4\sqrt{11}$$

$$\hat{n} = \frac{-4\vec{i} - 12\vec{j} + 4\vec{k}}{4\sqrt{11}} = \frac{-\vec{i} - 3\vec{j} + \vec{k}}{\sqrt{11}}$$

- **Find the angle between the normals to the surface $xy^3z^2 = 4$ at the points $(-1,-1,2)$ and $(4,1,-1)$.**

<u>**Solution:**</u>

we know that, $\quad \cos\theta = \dfrac{\vec{n_1}.\vec{n_2}}{|\vec{n_1}|.|\vec{n_2}|}$

Let $\vec{n_1} = \nabla\phi$ at $(-1,-1,2)$

$\vec{n_2} = \nabla\phi$ at $(4,1,-1)$ where $\phi = xy^3z^2 - 4$

$$\nabla\phi = \vec{i}(y^3z^2) + \vec{j}(3xy^2z^2) + \vec{k}(2xy^3z)$$

$$\nabla\phi \text{ at } (-1,-1,2) = -4\vec{i} - 12\vec{j} + 4\vec{k}$$

$$|\nabla\phi| = |\vec{n_1}| = \sqrt{(16+144+16)} = \sqrt{176}$$

$$\nabla\phi \text{ at } (4,1,-1) = \vec{i} + 12\vec{j} - 8\vec{k}$$

$$|\nabla\phi| = |\vec{n_2}| = \sqrt{(1+144+64)} = \sqrt{209}$$

$$\therefore \cos\theta = \frac{(-4\vec{i} - 12\vec{j} + 4\vec{k}).(\vec{i} + 12\vec{j} - 8\vec{k})}{\sqrt{176}.\sqrt{209}}$$

$$= \frac{-4 - 144 - 32}{\sqrt{176}.\sqrt{209}}$$

$$= \frac{-180}{\sqrt{176}.\sqrt{209}} = \frac{-45}{\sqrt{19.11}}$$

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- **Find the angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point $(-1, -2, 1)$.**

<u>**Solution:**</u>

we know that, $\text{Cos}\theta = \dfrac{\vec{n_1}.\vec{n_2}}{|\vec{n_1}|.|\vec{n_2}|}$

$\vec{n_1} = \nabla\phi_1$ at $(-1,-2,1)$

$\vec{n_2} = \nabla\phi_2$ at $(-1,-2,1)$

Let $\phi_1 = xy^2z - 3x - z^2$

$\nabla\phi_1 = \vec{i}(y^2z - 3) + \vec{j}(2xyz) + \vec{k}(xy^2 - 2z)$

$\nabla\phi_1$ at $(-1,-2,1) = \vec{i} - 4\vec{j} - 6\vec{k}$

$|\nabla\phi_1| = \sqrt{(1 + 16 + 36)} = \sqrt{53}$

Let $\phi_2 = 3x^2 - y^2 + 2z = 1$

$\nabla\phi_2 = \vec{i}\,(6x) + \vec{j}\,(-2y) + \vec{k}(2)$

$\nabla\phi_2$ at $(-1,-2,1) = -6\,\vec{i} + 4\vec{j} + 2\vec{k}$

$|\nabla\phi_2| = \sqrt{(36 + 16 + 4)} = \sqrt{56}$

$$\therefore \quad \text{Cos}\theta = \frac{\left(\vec{i} - 4\vec{j} - 6\vec{k}\right).(-6\,\vec{i} + 4\vec{j} + 2\vec{k})}{\sqrt{53}\,\sqrt{56}}$$

$$= \frac{-6 - 16 - 12}{\sqrt{53}\,\sqrt{56}} = \frac{-34}{\sqrt{53}\,\sqrt{56}}$$

- **Find the angle between the surfaces $x^2 - y^2 - z^2 = 11$ and $xy + yz - zx = 18$ at the point (6, 4, 3).**

<u>**Solution:**</u>

we know that, $\text{Cos}\theta = \dfrac{\vec{n_1}.\vec{n_2}}{|\vec{n_1}|.|\vec{n_2}|}$

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$\overrightarrow{n_1} = \nabla\phi_1$ at (6,4,3)

$\overrightarrow{n_2} = \nabla\phi_2$ at (6,4,3)

