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EC MATHEMATICS

10

This special guide is prepared on the basis of New Syllabus

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PREFACE

We all know that the Queen of Science is Mathematics. Such a wonderful subject should be dealt effectively by our students for the first time in Xth board examination. For that this book **"EC MATHEMATICS"** paves way to achieve the success of life by engraving the methods in the minds of the students.

- The teachers who have written this text have been effectively working in schools for many years.
- They know the mindset as the students very well.
- They have the experience to make the students learn maths easily and effectively.
- This book has been designed based on the new syllabus (2019 - 2020).
- All the exercises are given with the motivation of making the students learn by themselves easily.
- The additional unit exercises are given solutions along with the diagrams.
- The solutions for one mark questions are also given
- It will be helpful for the students to learn easily.
- I wish all the students who use this book for learning to get more marks and reach the peak as success in life
- I also appreciate the good hearts who have rendered support to this creation.

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CONTENTS

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1 Relations and Functions

Points to Remember

- The Cartesian Product of A with B is defined as A × B ={(a, b) / for all a∈b b∈B}
- A relation R from A to B is always a subset of A × B That is ⊆ A× B.
- A relation R is a function if for every x∈X there exists only one y∈Y.
- **One–one function**

  A function of f : A → B is called one–one function if distinct elements of A have distinct images in B
- **Many–one function**

  A function of f : A → B is called many one function if two or more elements of A have same image in B.
- **Onto function**

  A function of f : A → B is said to be onto function if the range of f is equal to the co–domain of f.
- **Into function**

  A function of f : A → B is called an into function if there exists at least one element in B which is not the image of any element of A.
- Identify function $f(x) = x$
- Reciprocal function $f(x) = \frac{1}{x}$
- Constant function $f(x) = c$
- Linear function $f(x) = ax + b \quad a \neq 0$
- Quadratic function $f(x) = ax^2 + bx + c, \; a \neq 0$
- Cubic function $f(x) = ax^3 + bx^2 + cx + d, \; a \neq 0$
- For three non – empty sets A, B and C if f : A → B and g: B → C are two functions then the composition of f and g is a function gof: A → C will be defined as $gof(x) = g[f(x)]$ for all $x \in A$.
- If f and g are any two functions then in general, fog ≠ gof
- If f, g and h are any three functions then fo(goh) = (fog)oh

Exercise 1.1

1. Find A × B, A × A and B × A **PTA - 1**

i) A = {2, –2, 3} and B = {1, –4} ii) A = B = {p, q} iii) A = {m, n}; B = ϕ

Solution:

i) A = {2, –2, 3} and B = {1, –4}

A × B = {2, –2, 3} × {1, –4}

= {(2, 1), (2, -4), (–2, 1), (–2, –4), (3, 1), (3, –4)}

A × A = {2, –2, 3} × {2, –2, 3}

= {(2, 2), (2, –2), (2, 3), (–2, 2), (–2, –2), (–2, 3), (3, 2), (3, –2), (3, 3)}

B × A = {1, –4} × {2, –2, 3}

= {(1, 2), (1, –2), (1, 3), (-4, 2), (–4, –2), (–4, 3)}

ii) A = B = {p, q}

A × B = {p, q} × {p, q}

= {(p, p), (p, q), (q, p), (q, q)}

A × A = {p, q} × {p, q}

= {(p, p), (p, q), (q, p), (q, q)}

B × A = {p, q} × {p, q}

= {(p, p), (p, q), (q, p), (q, q)}

Note:
HERE
A × A = A × B = B × A = B × B
Since the element of the set A and B are equal

iii) If A = {m, n}; B = ϕ

A × B = {}

A × A = {m, n} × {m, n}

= {(m, m), (m, n), (n, m), (n, n)}

B × A = {}

Note:
A × B = φ means
A = φ and B = φ

---

2. Let A = {1, 2, 3} and B = {$x \mid x$ is a prime number less than 10}. Find A × B and B × A

May - 2022

Solution:

Let A = {1, 2, 3} ; B = {2, 3, 5, 7}

A × B = {(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)}

B × A = {2, 3, 5, 7} × {1, 2, 3}

= {(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)}

---

3. If B × A = {(-2, 3) (-2, 4) (0, 3) (0, 4) (3, 3) (3, 4)}. Find A and B

April - 2023

Solution:

B = {set of all first co ordinates of elements of B × A}

∴ B = {(-2, 0, 3)}

A = {set of all second co ordinates of element of B × A}

∴ A = {3, 4}

---

4. If A = {5, 6} B = {4, 5, 6} C = {5, 6, 7}

Aug - 2022

Show that A × A = (B × B) ∩ (C × C)

Solution:

L.H.S = A × A

= {5, 6} × {5, 6}

A × A = {(5, 5) (5, 6), (6, 5) (6, 6)} ——— ❶

RHS = (B × B) ∩ (C × C)

B × B = {4, 5, 6} × {4, 5, 6}

= {(4, 4) (4, 5) (4, 6) (5, 4) (5, 5) (5, 6) (6, 4) (6, 5) (6, 6)}

C × C = {5, 6, 7} × {5, 6, 7}

= {(5, 5) (5, 6) (5, 7) (6, 5) (6, 6) (6, 7) (7, 5) (7, 6) (7, 7)}

(B × B) ∩ (C × C) = {(5, 5) (5, 6) (6, 5) (6, 6)} ——— ❷

(1) = (2)

∴ A × A = (B × B) ∩ (C × C)

5. Given A = {1, 2, 3}, B = {2, 3, 5}, C = {3, 4} and D = {1, 3, 5}, check if (A ∩ C) × (B ∩ D) = (A × B) ∩ (C × D) is true?

Solution:

L.H.S (A ∩ C) × (B ∩ D)

A ∩ C = {1, 2, 3} ∩ {3, 4}

= {3}

B ∩ D = {2, 3, 5} ∩ {1, 3, 5}

= {3, 5}

(A ∩ C) × (B ∩ D) = {3} × {3, 5}

= {(3,3) (3, 5)} ——— ❶

R.H.S (A × B) ∩ (C × D)

A × B = {1, 2, 3} × {2, 3, 5}

= { (1, 2) (1, 3) (1, 5) (2, 2) (2, 3) (2, 5) (3, 2) (3, 3) (3, 5)}

C × D = { (3, 4)} × {(1, 3, 5)}

= {(3, 1) (3, 3) (3, 5) (4, 1) (4, 3) (4, 5)}

(A × B) ∩ (C × D) = { (3, 3) (3, 5) } ——— ❷

(1) = (2)

∴ (A ∩ C) × (B ∩ D) = (A × B) ∩ (C ∩ D)

---

6. Let A = $\{x \in W / x < 2\}$, B = $\{x \in N / 1 < x \leq 4\}$ and C = {3, 5}. Verify that PTA - 2,3 & 5 SEP - 2021

i) A × (B ∪ C) = (A × B) ∪ (A × C)

ii) A × (B ∩ C) = (A × B) ∩ (A × C)

iii) (A ∪ B) × C = (A × C) ∪ (B × C)

Solution: Let A = { 0, 1 } B = {2, 3, 4} C = {3, 5}

i) L.H.S = A × (B ∪ C)

B ∪ C = {2, 3, 4} ∪ {3, 5}

= {2, 3, 4, 5}

A × (B ∪ C) = {0, 1} × {2, 3, 4, 5}

= {(0, 2) (0, 3) (0, 4) (0, 5) (1, 2) (1, 3) (1, 4) (1, 5)} ——— ❶

RHS = (A × B) ∪ (A × C)

A × B = {0, 1} × {2, 3, 4}

= {(0, 2) (0, 3) (0, 4) (1, 2) (1, 3) (1, 4)}

A × C = {0, 1} × {3, 5}

= {(0, 3) (0, 5) (1, 3) (1, 5) }

(A × B) ∪ (A × C) = { (0, 2) (0, 3) (0, 4) (0, 5) (1, 2) (1, 3) (1, 4) (1, 5)} ——— ❷

∴L.H.S = R.H.S

ii) A × (B ∩ C) = (A × B) ∩ (A × C)

L.H.S = A × (B ∩ C)

B ∩ C = {2, 3, 4} ∩ {3, 5}

= {3}

A × (B ∩ C) = {0, 1} × {3}

= {(0, 3) (1, 3) } ——— ❶

R.H.S = (A × B) ∩ (A × C)

A × B = {0, 1} × {2, 3, 4}

= {(0, 2) (0, 3) (0, 4) (1, 2) (1, 3) (1, 4)}

A × C = {0, 1} × {3, 5}

= {(0, 3) (0, 5) (1, 3) (1, 5)}

(A × B) ∩ (A × C) = {(0, 3) (1, 3) } ——— ❷

∴ L.H.S = R.H.S

iii) (A ∪ B) × C = (A × C) ∪ (B × C)

L.H.S = (A ∪ B) × C

A ∪ B = {0, 1} ∪ {2, 3, 4}

= {0, 1, 2, 3, 4}

(A ∪ B) × C = {0, 1, 2, 3, 4} × {3, 5}

= { (0, 3) (0, 5) (1, 3) (1, 5) (2, 3) (2, 5) (3, 3) (3, 5) (4, 3) (4, 5)} ——— ❶

R.H.S = (A × C) ∪ (B × C)

(A × C) = {(0, 1) × (3, 5)}

= {(0, 3) (0, 5) (1, 3) (1, 5)}

B × C = {2, 3, 4} × {3, 5}

= {(2, 3) (2, 5) (3, 3) (3, 5) (4, 3) (4, 5)}

(A × C) ∪ (B × C) = {(0, 3) (0, 5) (1, 3) (1, 5) (2, 3) (2, 5) (3, 3) (4, 3) (4, 5)} ——— ❷

L.H.S = R.H.S (from 1 and 2)

---

7. Let A=The set of all natural numbers less than 8, B=The set of all prime numbers less than 8. C =The set of even prime number verify that **PTA - 1**

i) (A ∩ B) × C = (A × C) ∩ (B × C) **SEP 20** ii) A × (B – C)= (A × B) – (A × C) **May 22**

Solution:

Given that A = {1, 2, 3, 4, 5, 6, 7} B = {2, 3, 5, 7} C = {2}

i) (A ∩ B) × C= (A × C) ∩ (B × C)

L.H.S = (A ∩ B) × C

A ∩ B = {1, 2, 3, 4, 5, 6, 7} ∩ {2, 3, 5, 7}

= {2, 3, 5, 7}

(A ∩ B) × C = {2, 3, 5, 7} × {2}

= {(2, 3) (3, 2) (5, 2) (7, 2)} ——— ❶

R.H.S = (A × C) ∩ (B × C)