Let $\phi_1 = x^2 - y^2 - z^2 - 11$

$\nabla\phi_1 = \vec{\imath}\,(2x) + \vec{\jmath}\,(-2y) + \vec{k}(-2z)$

$\nabla\phi_1$ at$(6, 4, 3) = 12\,\vec{\imath} - 8\vec{\jmath} - 6\vec{k}$

$|\nabla\phi_1| = \sqrt{(144 + 64 + 36)} = \sqrt{244}$

Let $\phi_2 = xy + yz - zx - 18$

$\nabla\phi_2 = \vec{\imath}\,(y - z) + \vec{\jmath}\,(x + z) + \vec{k}(y - x)$

$\nabla\phi_2$ at $(-1, -2, 1) = \vec{\imath} + 9\vec{\jmath} - 2\vec{k}$

$|\nabla\phi_2| = \sqrt{(1 + 81 + 4)} = \sqrt{86}$

$$\therefore \ \text{Cos}\theta = \frac{(12\,\vec{\imath} - 8\vec{\jmath} - 6\vec{k}).(\vec{\imath} + 9\vec{\jmath} - 2\vec{k})}{\sqrt{244}\,\sqrt{86}}$$

$$= \frac{12 - 72 + 124}{\sqrt{244}\,\sqrt{86}}$$

$$= \frac{-24}{\sqrt{5246}}$$

- **Find the values of $\boldsymbol{\lambda}$ and μ if the surfaces $\boldsymbol{\lambda x^2 - \mu yz = (\lambda + 2)x}$ and $\boldsymbol{4x^2y + z^3 = 4}$ cut orthogonally at the point $\boldsymbol{(1, -1, 2)}$.** [8 Marks]

Solution:

Let $\lambda x^2 - \mu yz - (\lambda + 2)x = 0$ ----------(1) and

$4x^2y + z^3 - 4 = 0$------------------------(2)

To prove that two surfaces cut orthogonally, we have to prove that

$$\vec{n_1}.\vec{n_2} = 0, \quad \text{since } \theta = \frac{\pi}{2}$$

$\vec{n_1} = \nabla\phi_1$ at $(1,-1,2)$

Let $\phi_1 = \lambda x^2 - \mu yz - (\lambda+2)x$

$\nabla\phi_1 = \vec{i}\,(2\lambda x - (\lambda+2)) + \vec{j}(-\mu z) + \vec{k}(-\mu y)$

$\nabla\phi_1$ at $(1,-1,2) = \vec{i}(2\lambda - \lambda - 2) + \vec{j}(-2\mu) + \vec{k}(\mu)$

$\nabla\phi_2 = \vec{i}(8xy) + \vec{j}(4x^2) + \vec{k}(3z^2) = \vec{n_2}$

$\nabla\phi_2$ at $(1,-1,2) = -8\vec{i} + 4\vec{j} + 12\vec{k}$

$\vec{n_1}.\vec{n_2} = -8(2\lambda - \lambda - 2) - 8\mu + 12\mu = 0$--------------(3)

Since $(1,-1,2)$ is the point of intersection of two surfaces ,it lies in (1)

$\lambda + 2\mu = (\lambda + 2) \Rightarrow \lambda + 2\mu - \lambda - 2 = 0 \Rightarrow 2\mu - 2 = 0$

$$\Rightarrow \mu = 1$$

Substitute $\mu=1$ in (3),we get,

$-8(\lambda - 2) - 8 + 12 = 0 \Rightarrow -8\lambda + 16 + 4 = 0$

$$\Rightarrow \lambda = \frac{5}{2}$$

- **Find the equation of the tangent plane to the surface $2xz^2 - 3xy - 4x = 7$ at the point $(1,-1,2)$.**

<u>**Solution:**</u>

Let $\emptyset = 2xz^2 - 3xy - 4x - 7$

$\nabla\emptyset = \vec{i}(2z^2 - 3y - 4) + \vec{j}(-3x) + \vec{k}(4xz)$

$\nabla\emptyset$ at $(1,-1,2) = 7\vec{i} - 3\vec{j} + 8\vec{k}$

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$\nabla\emptyset$ at $(1,-1,2)$ is a vector in the direction of the normal to the surface $\emptyset = c$.

D.R.'s of the normal to the surface $\emptyset = c$ at the point $(1,-1,2)$ and having the line whose D.R.'s are $(7,-3,8)$ as a normal .

Equation of the tangent plane is $7(x-1) - 3(y+1) + 8(z-2) = 0$

$$\Rightarrow \quad 7x - 3y + 8z - 26 = 0.$$

Directional derivative of a scalar point function.

A scalar quantity that is a function of the position of a point in space is called a scalar point function.

- **Find the directional derivative of $\emptyset = xy + yz + zx$ at the point (1, 2, 3) along the x-axis.**

Solution:

$\nabla\emptyset = \vec{i}(y+z) + \vec{j}(x+z) + \vec{k}(y+x)$.

Along the x- axis $y = z = 0$

Directional derivative $= \dfrac{\nabla\emptyset.\vec{a}}{|\vec{a}|}$, where $\vec{a}$ is the given unit vector.

$\vec{a} = \vec{i}$

$\therefore |\vec{a}| = \sqrt{1} = 1$

$$\therefore \frac{\nabla\emptyset.\vec{a}}{\vec{a}} at\ (1,2,3) = \frac{\left(5\,\vec{i} + 4\vec{j} + 3\vec{k}\right).\vec{i}}{1} = 5$$

- **Find the direction in which the directional derivative of $\emptyset = x^2y^2z^4$ from (3, 1,–2) is maximum.**

Solution:

$\nabla\emptyset = \vec{i}(2xy^2z^4) + \vec{j}(2x^2yz^4) + \vec{k}(4x^2y^2z^3)$.

$\nabla\emptyset_{(3,1,-2)} = \vec{i}(6\times1\times16) + \vec{j}(2\times9\times16) + \vec{k}[(4\times9\times(-8)]$

$= 96\vec{i} + 288\,\vec{j} - 288\,\vec{k} = 96(\vec{i} + 3\vec{j} - 3\vec{j})$

Max. directional derivative $|\nabla\emptyset| = 96\sqrt{19}$

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$$=\frac{96(\vec{i}+3\vec{j}-3\vec{j})}{96\sqrt{19}}$$

$$=\frac{\vec{i}+3\vec{j}-3\vec{j}}{\sqrt{19}}$$

- **Find the maximum directional derivative of $\emptyset = x^3y^2z$ at the point (1, 1, 1).**

  Solution:

  Max. directional derivative of $\emptyset=|\nabla\emptyset|$

  $\nabla\emptyset = \vec{i}(3x^2y^2z) + \vec{j}(2x^3yz) + \vec{k}(x^3y^2)$

  $\nabla\emptyset_{(1,1,1)}=3\vec{i}+2\vec{j}+\vec{k}$

  $|\nabla\emptyset|=\sqrt{3^2+2^2+1^2}=\sqrt{14}$

- **Prove that $\nabla r^n = nr^{n-2}\vec{r}$.**

  Solution:

  Let $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$

  $r^2=x^2+y^2+z^2$ ---------------(1)

$$\nabla r^n = \left(\vec{i}\frac{\partial}{\partial x}+\vec{j}\frac{\partial}{\partial y}+\vec{k}\frac{\partial}{\partial z}\right)r^n$$

$$=\vec{i}\frac{\partial}{\partial x}(r^n)+\vec{j}\frac{\partial}{\partial y}(r^n)+\vec{k}\frac{\partial}{\partial z}(r^n)$$

$$=\vec{i}nr^{n-1}\frac{\partial r}{\partial x}+\vec{j}\,nr^{n-1}\frac{\partial r}{\partial y}+\vec{k}\,nr^{n-1}\frac{\partial r}{\partial z}$$

$$=nr^{n-1}\left(\vec{i}\frac{\partial r}{\partial x}+\vec{j}\frac{\partial r}{\partial y}+\vec{k}\frac{\partial r}{\partial z}\right)$$

From (1), $2r\frac{\partial r}{\partial x}=2x \Rightarrow \frac{\partial r}{\partial x}=\frac{x}{r}$

$2r\frac{\partial r}{\partial y}=2y \Rightarrow \frac{\partial r}{\partial y}=\frac{y}{r}$

$2r\frac{\partial r}{\partial z}=2z \Rightarrow \frac{\partial r}{\partial z}=\frac{z}{r}$

$$\nabla r^n = nr^{n-1}\left(\vec{i}\frac{x}{r}+\vec{j}\frac{y}{r}+\vec{k}\frac{z}{r}\right)=nr^{n-1}\frac{\vec{r}}{r}=nr^{n-2}\vec{r}$$

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- **Find the directional derivative of the function $\emptyset = xy^2 + yz^2$ at the point $(2, -1, 1)$ in the direction of the normal to the surface $x\,logz - y^2 + 4 = 0$ at the point $(-1, 2, 1)$.**

Solution:

Directional derivative is given by $\nabla\emptyset$

$$\nabla\emptyset = \vec{i}\frac{\partial\emptyset}{\partial x} + \vec{j}\frac{\partial\emptyset}{\partial y} + \vec{k}\frac{\partial\emptyset}{\partial z}$$

$$= \vec{i}(y^2) + \vec{j}(2xy + z^2) + \vec{k}(2yz)$$

$$\nabla\emptyset_{at(2,-1,1)} = \vec{i} + \vec{j}(-4+1) + \vec{k}(-2)$$

$$= \vec{i} - 3\vec{j} - 2\vec{k}$$

Now we have the equation of the surface $x\,logz - y^2 + 4 = 0$ is identified with $\psi(x, y, z) = $c.

$$\therefore\ \psi(x, y, z) = x\,logz - y^2 + 4 \Rightarrow c = -4$$

The direction of the normal to this surface is the same as that of $\nabla\psi$.

$$\nabla\psi = logz\,\vec{i} - 2y\vec{j} + \frac{x}{z}\vec{k}$$

$$\nabla\psi \text{ at } (-1,2,1) = -4\vec{j} - \vec{k} = b(\text{say})$$

Directional derivative of ϕ in the direction of $\vec{b}$

$$\frac{\nabla\emptyset.\vec{b}}{|\vec{b}|} = \frac{[(\vec{i} - 3\vec{j} - 2\vec{k}).(-4\vec{j} - \vec{k})]}{\sqrt{17}} = \frac{14}{\sqrt{17}}$$

- **Find the directional derivative of $\emptyset = xy^2 + yz^3$ at the point P $(2, -1, 1)$ in the direction of PQ where Q is the point (3, 1, 3).**

Solution:

Given that $\emptyset = xy^2 + yz^3$

$$\nabla\emptyset = \vec{i}(y^2) + \vec{j}(2xy + z^3) + \vec{k}(3yz^2)$$

$$\nabla\emptyset_{\ (2,1,-1)} = \vec{i} + \vec{j}(-4+1) + \vec{k}(-3)$$

$$= \vec{i} - 3\vec{j} - 3\vec{k}$$

Directional derivative of $\emptyset$ in the direction of $\overrightarrow{PQ}$=component or projection of directional derivative of $\nabla\emptyset$ along

$$\overrightarrow{PQ} = \frac{\nabla\emptyset\,.\overrightarrow{PQ}}{|\overrightarrow{PQ}|}$$

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$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

$$\overrightarrow{PQ} = (-3\vec{i} + \vec{j} - 3\vec{k}) - (-2\vec{i} + \vec{j} - \vec{k}) = -\vec{i} - 2\vec{k}$$

$$|\overrightarrow{PQ}| = \sqrt{5}$$

$$\therefore \overrightarrow{PQ} = \frac{(-\vec{i} - 2\vec{k}).(\vec{i} - 3\vec{j} - 3\vec{k})}{\sqrt{5}} = \frac{(-1+0+6)}{\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5} \text{ units}$$

- **Find the directional derivative of $\nabla.(\nabla\phi)$ at the point $(1, -2, 1)$ in the direction of the normal to the surface $xy^2z = 3x + z^2$, where $\emptyset = x^2y^2z^2$.**

Solution:

$$\nabla\phi = \vec{i}(2xy^2z^2) + \vec{j}(2x^2yz^2) + \vec{k}(2x^2y^2z)$$

$$\nabla.(\nabla\phi) = \left(\vec{i}\frac{\partial\emptyset}{\partial x} + \vec{j}\frac{\partial\emptyset}{\partial y} + \vec{k}\frac{\partial\emptyset}{\partial z}\right).\left[\vec{i}(2xy^2z^2) + \vec{j}(2x^2yz^2) + \vec{k}(2x^2y^2z)\right]$$

$$= 2y^2z^2 + 2x^2z^2 + 2x^2y^2$$

$$= 2(x^2z^2 + y^2z^2 + x^2y^2)$$

$$\nabla.(\nabla\phi) \text{ at } (1,-2,1) = 2(1+4+4) = 18$$

The direction of the normal to the surface is the same as that of $\nabla\psi$.

$$\psi(x,y,z) = xy^2z - 3x - z^2$$

$$\nabla\psi = \vec{i}(y^2z - 3) + \vec{j}(2xyz) + \vec{k}(xy^2 + 2z)$$

$$\nabla\psi_{(1,2,-1)} = \vec{i}(-4-3) - \vec{j}(2) + \vec{k}(4-2)$$

$$= -7\vec{i} - 2\vec{j} + 2\vec{k}$$

$$|\nabla\psi| = \sqrt{(-7)^2 + (-2)^2 + 2^2} = \sqrt{57}$$

$$\text{Directional derivative} = \frac{\nabla.(\nabla\phi).\nabla\psi}{|\nabla\psi|} = \frac{18.\left(-7\vec{i} - 2\vec{j} + 2\vec{k}\right)}{\sqrt{57}}$$

$$= \frac{-126\vec{i} - 36\vec{j} + 36\vec{k}}{\sqrt{57}}$$

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Define Divergence of a vector.