A × C = {1, 2, 3, 4, 5, 6, 7} × {2}

= {(1, 2) (2, 2) (3, 2) (4, 2) (5, 2) (6, 2) (7, 2)}

B × C = {2, 3, 5, 7} × {2}

={(2, 2) (3, 2) (5, 2) (7, 2)}

(A × C) ∩ B × C = {(2, 2) (3, 2) (5, 2) (7, 2)} ——— ❷

∴ L.H.S = R.H.S

ii) A × (B – C) = (A × B) – (A × C)

L.H.S = A × (B – C)

B – C = {2, 3, 5, 7} – {2}

= {3, 5, 7}

A × (B – C) = {1, 2, 3, 4, 5, 6, 7} × {3, 5, 7}

= {(1, 3) (1, 5) (1, 7) (2, 3) (2, 5) (2, 7) (3, 3) (3, 5) (3, 7) (4, 3) (4, 5) (4, 7) (5, 3) (5, 5) (5, 7) (6, 3) (6, 5) (6, 7) (7, 3) (7, 5) (7, 7) } —— ❶

R.H.S = (A × B) – (A × C)

(A × B) = {1, 2, 3, 4, 5, 6, 7} × {2, 3, 5, 7}

= {(1, 2) (1, 3) (1, 5) (1, 7) (2, 2) (2, 3) (2, 5) (2, 7) (3, 2) (3, 3) (3, 5) (3, 7) (4, 2) (4, 3) (4, 5) (4, 7) (5, 2) (5, 3) (5, 5) (5, 7) (6, 2) (6, 3) (6, 5) (6, 7) (7, 2) (7, 3) (7, 5) (7, 7)}

A × C = {1, 2, 3, 4, 5, 6, 7} × {2}

= {(1, 2) (2, 2) (3, 2) (4, 2) (5, 2) (6, 2) (7, 2)}

(A × B) – (A × C)

={(1, 3) (1, 5) (1, 7) (2, 3) (2, 5) (2, 7) (3, 3) (3, 5) (3, 7) (4, 3) (4, 5) (4, 7) (5, 3) (5, 5) (5, 7) (6, 3) (6, 5) (6, 7) (7, 3) (7, 5) (7, 7)} —— ❷

L.H.S = R.H.S (From 1 and 2)

Exercise 1.2

1. Let A = {1, 2, 3, 7 } and B = {3, 0, –1, 7 } which of the following are relation from A to B?

i) R_1 = {(2, 1) (7, 1)}

ii) R_2 = {(–1, 1)}

iii) R_3 = {(2, –1) (7, 7) (1,3)}

iv) R_4 = {(7, –1) (0, 3) (3,3) (0, 7)}

Solution:–

A = {1, 2, 3, 7} B = {3, 0, -1, 7}

A×B = {1, 2, 3, 7} × {3, 0, -1, 7}

A × B = { (1, 3) (1, 0) (1, -1) (1, 7) (2, 3) (2, 0) (2, -1) (2, 7) (3, 3) (3, 0) (3, -1) (3, 7) (7, 3) (7,0) (7, -1) (7, 7)}

i) Here (2, 1) and (7, 1) $\notin$ A × B
Thus R_1 is not a relation from A to B

ii) Here (–1, 1) $\notin$ A × B
Thus R_2 is not a relation from A to B

iii) $R_3 \in$ A × B
Thus R_3 is a relation from A to B

iv) Here (0, 3), (0, 7) $\notin$ A × B
Thus R_4 is not a relation from A to B

2. Let A = {1, 2, 3, 4, ..., 45} and R be the relation defined as "is square of a number" on A. Write R as a subset of A × A. Also, find the domain and range of R.

Solution: SEP - 2021

$1^2 = 1$; $2^2 = 4$; $3^2 = 9$; $4^2 = 16$;
$5^2 = 25$; $6^2 = 36$ $7^2 = 49 \neq 45$

R = {(1, 1) (2, 4) (3, 9) (4, 16) (5, 25) (6, 36)}

R $\in$ A × A

Domain of R = {1, 2, 3, 4, 5, 6}

Range of R = {1, 4, 9, 16, 25, 36}

3. A relation R is given by the set {(x, y)/ y = x +3, $x \in$ {0, 1, 2, 3, 4, 5}} Determine its domain and range

Solution: PTA - 2 & 5

Given y = x + 3 x = 0, 1, 2, 3, 4, 5

Put x = 0; y = 0 + 3 = 3
Put x = 1; y = 1 + 3 = 4
Put x = 2; y = 2 + 3 = 5
Put x = 3; y = 3 + 3 = 6
Put x = 4; y = 4 + 3 = 7
Put x = 5; y = 5 + 3 = 8

Domain of R = { 0, 1, 2, 3, 4, 5}

Range of R = { 3, 4, 5, 6, 7, 8}

4. Represent each of the given relations by (a) an arrow diagram, (b) a graph and (c) a set in roster from, wherever possible.

i) $\{(x, y) \mid x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}\}$

ii) $\{(x, y) \mid y = x + 3,\ x, y$ are natural numbers $< 10\}$ **Aug 2022**

Solution:

i) $x = 2y \Rightarrow y = \frac{x}{2}$ $\quad \therefore x = 2, 3, 4, 5$

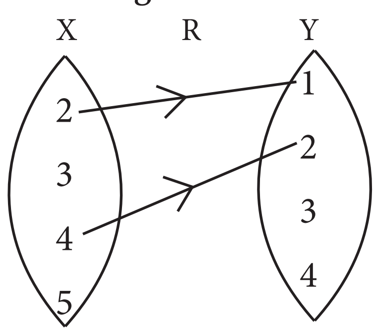
Put $x = 2$; $y = \frac{2}{2} = 1$ $\quad (2, 1)$

$x = 3$; $y = \frac{3}{2}$ $\quad (3, \frac{3}{2})$

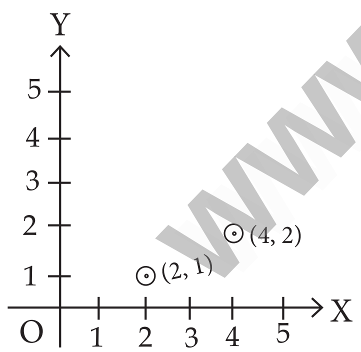
$x = 4$; $y = \frac{4}{2} = 2$ $\quad (4, 2)$

$x = 5$; $y = \frac{5}{2}$ $\quad (5, \frac{5}{2})$

a. Arrow Diagram



b. Graph



c. A set in roster form

$\{(2, 1)\ (4, 2)\}$

ii) $\{(x, y) \mid y = x + 3$ x *and* y are natural numbers < 10

$x = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$y = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Given $y = x + 3$

Put $x = 1$; $\quad y = 1 + 3 = 4$

$x = 2$; $\quad y = 2 + 3 = 5$

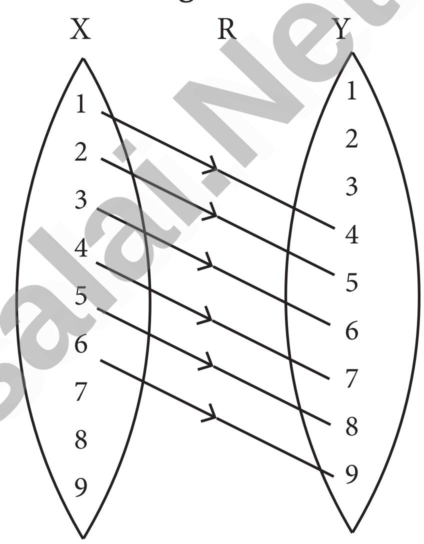
$x = 3$; $\quad y = 3 + 3 = 6$

$x = 4$; $\quad y = 4 + 3 = 7$

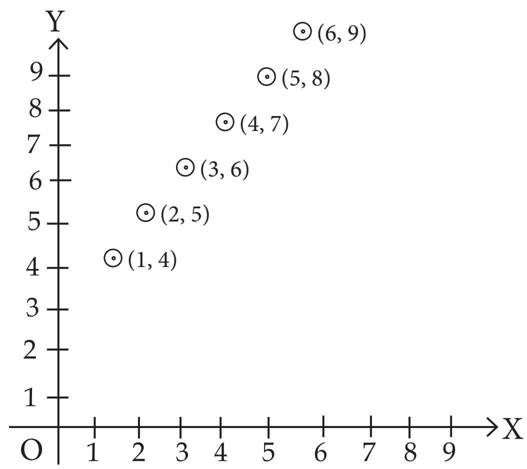
$x = 5$; $\quad y = 5 + 3 = 8$

$x = 6$; $\quad y = 6 + 3 = 9$

a. An arrow diagram



b. Graph



c. A set in roster

$\{(1, 4)\ (2, 5)\ (3, 6)\ (4, 7)\ (5, 8)\ (6, 9)\}$

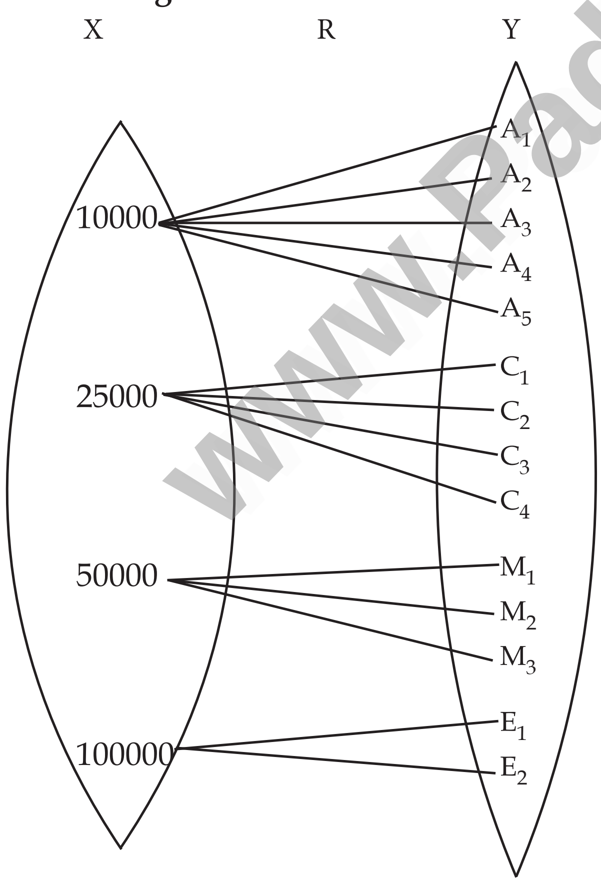
5. **A company has four categories of employees given by Assistants (A); clerks (C), Managers (M) and an Executive Officer (E). The company provide ₹ 10,000, ₹ 25,000, ₹ 50,000 and ₹ 1,00,000 as salaries to the people who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 were Assistants; C_1, C_2, C_3, C_4 were clerks, M_1, M_2, M_3 were Managers and E_1, E_2 were Executive officers and if the relation R is defined by x Ry where x is the salary given to person y, express the relation R through an ordered pair and an arrow diagram.**