If $\vec{F}(x,y,z)$ is a differentiable vector point function defined at each point (x,y,z) in some region of space ,then the divergence of $\vec{F}$,denoted as div$\vec{F}$ is defined as ,

$$\text{div}\,\vec{F} = \nabla.\vec{F}$$

$$= \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right).\vec{F}$$

$$= \vec{i}\frac{\partial \vec{F}}{\partial x} + \vec{j}\frac{\partial \vec{F}}{\partial y} + \vec{k}\frac{\partial \vec{F}}{\partial z}$$

- **If $\vec{F} = 3xyz^2\vec{i} + 2xy^3\vec{j} - x^2yz\vec{k}$, find $\nabla.\vec{F}$ at the point (1,−1,1).**

Solution:

$$\text{div}\,\vec{F} = \nabla.\vec{F} = \vec{i}\frac{\partial \vec{F}}{\partial x} + \vec{j}\frac{\partial \vec{F}}{\partial y} + \vec{k}\frac{\partial \vec{F}}{\partial z}$$

$$= (3yz^2 + 6xy^2 - x^2y)$$

$$= \nabla.\vec{F}_{(1,-1,1)} = 3(-1)(1) + 6(1)(1) - 1(-1) = 4$$

Define Solenoidal vector

If $\vec{F}$ is a vector such that $\nabla.\vec{F}=0$ at all points in a region, then it is said to be a Solenoidal vector in that region.

- **Show that $\vec{F} = (x+2y)\vec{i} + (y+3z)\vec{j} + (x-2z)\vec{k}$ is solenoid.**

Solution:

To prove that $\vec{F}$ is solenoidal, we have, $\nabla.\vec{F}=0$

$$\nabla.\vec{F} = \frac{\partial}{\partial x}(x+2y) + \frac{\partial}{\partial y}(y+3z) + \frac{\partial}{\partial z}(x-2z)$$

$$= 1 + 1 - 2 = 0$$

$\therefore \vec{F}$ is solenoidal.

- **Prove that the vector $\vec{F} = z\vec{i} + x\vec{j} + y\vec{k}$ is solenoidal. (L6)**

Solution:

To prove that $\vec{F}$ is solenoidal, we have

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$$\nabla.\vec{F} = \frac{\partial}{\partial x}(z) + \frac{\partial}{\partial y}(x) + \frac{\partial}{\partial z}(y) = 0$$

- **Find the value of λ, so that $\vec{F} = \lambda y^4 z^2 \vec{i} + 4x^3 z^2 \vec{j} + 5x^2 y^2 \vec{k}$ may be solenoidal. (L1)**

  Solution:

  $\vec{F}$ is solenoidal ,if $\nabla.\vec{F}$=0

$$\Rightarrow \nabla.\vec{F} = \frac{\partial}{\partial x}(\lambda y^4 z^2) + \frac{\partial}{\partial y}(4x^3 z^2) + \frac{\partial}{\partial z}(5x^2 y^2)$$
$$= 0$$

  Since all the values are zero , λ can take any value.

- **If $\nabla\emptyset$ is solenoidal, then find $\nabla^2\emptyset$**

  Solution:

  Since $\nabla\emptyset$ is solenoidal, $\nabla.\nabla\emptyset$ =0

$$\Rightarrow \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)\left(\vec{i}\frac{\partial\emptyset}{\partial x} + \vec{j}\frac{\partial\emptyset}{\partial y} + \vec{k}\frac{\partial\emptyset}{\partial z}\right) = 0$$
$$\Rightarrow \frac{\partial^2\emptyset}{\partial x^2} + \frac{\partial^2\emptyset}{\partial y^2} + \frac{\partial^2\emptyset}{\partial z^2} = 0$$
$$\Rightarrow \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\emptyset = 0$$
$$\Rightarrow \nabla^2\emptyset = 0$$

Define Curl of a vector

If $\vec{F}(x, y, z)$ is a differentiable vector point function defined at each point (x, y, z) in some region of space,then the curl of $\vec{F}$,denoted as curl $\vec{F}$ is defined as ,curl$\vec{F}$ or rot$\vec{F}$. It is given by,

$$\text{Curl } \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

Define irrotational vector

If curl $\vec{F}$ = 0 (i.e) ∇x $\vec{F}$=0, then $\vec{F}$ is said to be irrotational.