Solution:

Ordered pair

{(10000, A_1) (10000, A_2) (10000, A_3) (10000, A_4) (10000, A_5) (25000, C_1) (25000, C_2) (25000, C_3) (25000, C_4) (50000, M_1) (50000, M_2) (50000, M_3) (100000, E_1) (100000, E_2)}

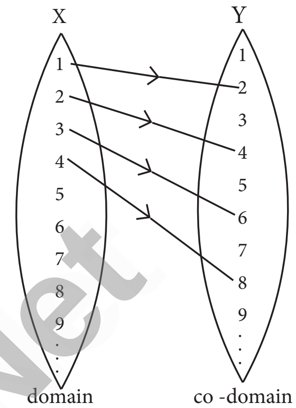
Arrow diagram



Exercise 1.3

1. **Let f = { (x, y) | $x, y \in$ N and y = $2x$ } be a relation on N. Find the domain co-domain and range Is this relation a function?**

Solution:



x, y ∈ N

x = {1,2,3, ...}

y = {1,2,3, ...}

y = $2x$

Put

x = 1, y = 2 × 1 = 2

x = 2, y = 2 × 2 = 4

x = 3, y = 2 × 3 = 6

x = 4, y = 2 × 4 = 8

f = {(1,2), (2,4), (3,6), (4,8), ... }

Domain = {1,2,3,4, ...}

Co domain = {1,2,3,4 .....}

Range = {2,4,6,8, ...}

Yes, this relation is a function.

2. **Let X = {3, 4, 6, 8}. Determine whether the relation R ={$(x, f(x)) \mid x \in X, f(x) = x^2 + 1$} is a function from X to N?**

Solution:

Given $f(x) = x^2+1$ where x = {3, 4, 6, 8}

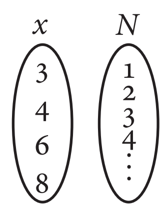
Put x

$f(3) = 3^2 + 1 = 9 + 1 = 10$

$f(4) = 4^2 + 1 = 16 + 1 = 17$

$f(6) = 6^2 + 1 = 36 + 1 = 37$

$f(8) = 8^2 + 1 = 64 + 1 = 65$



Yes R is a function

Reason : Each element in the domain of f has a unique image

3. **Given the function f : $x \to x^2 - 5x + 6$, evaluate (i) f(–1) (ii) f(2a) (iii) f(2) (iii) f(x–1)**

Solution:

$f(x) = x^2 - 5x + 6$

i) Replacing x with –1 we get

$f(-1) = (-1)^2 - 5(-1) + 6$

$= 1 + 5 + 6$

$= 12$

ii) **Replacing x with 2a we get**

$f(2a) = (2a)^2 - 5(2a) + 6$

$= 4a^2 - 10a + 6$

iii) **Replacing x with 2 we get**

$f(2) = (2)^2 - 5(2) + 6$

$= 4 - 10 + 6$

$= 0$

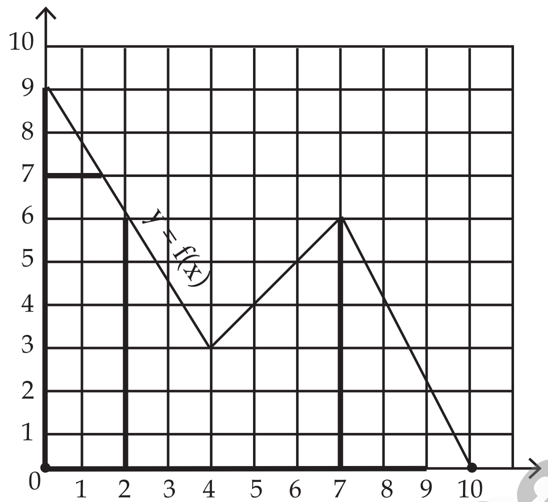
iv) **Replacing x with x–1 we get**

$f(x-1) = (x-1)^2 - 5(x-1) + 6$

$= x^2 - 2x + 1 - 5x + 5 + 6$

$= x^2 - 7x + 12$

4. **A graph representing the function f(x) is given below it is clear that f(9) = 2**



i) **Find the following values of the function.**
(a) f (0) (b) f (7) (c) f (2) (d) f (10)

ii) **For what value of x is f (x) = 1?**

iii) **Describe the following**
(i) Domain (ii) Range

iv) **What is the image of 6 under f?**

Solution:

i) (a) f (0) = 9 (b) f (7) = 6
(c) f (2) = 6 (d) f (10) = 0

ii) if f (x) = 1 the value of x is 9.5

iii) Domain = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10} |
Range = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

iv) The image of 6 under f is 5

5. **Let f(x) = 2x + 5. If $x \neq 0$ then find $\frac{f(x+2) - f(2)}{x}$**

Solution:

Given $f(x) = 2x + 5$

$f(x + 2) = 2(x+2) + 5$

$= 2x + 4 + 5$

$= 2x + 9$

$f(2) = 2(2) + 5$

$= 4 + 5 = 9$

$$\frac{f(x+2) - f(2)}{x} = \frac{2x + 9 - 9}{x} = \frac{2x}{x} = 2$$

6. **A function f is defined by f(x) = 2x – 3**

i) **find $\frac{f(0)+f(1)}{2}$**

ii) **find x such that f(x) = 0**

iii) **find x such that f(x) = x**

iv) **find x such that f(x) = f(1–x)**

Solution:

Given $f(x) = 2x - 3$

$f(0) = 2(0) - 3 = 0 - 3 = -3$

$f(1) = 2(1) - 3 = 2 - 3 = -1$

i) $\frac{f(0)+f(1)}{2} = \frac{-3-1}{2} = \frac{-4}{2} = -2$

ii) Given $f(x) = 0$

$2x - 3 = 0$

$2x = 3 \Rightarrow x = \frac{3}{2}$

iii) Given $f(x) = x$

$2x - 3 = x$

$2x - x = 3 \Rightarrow (2 - 1)x = 3$

$x = 3$

iv) Given $f(x) = f(1-x)$

$2x - 3 = 2(1-x) - 3$

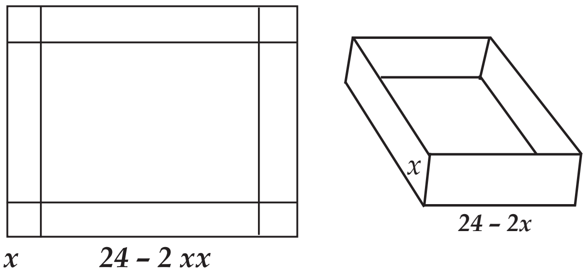
$2x - 3 = 2 - 2x - 3$

$2x - 3 = -1 - 2x$

$2x + 2x = -1 + 3$

$4x = 2 \Rightarrow x = \frac{2}{4} = \frac{1}{2}$

7. **An open box is to be made from a square piece of material, 24cm on a side by cutting equal squares from the corners and turning up the sides as shown. Express the volume V of the box as a function of x.**



Solution:

Given : length $= 24 - 2x$

breadth $= 24 - 2x$

height $= x$

Volume of the box = l × b × h

$= (24 - 2x) \times (24 - 2x) \times x$

∴ $= (24 - 2x)^2 \times x$

$[\because (a-b)^2 = a^2 - 2ab + b^2]$

$= (576 - 96x + 4x^2) \times x$

$= 576x - 96x^2 + 4x^3$

$= 4x^3 - 96x^2 + 576x$

8. A function f is defined by $f(x) = 3 - 2x$ Find x such that $f(x^2) = (f(x))^2$

Solution:

Given $f(x) = 3 - 2x$

$f(x^2) = 3 - 2x^2$

$[f(x)]^2 = (3 - 2x)^2$

Given $f(x^2) = [f(x^2)]^2$

$3 - 2x^2 = (3 - 2x)^2$

$3 - 2x^2 = 9 - 12x + 4x^2$

$\Rightarrow 3 - 2x^2 - 9 + 12x - 4x^2 = 0$

$-6x^2 + 12x - 6 = 0$

$\div (-6) \Rightarrow x^2 - 2x + 1 = 0$

36	
-6	-6
6x	6x

Squaring both sides

$\Rightarrow (x - 1)^2 = 0^2$

$\Rightarrow x - 1 = 0$

$\therefore x = 1$

9. A plane is flying at a speed of 500 km per hour. Express the distance 'd' travelled by the plane as function of time t in hours.

Solution:

Given speed = 500 km / hr

time = t hours

Distance = speed × time

= 500 × t = 500 t

10. The data in the adjacent table depicts the length of a person forehand and their corresponding height. Based on this data, a student finds a relationship between the height (y) and the forehand length (x) as y = ax + b where a, b are constants.

i) Check if this relation is a function

ii) Find a and b

iii) Find the height of a Person whose forehand length is 40 cm

iv) Find the length of forehand of a Person if the height is 53.3 inches

Length of forehand (cm) X	Height (inch)
35	56
45	65
50	69.5
55	74

PTA - 4

Solution:

The relation is $y = 0.9x + 24.5$

i) Yes the relation is a function

ii) When compare with $y = ax + b$

$a = 0.9, b = 24.5$

iii) When the forehead length is 40 cm, then height is 60.5 inches.

Hint :

$y = 0.9 \times 40 + 24.5$

$= 36 + 24.5$

$= 60.5$

iv) When the height is 53.3 inches their forehead length is 32 cm

Hint : $y = 0.9x + 24.5$

$53.3 = 0.9x + 24.5$

$0.9x = 53.3 - 24.5$

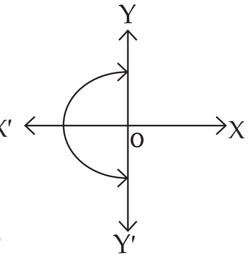
$= 28.8$

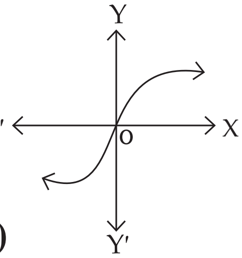
$x = 28.8 / 0.9$

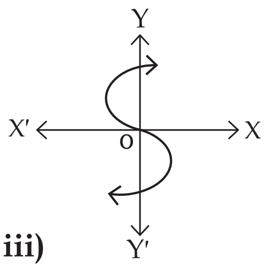
$\therefore x = 32$

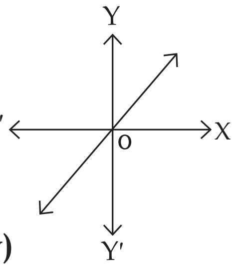
Exercise 1.4

1. Determine whether the graph given below represent functions Give reason for your answers concerning each graph.

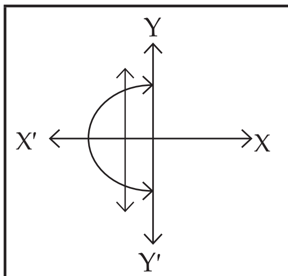
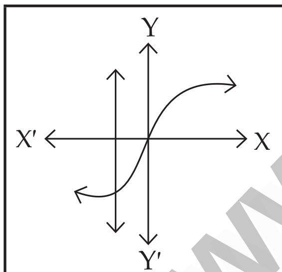
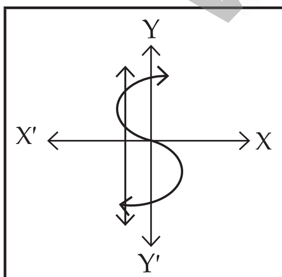
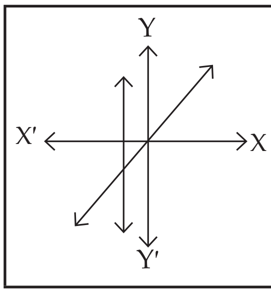
i) 

ii) 

iii) 

iv) 

Solution:

	Do not represent a function as the vertical line meet the curves in two points.
	Represent a function as the vertical line meet the curve in one point.
	Do not represent a function as the vertical line meet the curves in two points.
	Represent a function as the vertical line meet the curve in one point.

2. Let f : A →B be a function defined by $f(x) = \frac{x}{2} - 1$ where A = {2, 4, 6, 10, 12} B = {0, 1, 2, 4, 5, 9} Represent f by i) Set of ordered pairs (ii) a table (iii) an arrow diagram (iv) a graph.

Solution: **GMQ** **April 2023**

Given $f(x) = \frac{x}{2} - 1$

$f(2) = \frac{2}{2} - 1 = 1 - 1 = 0$

$f(4) = \frac{4}{2} - 1 = 2 - 1 = 1$

$f(6) = \frac{6}{2} - 1 = 3 - 1 = 2$

$f(10) = \frac{10}{2} - 1 = 5 - 1 = 4$

$f(12) = \frac{12}{2} - 1 = 6 - 1 = 5$

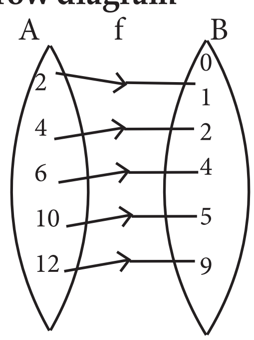
i) Set of ordered pairs

f = {(2, 0) (4, 1) (6, 2) (10, 4) (12, 5)}

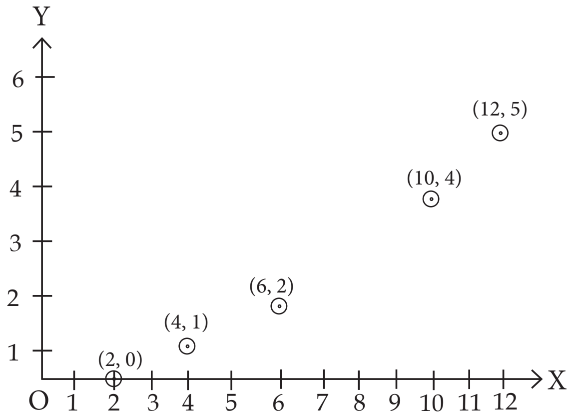
ii) a table

x	2	4	6	10	12
$f(x)$	0	1	2	4	5

iii) an arrow diagram



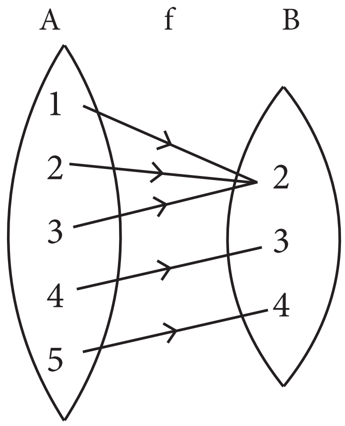
iv) a graph



3. Represent the function f = {(1, 2) (2, 2) (3, 2) (4, 3) (5, 4)} through i) an arrow diagram ii) a table form iii) a graph

Solution:

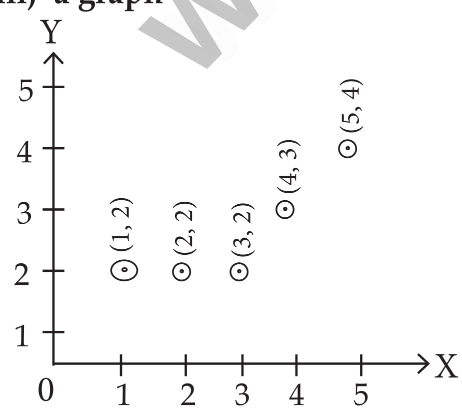
i) an arrow diagram



ii) a table

x	1	2	3	4	5
$f(x)$	2	2	2	3	4

iii) a graph



4. Show that the function f : N →N defined by f(x) = 2x – 1 is one - one but not onto

Solution:

f(x) = 2x – 1

f(1) = 2(1) – 1 = 2 – 1 = 1

f(2) = 2(2) – 1 = 4 – 1 = 3

f(3) = 2(3) – 1 = 6 – 1 = 5

f(4) = 2(4) – 1 = 8 – 1 = 7 .... etc

Co–domain = {1, 2, 3, 4, 5.................}

Range = {1, 3, 5, 7.................}

It is one – one because distinct elements of first set have distinct images in 2nd set. It is not onto because the co–domain and the range are not same.

5. Show that the function f : N →N defined by f(m) = $m^2 + m + 3$ is one - one function.

Solution: **SEP 20**

Given f(m) = $m^2 + m + 3$

f(1) = $(1)^2 + 1 + 3 = 5$

f(2) = $(2)^2 + 2 + 3 = 9$

f(3) = $(3)^2 + 3 + 3 = 15$..................etc

So the function f is one–one. Since every element in 1st set have distinct image in 2nd set.

6. Let A = {1, 2, 3, 4} and B = N. Let f : A →B be defined by f(x) = x^3 then

i) Find the range of f.

ii) Identify the type of function **PTA - 5**

Solution:

Given f(x) = x^3

f(1) = $1^3 = 1$

f(2) = $2^3 = 8$

f(3) = $3^3 = 27$

f(4) = $4^3 = 64$

i) Range = {1, 8, 27, 64}

ii) Type of function is one-one and into function

7. In each of the following cases state whether the function is bijective or not justify your answer

i) f : R → R defined by f(x) = 2x + 1

ii) f : R → R defined by f(x) = 3 - 4x^2

Solution:

i) f(x) = 2(x) + 1

$f(0) = 2 \times 0 + 1 = 1$

$f(1) = 2 \times 1 + 1 = 3$

$f(2) = 2 \times 2 + 1 = 5$

$f(3) = 2 \times 3 + 1 = 7$

Here, Different elements has different images

∴ It is an one - one function

It is also an onto function

∴ It is bijective

ii) f(x) = 3 – 4x^2

$f(1) = 3 - 4(1^2) = 3 - 4 = -1$

$f(2) = 3 - 4(2^2) = 3 - 4\ (4) = 3 - 16 = -13$

$f(3) = 3 - 4(3^2) = 3 - 4\ (9) = 3 - 36 = -33$

$f(4) = 3 - 4(4^2) = 3 - 4\ (16) = 3 - 64 = -61$

$f(-1) = 3 - 4(-1)^2 = 3 - 4\ (1) = 3 - 4 = -1$

Here f(1) = f(-1)

but $1 \neq -1$

∴ It is not one - one function

∴ It is not bijective

8. Let A = {-1, 1} and B = {0, 2} If the function f : A → B defined by f(x) = ax + b is an onto function? Find a and b

Solution:

f(x) = ax + b

Given $f(-1) = a(-1) + b = 0$

$-a + b = 0$ ——— ❶

Also f(1) = 2

⇒ a(1) + b = 2

$a + b = 2$ ——— ❷

❶ + ❷

- a + b + a+b = 0 + 2

⇒ 2b = 2

b = 1

Substitute b = 1 in (2), we get

a = 1

9. If the function f is defined by

$$f(x) = \begin{cases} x + 2\ ; & x > 1 \\ 2\ ; & -1 \le x \le 1 \\ x - 1\ ; & -3 < x < -1 \end{cases}$$

find the values of

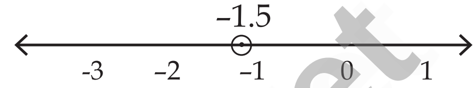
i) f(3) **ii) f(0)**

iii) f(–1.5) **iv) f(2) + f(–2)**

Solution:

$x - 1$ if $-3 < x < -1$

–1.5



$$f(x) = \begin{cases} x + 2 & \text{if } x = \{2, 3, 4, 5......\} \\ 2 & \text{if } x = \{-1, 0, 1\} \\ x - 1 & \text{if } x \quad x = \{-2\} \end{cases}$$

i) f(3) = x + 2

= 3 + 2 = 5

ii) f(0) = 2

iii) f(–1.5) = x – 1

= –1.5 – 1

= –2.5

iv) f(2) + f(–2)

= x + 2 + x – 1

= 2 + 2 + (– 2) – 1

= 4 – 3

= 1

10. A function f : [–5, 9] → R is defined as follows PTA - 4

$$f(x) = \begin{cases} 6x + 1\ ; & -5 \le x < 2 \\ 5x^2 - 1; & 2 \le x < 6 \\ 3x - 4\ ; & 6 \le x \le 9 \end{cases}$$