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- **Show that** $\vec{F}=(\sin y+z)\vec{i}+(x\cos y-z)\vec{j}+(x-y)\vec{k}$ is irrotational.

$\vec{F}$ is said to be irrotational if $\nabla\times\vec{F}=0$

$$\nabla\times\vec{F}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ F_1 & F_2 & F_3\end{vmatrix}$$

$$=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ (siny+z) & (xcosy-z) & (x-y)\end{vmatrix}=0$$

$$=\vec{i}(-1+1)-\vec{j}(1-1)+\vec{k}(cosy-cosy)$$

$\therefore$ $\vec{F}$ is irrotational.

- **Find the values of a , b , c so that the vector** $\vec{F}=(x+y+az)\vec{i}+(bx+2y-z)\vec{j}+(-x+cy+2z)\vec{k}$ **may be irrotational.**

<u>Solution:</u>

$\vec{F}$ is said to be irrotational if $\nabla\times\vec{F}=0$

$$\nabla\times\vec{F}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ F_1 & F_2 & F_3\end{vmatrix}$$

$$=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ (x+y+az) & (bx+2y-z) & (-x+cy+2z)\end{vmatrix}=0$$

$\Rightarrow \vec{i}(c+1)-\vec{j}(-1-a)+\vec{k}(b-1)=0$

$\Rightarrow c+1=0\,;a+1=0;\,b-1=0$

$\Rightarrow c=-1\,;\ a=-1\,;\ b=1$

- **Prove that the gradient of any scalar point function is irrotational.**

<u>Solution:</u>

Let $\nabla\emptyset$ be the gradient of a scalar point function $\emptyset$ then to prove that it is irrotational ,we have $\nabla\times\nabla\emptyset=0$

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$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial \emptyset}{\partial x} & \frac{\partial \emptyset}{\partial y} & \frac{\partial \emptyset}{\partial z} \end{vmatrix} = 0$$

- **If $\vec{F}$ is irrotational, prove that it is conservative.**

Solution:

Let $\vec{F} = \nabla\emptyset$

$$\int_A^B \vec{F}.d\vec{r} = \int_A^B \nabla\emptyset.d\vec{r}$$

$$= \int_A^B \left(\vec{i}\frac{\partial\emptyset}{\partial x} + \vec{j}\frac{\partial\emptyset}{\partial y} + \vec{k}\frac{\partial\emptyset}{\partial z}\right).(dx\vec{i} + dy\vec{j} + dz\vec{k})$$

$$= \int_A^B \left(\frac{\partial\emptyset}{\partial x}dx + \frac{\partial\emptyset}{\partial y}dy + \frac{\partial\emptyset}{\partial z}dz\right) = \int_A^B d\emptyset$$

$$= (\emptyset)_A^B \quad = \emptyset(B) - \emptyset(A)$$

$\therefore$ $\vec{F}$ is conservative.

- **Find the values of a , b , c so that the vector $\vec{F} = (x+y+az)\vec{i} + (bx+2y-z)\vec{j} + (-x+cy+2z)\vec{k}$ may be irrotational.**

Solution:

To prove that $\vec{F}$ is conservative , ∇x $\vec{F}$=0

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (2x+yz) & (xz-3) & (xy) \end{vmatrix}$$

$=\vec{i}(x-x) - \vec{j}(y-y) + \vec{k}(z-z) = 0.$

- **If $\vec{F} = (x+y+1)\vec{i} + \vec{j} - (x+y)\vec{k}$, show that $\vec{F}$ is perpendicular to curl$\vec{F}$.**

Solution:

To prove that $\vec{F}$ is perpendicular we have to prove that

$\vec{F}$. curl $\vec{F}$ =0.