Find (i) f(–3) + f(2) **ii) f(7) – f(1)**

iii) 2f(4) + f(8) **iv) $\dfrac{2f(-2) - f(6)}{f(4) + f(-2)}$**

Solution:

$$f(x) = \begin{cases} 6x + 1 & \text{if } x = \{-5, -4, -3, -2, -1, 0, 1\} \\ 5x^2 - 1 & \text{if } x = \{2, 3, 4, 5\} \\ 3x - 4 & \text{if } x = \{6, 7, 8, 9\} \end{cases}$$

f(-3) = 6x + 1
= 6 (-3)+1 = -18 + 1 = -17
f(2) = $5x^2$ - 1
= $5(2^2)$ - 1 = 5 × 4 - 1 = 20 - 1 = 19
f(7) = 3x - 4
= 3(7) -4 = 21 - 4 = 17
f(1) = 6x + 1
= 6 (1) + 1 = 7
f(4) = $5x^2$ - 1
= 5 × 4^2 - 1 = 5 × 16-1 = 80 - 1 = 79
f(8) = 3x - 4
= 3(8) - 4 = 24 - 4 = 20
f(-2) = 6x + 1
= 6 (-2) + 1 = -12 + 1 = -11
f(6) = 3x - 4
= 3(6) - 4 = 18 - 4 = 14

i) f(-3) + f(2) = -17 + 19 = 2

ii) f (7) - f(1) = 17 - 7 = 10

iii) 2f(4) + f(8) = 2 × 79 + 20
= 158 + 20
= 178

iv) $\frac{2f(-2) - f(6)}{f(4) + f(-2)} = \frac{2(-11) - 14}{79 + (-11)}$

$= \frac{-22 - 14}{79 - 11} = \frac{-36}{68} = \frac{-9}{17}$

11. The distance S an object travels under the influence of gravity in time t seconds is given by S(t) = $\frac{1}{2}gt^2 + at + b$ where (g is the acceleration due to gravity) a, b are constants. Verify wheather the function S(t) is one-one or not **PTA - 3**

Solution:

s(t) = $\frac{1}{2}gt^2 + at + b$

If t = 0, then S(0) = b

It t = 1, then S(1) = = $\frac{1}{2}g\times 1^2 + a\times 1 + b$

If t = 2, then S(2) = $\frac{1}{2}g(2^2) + a \times 2 + b$

$= \frac{4g}{2} + 2a + b$

= 2g + 2a + b

Here, for every different value of t, there will be different distance.

∴ It is an one - one function.

12. The function 't' which maps temperature in Celsius (C) into temperature in Fahrenheit (F) is defined by t(C) = F where F = $\frac{9C}{5}$+32 find

(i) t(0) (ii) t(28) (iii) t(–10)

iv) the value of C when t(C) = 212

v) the temperature when the Celsius value is equal to the Farenheit value

Solution: **PTA - 1**

Given t(C) = F

$F = \frac{9c}{5} + 32$ ∴ $t(C) = \frac{9C}{5} + 32$

i) $t(0) = \frac{0}{5} + 32 = 32°F$

ii) $t(28) = \frac{9 \times 28}{5} + 32$

$= \frac{252}{5} + 32$

= 50.4 + 32 = 82.4°F

iii) $t(-10) = \frac{9(-10)}{5} + 32$

$\Rightarrow \frac{-90}{5} + 32 = 24$

iv) t(C) = 212

$C = \frac{9C}{5} + 32 = 212$

$\frac{9C}{5} = 212 - 32$

= 180

9C = 180 × 5

= 900

∴ C = 100°C

v) The temperature when the celsius value is equal to the Farenheit value

$C = \frac{9C}{5} + 32$

$$C - 32 = \frac{9C}{5}$$

$5(C-32) = 9C$
$5C - 160 = 9C$
$5C - 9C = 160$
$-4C = 160$
$C = -40$

Exercise 1.5

1. Using the functions f and g given below, find fog and gof, check whether fog = gof

i) $f(x) = x-6$, $g(x) = x^2$

ii) $f(x) = \frac{2}{x}$, $g(x) = 2x^2 - 1$

iii) $f(x) = \frac{x+6}{3}$, $g(x) = 3 - x$

iv) $f(x) = 3 + x$, $g(x) = x - 4$

v) $f(x) = 4x^2 - 1$, $g(x) = 1 + x$

Solution:

i) $f(x) = x-6$, $g(x) = x^2$ $[\because f(x) = x - 6]$

$fog(x) = f(g(x)) = f(x^2)$
$= x^2 - 6$

$gof(x) = g(f(x)) = g(x-6)$
$= (x-6)^2$

$\therefore fog \neq gof$

ii) $f(x) = \frac{2}{x}$; $g(x) = 2x^2 - 1$

$fog(x) = f(g(x)) = f(2x^2-1)$

$$= \frac{2}{2x^2-1}$$

$$gof(x) = g(f(x)) = g\left(\frac{2}{x}\right)$$

$$= 2\left(\frac{2}{x}\right)^{2} - 1$$

$$= 2 \times \frac{4}{x^2} - 1$$

$$= \frac{8}{x^2} - 1$$

$\therefore fog \neq gof$

iii) $f(x) = \frac{x+6}{3}$, $g(x) = 3 - x$

$fog(x) = f(g(x)) = f(3-x)$

$$= \frac{3-x+6}{3}$$

$$= \frac{9-x}{3}$$

$$gof(x) = g(f(x)) = g\left(\frac{x+6}{3}\right)$$

$$= 3 - \left(\frac{x+6}{3}\right)$$

$$= \frac{9-x-6}{3}$$

$$= \frac{3-x}{3}$$

$\therefore fog \neq gof$

iv) $f(x) = 3 + x$; $g(x) = x - 4$ GMQ

$fog(x) = f(g(x)) = f(x-4)$
$= 3 + x - 4$
$= x - 1$

$gof(x) = g(f(x)) = g(3 + x)$
$= 3 + x - 4$
$= x - 1$

So fog = gof

v) $f(x) = 4x^2 - 1$, $g(x) = 1 + x$

$fog(x) = f(g)(x)) = f(1 + x)$
$= 4(1 + x)^2 - 1$
$= 4(1 + 2x + x^2) - 1$
$= 4 + 8x + 4x^2 - 1$
$= 4x^2 + 8x + 3$

$gof(x) = g(f(x)) = g(4x^2 - 1)$
$= 1 + 4x^2 - 1$
$= 4x^2$

So fog $\neq$ gof

2. Find the value of k such that fog = gof

i) $f(x) = 3x + 2$, $g(x) = 6x - k$

ii) $f(x) = 2x - k$, $g(x) = 4x + 5$

Solution :

i) $f(x) = 3x + 2 \qquad g(x) = 6x - k$

$fog = gof$ (Given)

$fog(x) = gof(x)$

$f(g(x)) = g(f(x))$

$f(6x - k) = g(3x + 2)$

$3(6x - k) + 2 = 6(3x + 2) - k$

$18x - 3k + 2 = 18x + 12 - k$

$2k = -10$

$k = -5$

ii) $f(x) = 2x - k,\ g(x) = 4x + 5$

Given $fog = gof$

$fog(x) = gof(x)$

$f(g)(x) = g(f(x)$

$f(4x + 5) = g(2x - k)$

$2(4x + 5) - k = 4(2x - k) + 5$

$8x + 10 - k = 8x - 4k + 5$

$3k = -5$

$k = -\frac{5}{3}$

3. If $f(x) = 2x - 1$; $g(x) = \frac{x+1}{2}$ show that $fog = gof = x$

Solution:

$fog(x) = f(g(x))$

$= f\left(\frac{x+1}{2}\right)$

$= 2\left(\frac{x+1}{2}\right) - 1$

$= x + 1 - 1 = x$

$gof = gof(x)$

$= g(f(x))$

$= g(2x - 1)$

$= \frac{2x - 1 + 1}{2}$

$= \frac{2x}{2} = x$

So $fog = gof = x$

Hence proved

4. If $f(x) = x^2 - 1$, $g(x) = x - 2$ Find a, if $gof(a) = 1$ PTA - 2 & 4

Solution:

$f(x) = x^2 - 1; \quad g(x) = x - 2$

Given $gof(a) = 1$

$g(f(a)) = 1$

$g(a^2 - 1) = 1$

$a^2 - 1 - 2 = 1$

$a^2 - 3 = 1$

$a^2 = 4$

$a = \sqrt{4}$

$a = \pm 2$

5. Let $A, B, C \subseteq N$ and a function $f: A \to B$ be defined by $f(x) = 2x + 1$ and $g: B \to C$ be defined by $g(x) = x^2$. Find the range of fog and gof.

Solution:

Given $f(x) = 2x + 1 \qquad g(x) = x^2$

$fog = fog(x)$

$= f(g(x))$

$= f(x^2)$

$= 2x^2 + 1$

$gof = gof(x)$

$= g(f(x))$

$= g(2x + 1)$

$= (2x + 1)^2$

Range of fog and gof is

$\{y / y = 2x^2 + 1, x \in N\}; \{y / y = (2x + 1)^2, x \in N\}$

6. Let $f(x) = x^2 - 1$. Find i) fof ii) fofof

Solution:

Given $f(x) = x^2 - 1$

i) $fof(x) = f(f(x))$

$= f(x^2 - 1)$

$= (x^2 - 1)^2 - 1$

$= x^4 - 2x^2 + 1 - 1$

$= x^4 - 2x^2$

ii) $fofof = fofof(x)$

$= fof(f(x))$

$= fof(x^2 - 1)$

$= f(f(x^2 - 1))$

$= f[(x^2 -1)^2 -1]$
$= f[x^4 -2x^2 + 1 -1]$
$= f[x^4 -2x^2] \Rightarrow (x^4 - 2x^2)^2 -1$

7. If f : R → R and g : R → R are defined by f(x) = x^5 and g(x) = x^4 then check if f,g are one-one and fog is one - one? **PTA - 6**

Solution:

If f(x) = f(y)

$x^5 = y^5$

and hence x = y

Thus f is one - one

If $g(x) = g(y)$

$x^4 = y^4$

and hence $x \neq \pm y$

Thus g is not one - one

$fog = fog(x)$
$= f(g(x))$
$= f(x^4)$
$= (x^4)^5$
$= x^{20}$

If fog(x) = fog(y)

$x^{20} = y^{20}$ hence $x \neq \pm y$

Thus fog is not one - one

8. Consider the functions f(x), g(x), h(x) as given below. Show that (fog)oh = fo(goh) in each case **PTA - 2**

i) $f(x) = x-1$, $g(x) = 3x + 1$ and $h(x) = x^2$

ii) $f(x) = x^2$, $g(x) = 2x$ and $h(x) = x + 4$

iii) $f(x) = x-4$ $g(x) = x^2$ and $h(x) = 3x -5$

Solution:

i) $f(x) = x-1$ $g(x) = 3x + 1$ $h(x) = x^2$

$fog(x) = f(g(x)) = f(3x+1)$
$= (3x + 1 -1)$
$= 3x$

$(fog)oh = (fog)\ oh\ (x)$
$= fog(h(x))$
$= fog(x^2)$
$= 3x^2$ ——— ❶

$goh(x) = g(h(x)) = g(x^2)$
$= 3x^2 + 1$

$fo(goh) = fo(goh(x))$
$= f(3x^2+1)$

$fo(goh)= fo(goh(x))$
$= f(3x^2 +1)$
$= 3x^2 +1 -1$
$= 3x^2$ ——— ❷

from ❶ and ❷

(fog)oh = fo(goh)

ii) $f(x) = x^2$ $g(x) = 2x$ $h(x) = x+4$

$fog(x) = f(g(x)= f(2x)$
$= (2x)^2$
$= 4x^2$

$(fog)oh = (fog)\ oh(x)$
$= fog(h(x)$
$= fog(x+4)$
$= 4(x+4)^2$
$= 4(x^2 + 8x + 16)$
$= 4x^2 + 32x + 64$ ——— ❶

goh $= goh = (x) = g(h(x))$
$= g\ (x + 4)$
$= 2\ (x + 4) = 2x + 8$

$fo\ (goh)= fo\ (goh)(x)$
$= fo\ (2x+8)$
$= (2\ x + 8)^2$
$= 4x^2 + 32x + 64$ ——— ❷

from ❶ and ❷

iii) (fog)oh = fo(goh).

$f(x) = x - 4$ $g(x) = x^2$

$h(x)=3x-5$

$fog(x) = fo(x^2)$
$= x^2 - 4$

$(fog)oh = (fog)oh(x)$
$= fog(3x -5)$
$= (3x -5)^2 - 4$
$= 9x^2 - 30x + 25 -4$
$= 9x^2 -30x + 21$ ——— ❶

$goh(x) = go(3x-5)$
$=(3x-5)^2$
$= 9x^2-30x+25$
$fo(goh)x = fo(9x^2-30x+25)$
$= 9x^2-30x+25-4$
$= 9x^2-30x+21$ ——— ❷

from ❶ and ❷

(fog)oh = fo(goh)

9. Let f = {(-1, 3), (0, -1), (2, -9)} be a linear function from Z into Z. Find f(*x*)

Solution:

The linear equation is f(x) = ax + b

Given f(-1) = 3

a(-1) + b = 3

-a + b = 3 ——— ❶

Also f(0) = -1

a(0) + b = -1

b = -1

substitute b = -1 in (1)

we get a = -4

The linear equation is *f(x) = -4x -1*

10. In electrical circuit theory, a circuit C(t) is called a linear circuit if it satisfies the superposition principle given by $C(at_1+bt_2) = aC(t_1)+bC(t_2)$ where a, b are constants Show that the circuit C(t) = 3t is linear.

Solution:

Given C(t) = 3t

$C(at_1) = 3at_1$ ——— ❶

$C(bt_2) = 3bt_2$ ——— ❷

❶ + ❷

$C(at_1) + C(bt_2) = 3at_1 + 3bt_2$

$C(at_1 + bt_2) = 3at_1 + 3bt_2$

$= C(at_1) + c(bt_2)$

$= C(at_1 + bt_2)$

Superposition principle is satisfied

∴ C(t) = 3t is a linear function.

Exercise 1.6

Multiple Choice Questions

1. If n(A × B) = 6 and A = {1, 3} then n(B) is SEP - 2021

A) 1 B) 2
C) 3 D) 6

n(A × B) = 6

n(A) = 2

$n(B) = \frac{n(A\times B)}{n(A)} = \frac{6}{2} = 3$ **Ans : C) 3**

2. A = {a, b, p } B = {2, 3 } C = {p, q, r, s} then n[(A ∪ C) × B] is

A) 8 B) 20 C) 12 D) 16

A ∪ C = {a, b, p, q, r, s} ⇒ n(AUC) = 6

B = {2, 3} ⇒ n (B) = 2

n[(A ∪ C × B)] = 6 × 2 = 12 **Ans : C) 12**

3. If A = {1, 2} , B = {1, 2, 3, 4} C = {5, 6} and D = {5, 6, 7, 8} then state which of the following statement is true.

A) (A × C) ⊂ (B × D)
B) (B × D) ⊂ (A × C)
C) (A × B) ⊂ (A × D)
D) (D × A) ⊂ (B × A)

A × C = {1, 2} × {5, 6}

= {(1, 5) (1, 6) (2, 5) (2, 6)}

B × D = {1, 2, 3, 4} × {5, 6, 7, 8}

= {(1, 5) (1, 6) (1, 7) (1, 8) (2, 5) (2, 6) (2, 7) (2,8), (3, 5) (3, 6) (3, 7) (3, 8) (4, 5) (4, 6) (4, 7) (4, 8)}

∴ (A × C) ⊂ (B × D)

Ans : A) (A × C) ⊂ (B × D)

4. If there are 1024 relations from a set A = {1, 2, 3, 4, 5} to a set B, then the number of elements in B is Aug 2022

A) 3 B) 2
C) 4 D) 8

$2^{pq} = 1024$ n(A) = 5 = p

$2^{5q} = 2^{10}$ n(B) = ? = q

5q = 10

q = 2

Ans : B) 2

5. The range of the relation R = {(x, x^2) | x is a prime number less than 13} is **Aug 2022**

A) {2, 3, 5, 7}

B) {2, 3, 5, 7, 11}

C) {4, 9, 25, 49, 121}

D) {1, 4, 9, 25, 49, 121}

Prime number less than 13 are

{2, 3, 5, 7, 11}

Given $f(x) = x^2$

$f(2) = 2^2 = 4$

$f(3) = 3^2 = 9$

$f(5) = 5^2 = 25$

$f(7) = 7^2 = 49$

$f(11) = 11^2 = 121$

Range = {4, 9, 25, 49, 121}

Ans : C) {4, 9, 25, 49, 121}

6. If the ordered pairs (a + 2, 4) and (5, 2a + b) are equal then (a, b) is **May 2022**

A) (2, –2) B) (5, 1)

C) (2, 3) D) (3, –2)

a + 2 = 5

a = 5 – 2

a = 3

2a + b = 4

2(3) + b = 4

6 + b = 4

b = 4 – 6

b = –2

Ans: D) (3, –2)

7. Let n(A) = m and n(B) = n then the total number of non empty relations that can be defined from A to B is

A) m^n B) n^m

C) $2^{mn} - 1$ D) 2^{mn}

Total number of relations = $2^{pq} = 2^{mn}$

Ans: D) 2^{mn}

8. If {(a, 8) {6, b}} represents an identify function then the value of a and b are respectively

A) (8, 6) B) (8, 8)

C) (6, 8) D) (6, 6)

Ans: C) (6, 8)

9. Let A = {1, 2, 3, 4} and B = {4, 8, 9, 10} A function f : A → B given by

f = {(1, 4) (2, 8) (3, 9) (4, 10)} is a

A) Many one function

B) Identify function

C) One to one function

D) Into function

Ans: C) One to one function

10. If $f(x) = 2x^2$ and $g(x) = \frac{1}{3x}$ Then fog is

A) $\frac{3}{2x^2}$ B) ——

C) $\frac{2}{9x^2}$ D) $\frac{1}{6x^2}$

$fog(x) = f(g(x)) = f\left(\frac{1}{3x}\right)$

$= 2\left(\frac{1}{3x}\right)^2$

$2 \times \left(\frac{1}{9x^2}\right) \Rightarrow \frac{2}{9x^2}$

Ans : C) $\frac{2}{9x^2}$

11. If f: A →B is a bijective function and if n(B) = 7 then n(A) is equal to

A) 7 B) 49

C) 1 D) 14

Ans : A) 7

12. Let f and g be two functions given by

f = {(0, 1), (2, 0), (3, -4), (4, 2), (5, 7)}

g = {(0, 2), (1, 0), (2, 4), (-4, 2), (7, 0)}

then the range of fog is

A) {0, 2, 3, 4, 5} B) {-4, 1, 0, 2, 7}
C) {1, 2, 3, 4, 5} D) {0, 1, 2}

Every image of g has an image in f

So fog = {0, 1, 2} **Ans : D) {0,1,2}**

13. Let f(x) = $\sqrt{1+x^2}$ then

A) f(xy) = f(x). f(y)
B) f(xy) ≥ f(x). f(y)
C) f(xy) ≤ f(x). f(y)
D) None of these

Let $f(x) = \sqrt{1+x^2}$

$f(y) = \sqrt{(1+y^2)}$

$f(xy) = \sqrt{(1+x^2y^2)}$

$f(xy) = f(x).\ f(y)$

$\sqrt{(1+x^2y^2)} = \sqrt{1+x^2}\ .\ \sqrt{(1+y^2)}$

$\sqrt{1+x^2y^2} = \sqrt{(1+x^2)(1+y^2)}$

square on both sides

$1 + x^2y^2 = (1+x^2)(1+y^2)$

$1 + x^2y^2 = 1 + x^2 + y^2 + x^2y^2$

so $1 + x^2y^2 \leq 1 + x^2 + y^2 + x^2y^2$

Ans : C) f(xy) ≤ f(x). f(y)

14. If g = {(1, 1), (2, 3), (3, 5), (4, 7)} is a function given by g(x) = αx + β then the values α and β are

A) (–1, 2) B) (2, –1)
C) (–1, –2) D) (1, 2)

$g(x) = \alpha x + \beta$

x y
(1, 1)

$g(1) = \alpha + \beta = 1$ ——— ❶

$g(2) = 2\alpha + \beta = 3$ ——— ❷

x y
(2, 3)

Solving ❶ and ❷

$\alpha = 2 \quad \beta = -1$ **Ans : B) (2, –1)**

15. f(x) = (x + 1)^3 – (x –1)^3 represents a function which is

A) linear
B) cubic
C) reciprocal
D) quadratic

$f(x) = (x+1)^3 - (x-1)^3$
$= x^3 + 3x^2 +$
$3x + 1 - x^3 + 3x^2 - 3x + 1$
= 6x2 + 2 is a quadratic function

Ans : D) quadratic

Unit Exercise - 1

1. If the ordered pairs (x^2 – 3x, y^2 +4y) and (–2, 5) are equal then find x and y.

Solution:

Given $x^2 - 3x = -2$
$x^2 - 3x + 2 = 0$
$(x-1)(x-2) = 0$
$x = 1$ and $x = 2$

(+2 ; $\frac{-1}{x}$ | $\frac{-2}{x}$)

Given $y^2 + 4y = 5$
$y^2 + 4y - 5 = 0$
$(y-1)(y+5) = 0$
y – 1 and y = –5

(–5 ; $\frac{-1}{y}$ | $\frac{+5}{y}$)

The value of x is 1 and 2
The value of y is 1 and –5

2. The cartesian product A × A has 9 elements among which (–1, 0) and (0, 1) are found. Find the set A and the remaining elements of A × A.