To find curl$\vec{F}$,

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$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (x+y+1) & 1 & -(x+y) \end{vmatrix}$$

$$= \vec{i}(-1-0) - \vec{j}(-1-0) + \vec{k}(0-1)$$

$$= -\vec{i} + \vec{j} - \vec{k}$$

$$\vec{F}.\,\text{curl}\,\vec{F} = [(x+y+1)\vec{i} + \vec{j} - (x+y)\vec{k}].[(-\vec{i} + \vec{j} - \vec{k})]$$

$$= -x - y - 1 + 1 + x + y = 0$$

- **If $\vec{F} = z\vec{i} + x\vec{j} + y\vec{k}$, prove that curl (curl $\vec{F}$) = 0.**

<u>**Solution:**</u>

$$\text{curl}\,\vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z & x & y \end{vmatrix}$$

$$= \vec{i}(1-0) - \vec{j}(0-1) + k$$

$$= \vec{i} + \vec{j} + \vec{k}$$

$$\text{curl}\,(\text{curl}\vec{F}) = \nabla \times \text{Curl}\,\vec{F}.$$

$$= \nabla \times (\vec{i} + \vec{j} + \vec{k})$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 1 & 1 & 1 \end{vmatrix} = 0$$

- **If $\vec{F} = 3x^2\vec{i} + 5xy^2\vec{j} + xyz^3\vec{k}$, find $\nabla(\nabla.\vec{F})$ and $\nabla.(\nabla \times \vec{F})$ at the point (1,2,3).**

<u>**Solution:**</u>

(i) $\nabla.\vec{F} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right).(3x^2\vec{i} + 5xy^2\vec{j} + xyz^3\vec{k})$

$$= 6x + 10xy + 3xyz^2$$

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$$\nabla(\nabla.\vec{F}) = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right).(6x + 10xy + 3xyz^2)$$

$$= (6 + 10y + 3yz^2)\,\vec{i} + (10x + 3xz^2)\vec{j} + (6xyz)$$

$$\nabla(\nabla.\vec{F})_{(1,2,3)} = (6 + 20 + 18)\,\vec{i} + (10 + 27)\,\vec{j} + 36\,\vec{k}$$

$$= 44\,\vec{i} + 37\vec{j} + 36\,\vec{k}$$

(ii) $$\nabla x\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2 & 5xy^2 & xyz^3 \end{vmatrix}$$

$$= \vec{i}(xz^3 - 0) - \vec{j}(yz^3 - 0) + \vec{k}(5y^2 - 0)$$

$$= xz^3\vec{i} - yz^3\vec{j} + 5y^2\vec{k}$$

$$\nabla.(\nabla x\vec{F}) = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right).\left(xz^3\vec{i} - yz^3\vec{j} + 5y^2\vec{k}\right)$$

$$= z^3 - z^3 + 0 = 0$$

- **If $u = x^2yz$ and $v = xy - 3z^2$, find (i) $\nabla.(\nabla u \times \nabla v)$ and (ii) $\nabla \times (\nabla u \times \nabla v)$ at the point (1,1,0).**

<u>**Solution:**</u>

(i) $\nabla.(\nabla u \times \nabla v)$

$$\nabla u = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)(x^2yz)$$

$$= \vec{i}2xyz + \vec{j}x^2z + \vec{k}x^2y$$

$$\nabla v = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)(xy - 3z^2)$$

$$= \vec{i}(y) + \vec{j}\,(x) - \vec{k}(6z)$$

$$(\nabla u \times \nabla v) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2xyz & x^2z & x^2y \\ y & x & -6z \end{vmatrix}$$

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$$= \vec{i}(-6x^2z^2 - x^3y) - \vec{j}(-12xyz^2 - x^2y^2) + \vec{k}(2x^2yz - x^2zy)$$

$\therefore \quad \nabla.(\nabla u \times \nabla v)$

$$= \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right).[\vec{i}(-6x^2z^2 - x^3y)$$

$$+ \vec{j}(12xyz^2 + x^2y^2) + \vec{k}(2x^2yz - x^2zy)]$$

$$= \frac{\partial}{\partial x}(-6x^2z^2 - x^3y) + \frac{\partial}{\partial y}(-12xyz^2 - x^2y^2) + \frac{\partial}{\partial z}(2x^2yz - x^2zy)$$

$$= (-12xz^2 - 3x^2y) - (12xz^2 + 2x^2y) + (2x^2y - x^2y)$$

$$= (-12xz^2 - 3x^2y) - (12xz^2 + 2x^2y) + (x^2y)$$

$\nabla.(\nabla u \times \nabla v)_{(1,1,0)} = -3 - 2 + 1 = -4$

(ii) $\nabla \times (\nabla u \times \nabla v)$

$(\nabla u \times \nabla v) = \vec{i}(-6x^2z^2 - x^3y) + \vec{j}(12xyz^2 + x^2y^2) + \vec{k}(x^2yz)$