Solution:

The set A = {5, 6, 7, 8}

The remaining elements of A × A is

{(–1, –1) (–1, 1) (0, –1) (0, 0) (1, –1) (1, 0) (1, 1)}

3. Given that $f(x) = \begin{cases} \sqrt{x-1} & x \geq 1 \\ 4 & x < 1 \end{cases}$ **find**

i) f(0) ii) f(3)
iii) f(a+1) in terms of a (Given that a ≥0)

Solution:

$f(x) = \begin{cases} \sqrt{x-1} & \text{if } x = \{1, 2, 3, 4.........\} \\ 4 & \text{if } x = \{0, -1, -2.........\} \end{cases}$

i) f(0) = 4

ii) f(3) $= \sqrt{x-1} = \sqrt{3-1} = \sqrt{2}$

iii) f(a+1) = $\sqrt{x-1} = \sqrt{a+1-1} = \sqrt{a}$

4. Let A = {9, 10, 11, 12, 13, 14, 15 ,16, 17} and let f : A→N be defined by f(n) = the highest prime factor of $n \in A$. Write f as a set of ordered pairs and find the range of f

Solution:

f(n) = the highest prime factor

f(9) = 3 (factors 1, 3, 9)

f(10) = 5 (factors 1, 2, 5)

f(11) = 11 (factors 1, 11)

f(12) = 3 (factors 1, 2, 3, 4, 6, 12)

f(13) = 13 (factors 1, 13)

f(14) = 7 (factors 1, 2, 7, 14)

f(15) = 5 (factors 1, 3, 5, 15)

f(16) = 2 (factors 1, 2, 4, 8, 16)

f(17) = 17 (factors 1, 17)

Set of ordered pair {(9, 3) (10, 5) (11, 11) (12, 3) (13, 13) (14, 7) (15, 5) (16, 2) (17, 7)}

Range of f = {(2, 3, 5, 11, 13, 17}

5. Find the domain of the function.

$$f(x) = \sqrt{1+\sqrt{1-\sqrt{1-x^2}}}$$

Here

$$\sqrt{1-x^2} = \sqrt{(1+x)(1-x)}$$

$$\Rightarrow x = 1 \text{ (or) } x = -1$$

$$\Rightarrow -1 \le x \le 1$$

∴ Domain of f(x) - {-1, 0, 1}

6. If f(x) = x^2, g(x) = 3x and h(x) = x–2 Prove that (fog)oh = fo(goh)

Solution:

fog(x) = f(g(x)) = f(3x)

= $(3x)^2$

= $9x^2$

(fog)oh(x) = fog(h(x)

= fog(x–2)

= $9(x-2)^2$

= $9[x^2 - 4x + 4]$

= $9x^2 - 36x + 36$ ——— ❶

goh(x) = g(h(x) = g(x–2)

= 3(x–2)

= 3x –6

fo(goh)(x) = fo(3x – 6)

= $(3x-6)^2$

= $9x^2 - 36x + 36$ ——— ❷

from ❶ and ❷ we get

(fog)oh = fo(goh)

7. Let A = {1, 2} and B = {1, 2, 3, 4} C = {5, 6} and D = {5, 6, 7, 8} verify whether A × C is a subset of B × D?

Solution:

A × C = {1, 2}× {5, 6}

= {(1, 5) (1,6) (2, 5) (2, 6)} ——— ❶

B × D = {1, 2, 3, 4}× {5, 6, 7, 8}

= {(1, 5) (1, 6) (1, 7) (1, 8) (2, 5) (2, 6) (2, 7) (2, 8) (3, 5) (3, 6) (3, 7) (3, 8) (4, 5) (4, 6) (4, 7) (4, 8)} ——— ❷

from (1) & (2) it is clear that

A × C ⊂ B × D

8. If f(x) = $\frac{x-1}{x+1}$ $x \neq -1$ show that $f(f(x)) = \frac{-1}{x}$ provided $x \neq 0$

Solution:

Given f(x) = $\frac{x-1}{x+1}$

$$f(f(x)) = f\left(\frac{x-1}{x+1}\right)$$

$$= \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1}$$

$$= \frac{\frac{x-1-(x+1)}{x+1}}{\frac{x-1+x+1}{x+1}}$$

$= \frac{-2}{-2x} \Rightarrow = \frac{-1}{x}$ proved

9. The functions f and g are defined by $f(x) = 6x + 8$, $g(x) = \frac{x-2}{3}$

i) Calculate the value of $gg\left(\frac{1}{2}\right)$

ii) Write an expression for gf(x) in its simplest form

Solution:

i) Given $f(x) = 6x + 8$

$g(x) = \frac{x-2}{3}$

$gg(x) = g\left(\frac{x-2}{3}\right)$

$gg\,\frac{1}{2} = g\left(\frac{\frac{1}{2}-2}{3}\right) = g\left(\frac{-3/2}{3}\right)$

$= g\left(\frac{-1}{2}\right)$

$= \frac{x-2}{3}$ where $x = -\frac{1}{2}$

$= \frac{-\frac{1}{2}-2}{3}$

$= \frac{-\frac{5}{2}}{3} \Rightarrow \frac{-5}{2}\times\frac{1}{3} = \frac{-5}{6}$

ii) Write an expression for g f(x) in its simplest form

Given : $f(x) = 6x + 8$

$g(x) = \frac{x-2}{3}$

$f(x) = g(6x + 8)$

$= \frac{x-2}{3}$ where $x = 6x + 8$

$\frac{6x+8-2}{}$

$= \frac{6x+6}{3} \Rightarrow \frac{6(x+1)}{3}$

$= 2(x + 1)$

10. Write the domain of the following real functions **PTA - 6**

i) $f(x) = \frac{2x+1}{x-9}$ ii) $p(x) = \frac{-5}{4x^2+1}$

iii) $g(x) = \sqrt{x-2}$ iv) $h(x) = x + 6$

Solution:

i) $f(x) = \frac{2x+1}{x-9}$

Domain = R – {9}

HINT

If $x = 9$

$f(x) = \frac{2(9)+1}{9-9}$

$= \frac{18+1}{0}$

= Not defined

ii) $p(x) = \frac{-5}{4x^2+1}$

Domain = R

iii) $g(x) = \sqrt{x-2}$

Domain = {2, 3, 4, 5..........}

HINT

If x = 0 and less than 0

$g(0) = \sqrt{0-2} = \sqrt{2} \notin R$

iv) $h(x) = x + 6$

Domain = R

2 MARK QUESTIONS

1. If $A \times B$ = {(3,2), (3,4), (5,2), (5,4)} then find A and B. **SEP 20**

Solution: $A \times B$ ={(3,2), (3,4), (5,2), (5,4)}

We have A= {set of all first coordinates of elements of $A \times B$ }. Therefore, A= {3,5}

B= {set of all second coordinates of elements of $A \times B$ }. Therefore, B = {2,4}

Thus A= {3,5} and B = {2,4}.

2. Let A= {3,4,7,8} and B= {1,7,10}. Which of the following sets are relations from A to B?

(i) $\mathbb{R}_1$ ={(3,7), (4,7), (7,10), (8,1)} (ii) $\mathbb{R}_2$ = {(3,1), (4,12)}

(iii) $\mathbb{R}_3$ = {(3,7), (4,10), (7,7), (7,8), (8,11), (8,7), (8,10)}

{(3,1), (3,7), (3,10), (4,1), (4,7), (4,10), (7,1), (7,7), (7,10), (8,1), (8,7), (8,10)}

Solution: $A \times B$ = {(3,1), (3,7), (3,10), (4,1), (4,7), (4,10), (7,1), (7,7), (7,10), (8,1), (8,7), (8,10)}

(i) We note that, $\mathbb{R}_1 \subseteq A \times B$. Thus, $\mathbb{R}_1$ is a relation from A to B.

(ii) Here, $(4, 12) \in \mathbb{R}_2$, but $(4, 12) \notin A \times B$. So, $\mathbb{R}_2$ is not a relation from A to B.

(iii) Here, $(7, 8) \in \mathbb{R}_3$, but $(7, 8) \notin A \times B$. So, $\mathbb{R}_3$ is not a relation from A to B.

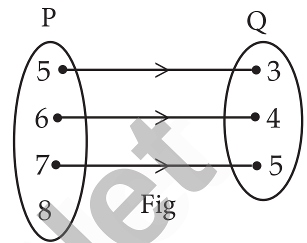
3. The arrow diagram shows (Fig) a relationship between the sets P and Q. Write the relation in (i) Set builder form (ii) Roster form (iii) What is the domain and range of $\mathbb{R}$.

Solution:

(i) Set builder form of $\mathbb{R} = \{(x, y) \mid y = x - 2, x \in P, y \in Q\}$

(ii) Roster form $\mathbb{R}$ ={(5, 3), (6, 4), (7, 5)}

(iii) Domain of $\mathbb{R}$ = {5,6,7}; range of $\mathbb{R}$ = {3, 4, 5}



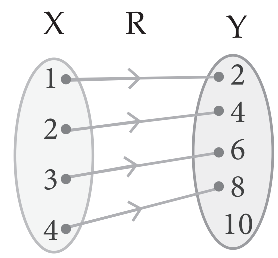
Fig

4. Let X={1, 2, 3, 4} and Y={2, 4, 6, 8, 10} and $\mathbb{R}$ = {(1,2),(2,4),(3,6),(4,8)}. Show that $\mathbb{R}$ is a function and find its domain, co-domain and range?

Solution:

Pictorial representation of $\mathbb{R}$ is given in fig From the diagram, we see that for each $x \in X$, there exists only one $y \in Y$. Thus all elements in X have only image in Y. Therefore $\mathbb{R}$ is a function.

Domain X = {1,2,3,4}; Co-domain Y = {2,3,6,8,10}; Range of f = {2,4,6,8}.