$$\nabla \times (\nabla u \times \nabla v) \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (-6x^2z^2 - x^3y) & (12xy\,z^2 + x^2\,y^2) & (x^2yz) \end{vmatrix}$$

$$= \vec{i}(x^2z - 24xyz) - \vec{j}(2xyz + 12x^2z) + \vec{k}(12yz^2 + 2xy^2 + x^3)$$

$$\nabla \times (\nabla u \times \nabla v)_{(1,1,0)} = 3\vec{k}$$

Work Done By The Force

- Work done by the force $\vec{F}$ through the displacement $d\vec{r}$ is $\vec{F}.d\vec{r}$
- Work done by the force $\vec{F}$ through the displacement $d\vec{r}$ along the curve C is $\int_c \vec{F}.d\vec{r}$
- If the work done by a force does not depend on the path C, but only on the end points of C, then the force $\vec{F}$ is said to be conservative.
- If $\vec{F}$ is conservative vector, then there corresponds a scalar point function $\emptyset$ such that $\vec{F} = \nabla\emptyset$.

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- $\emptyset$ is called the scalar potential of $\vec{F}$.
- If $\vec{F}$ is irrotational,then $\vec{F}$ is conservative
- **Find the work done by the force $\vec{F} = x\vec{i} + 2y\vec{j}$ when it moves a particle on the curve 2y = x^2 from (0, 0) to (2, 2).**

Solution:

Work done by a force $\vec{F} = \int_c \vec{F}.d\vec{r}$

$$\vec{F}.d\vec{r} = (x\,\vec{i} + 2y\vec{j}).(dx\vec{i} + dy\vec{j} + dz\vec{k})$$

$$= xdx + 2ydy$$

$$\int_c \vec{F}.d\vec{r} = \int_0^2 xdx + 2ydy$$

$$2y = x^2 \Rightarrow 2dy = 2xdx$$

$$= \int_0^2 xdx + \frac{x^2}{2} 2xdx$$

$$= \int_0^2 xdx + x^3 dx$$

$$= \left(\frac{x^2}{2} + \frac{x^4}{4}\right)_0^2 = 2 + 4 = 6$$

- **Find the scalar point function whose gradient is $(y^2 - 2xyz^3)\vec{i} + (3 + 2xy - x^2z^3)\vec{j} + (6z^3 - 3x^2yz^2)\vec{k}$**

Solution:

Given that

$$\nabla\emptyset = (y^2 - 2xyz^3)\vec{i} + (3 + 2xy - x^2z^3)\vec{j} + (6z^3 - 3x^2yz^2)\vec{k}$$

$$\Rightarrow \vec{i}\frac{\partial\emptyset}{\partial x} + \vec{j}\frac{\partial\emptyset}{\partial y} + \vec{k}\frac{\partial\emptyset}{\partial z}$$

$$= (y^2 - 2xyz^3)\vec{i} + (3 + 2xy - x^2z^3)\vec{j} + (6z^3 - 3x^2yz^2)\vec{k}$$

Comparing and equating the coefficients of $\vec{i}, \vec{j}, \vec{k}$ on both sides and integrating partially with respect to x, y, z respectively we get

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$$\frac{\partial \emptyset}{\partial x} = y^2 - 2xyz^3 \implies \emptyset = y^2x - 2\frac{x^2}{2}yz^3 + f(y,z)$$

$$= y^2x - x^2yz^3 + f(y,z) ---------- (1)$$

$$\frac{\partial \emptyset}{\partial y} = 3 + 2xy - x^2z^3 \implies \emptyset = 3y + 2x\frac{y^2}{2} - x^2z^3y + f(x,z)$$

$$= 3y + xy^2 - x^2z^3y + f(x,z) ---------- (2)$$

$$\frac{\partial \emptyset}{\partial z} = 6z^3 - 3x^2yz^2 \implies \emptyset = 6\frac{z^4}{4} + 3x^2y\frac{z^3}{3} + f(x,y)$$

$$= 3\frac{z^4}{2} + x^2yz^3 + f(x,y) -------- (3)$$

Combining (1),(2),(3) we get,

$$\emptyset = y^2x - x^2yz^3 + 3y + 3\frac{z^4}{2} + C$$

TO BE CONTINUED……

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