Fig

5. Using vertical line test, determine which of the following curves (fig(a), (b), (c), (d)) represent a function?

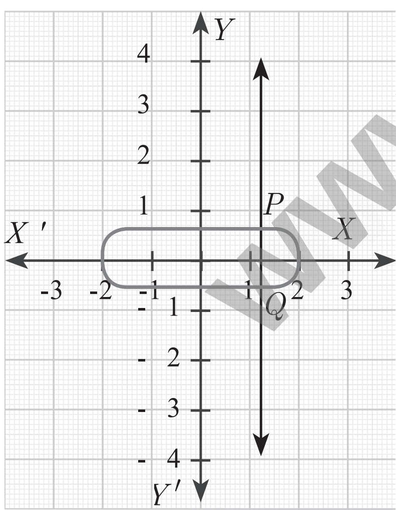


Fig (a)

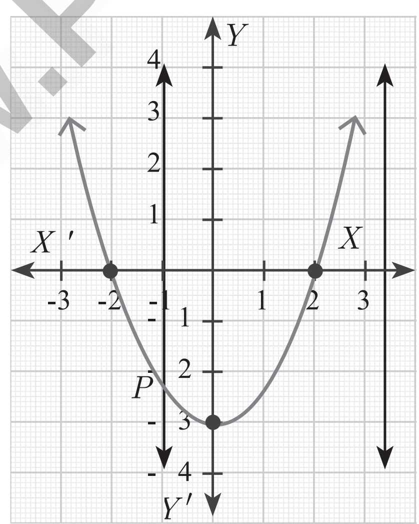


Fig (b)

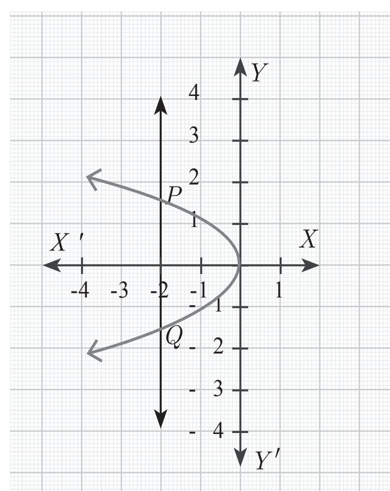
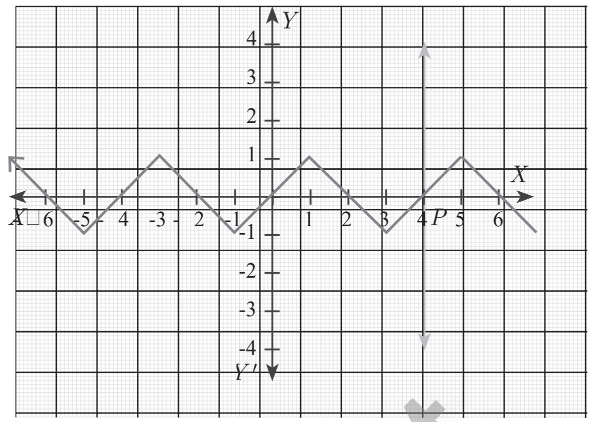


Fig (c)

Solution

The curves in Fig(a) and Fig(c) do not represent a function as the vertical lines meet the curves in two points P and Q.

The curves in Fig(b) and Fig(d) represent a function as the vertical lines meet the curve in at most one point



6. Using horizontal line test (Fig(a), (b), (c)), determine which of the following functions are one – one.

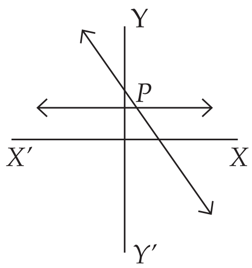


Fig (a)

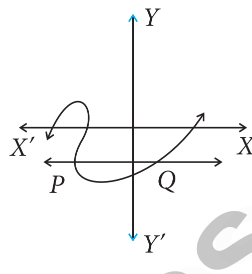


Fig (b)

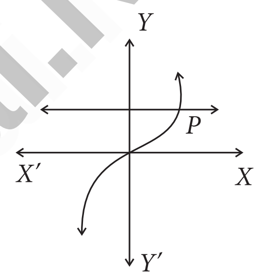


Fig (c)

Solution:

The curves in Fig. (a) and Fig.(c) represent a one–one function as the horizontal lines meet the curves in only one point P.

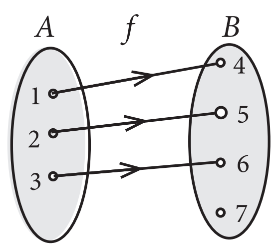
The curve in Fig.(b) does not represent a one–one function, since, the horizontal line meet the curve in two points P and Q.

7. Let A={1, 2, 3} , B={4, 5, 6, 7} and f={(1,4), (2, 5) (3, 6) } be a function from A to B. Show that f is one – one but not onto function.

Solution:

A={1, 2, 3}, B={4, 5, 6, 7}; f={(1,4) }, (2,5),(3, 6)}

Then f is a function from A to B and for different elements in A, there are different images in B. Hence f is one–one function. Note that the element 7 in the co-domain does not have any pre-image in the domain. Hence f is not onto. (Fig)



Therefore f is one–one but not an onto function

8. **Represent the function $f(x)=\sqrt{2x^2-5x+3}$ as a composition of two functions.**
Solution:
We set $f_2(x) = 2x^2 - 5x + 3$ and) $f_1(x) = \sqrt{x}$

Then,
$$f(x)=\sqrt{2x^2-5x+3}$$
$$=\sqrt{f_2(x)}$$
$$=f_1[f_2(x)]$$
$$=f_1f_2(x)$$

3 MARK QUESTIONS

1. **If $A= \{1,3,5\}$ and $B= \{2,3\}$ then**
(i) find $A \times B$ and $B \times A$ (ii) Is $A \times B = B \times A$? Why?
(iii) Show that $n(A \times B) = n(B \times A) = n(A) \times n(B)$

Solution :
Given that $A= \{1,3,5\}$ and $B= (2,3)$
(i) $A \times B = \{1,3,5\} \times \{2,3\} = \{(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)\}$ ...(1)
$B \times A = \{2,3\} \times \{1,3,5\} = \{(2,1), (2,3), (2,5), (3,1), (3,3), (3,5)\}$ ...(2)
(ii) From (1) and (2) we conclude that $A \times B \neq B \times A$ as $(1,2) \neq (2, 1)$ and $(1, 3) \neq$) $(3, 1)$, etc.
(iii) $n(A)=3$; $n(B) = 2$.
From (1) and (2) we observe that, $n(A\times B) = n(B \times A) = 6$;
we see that, $n(A) \times n(B) = 3 \times 2 = 6$ and $n(B) \times n(A) = 2 \times 3 = 6$
Hence, $n(A \times B) = n(B \times A) = n(A) \times n(B) = 6$.
Thus, $n(A \times B) = n(B \times A) = n(A) \times n(B)$.

2. **A relation 'f' is defined by $f(x) = x^2 - 2$ where, $x \in \{-2, -1, 0, 3\}$** PTA - 1
(i) List the elements of f (ii) Is f a function?
Solution :
$f(x) = x^2 - 2$ where $x \in \{-2, -1, 0, 3\}$
(i) $f(-2) = (-2)^2 - 2 = 2$; $f(-1) = (-1)^2 - 2 = -1$
$f(0) = (0)^2 - 2 = -2$; $f(3) = (3)^2 - 2 = 7$
Therefore, $f = \{(-2, 2), (-1, -1), (0, -2), (3, 7)\}$
(ii) We note that each element in the domain of f has a unique image (Fig.1.12). Therefore f is a function.

3. **If $x= \{-5,1,3,4\}$ and $Y = \{a,b,c\}$, then which of the following relations are functions from X to Y?**
(i) $\mathbb{R}_1 = \{(-5, a), (1, a), (3, b)\}$ (i) $\mathbb{R}_2 = \{(-5, b), (1, b), (3, a), (4, c)\}$
(iii) $\mathbb{R}_3 = \{(-5, a), (1, a), (3, b), (4, c), (1, b)\}$

Solution :

(i) $\mathbb{R}_1 = \{(-5, a), (1, a), (3, b)\}$

We may represent the relation $\mathbb{R}_1$ in an arrow diagram (Fig.1.15(a)).

$\mathbb{R}_1$ is not a function as $4 \in X$ does not have an image in Y.

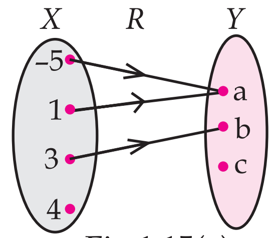


Fig 1.15(a)

(ii) $\mathbb{R}_2 = \{(-5, b), (1, b), (3, a), (4, c)\}$

Arrow diagram of $\mathbb{R}_2$ is shown in Fig. 1.15 (b).

$\mathbb{R}_2$ is a function as each element of X has an unique image in Y.

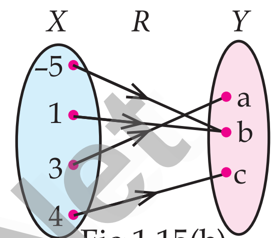


Fig 1.15(b)

(iii) $\mathbb{R}_3 = \{(-5, a), (1, a), (3, b), (4, c), (1, b)\}$

Representing $\mathbb{R}_3$ in arrow diagram (Fig. 1.15 (c)).

$\mathbb{R}_3$ is not a function as $1 \in X$ has two images $a \in Y$ and $b \in Y$.

Note that the image of an element should always be unique.

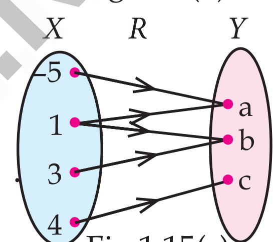


Fig 1.15(c)

4. If $A = \{-2, -1, 0, 1, 2\}$ and $f : A \to B$ is an onto function defined by $f(x) = x^2 + x + 1$ then find B

Solution : $A = \{-2, -1, 0, 1, 2\}$ and $f(x) = x^2 + x + 1$

$f(-2) = (2)^2 + (-2) + 1 = 3$

$f(-1) = (-1)^2 + (-1) + 1 = 1$

$f(0) = 0^2 + 0 + 1 = 1$

$f(1) = 1^2 + 1 + 1 = 3$

$f(2) = 2^2 + 2 + 1 = 7$

Since, f is an onto function range of $f = B$ Co–domain

Therefore, B = {1, 3, 7}

5. Let f be a function $f : \mathbb{N} \to \mathbb{N}$ defined by $f(x) = 3x + 2, x \in \mathbb{N}$

(i) Find the images of 1, 2, 3 **(ii) Find the pre–images 29, 53**

(iii) Identify the type of function

PTA - 3 & GMQ

Solution :

The function $f : \mathbb{N} \to \mathbb{N}$ defined by $f(x) = 3x + 2$

(i) If $x = 1, f(1) = 3(1) + 2 = 5$

If $x = 2, f(2) = 3(2) + 2 = 8$

If $x = 3, f(3) = 3(3) + 2 = 11$

The images of 1, 2, 3 are 5, 8. 11 respectively

(ii) If x is the pre–image of 29, then $f(x)$ =29. Hence $3x + 2 = 29$
$3x = 27 \Rightarrow x = 9$
Similarly, if x is the pre–image of 53, then $f(x) = 53$. Hence $3x + 2 = 53$
$3x = 51 \Rightarrow x = 17$
Thus the pre–images of 29 and 53 are 9 and 17 respectively

(iii) Since different elements of $\mathbb{N}$ have different images in the co-domain, the function is one – one function.
The co–domain of f is $\mathbb{N}$.
But the range of $f = \{5, 8, 11, 14, 17, ...\}$ is a subset of $\mathbb{N}$.
Therefore f is not an onto function. That is, f is an into function
Thus f is one – one and into function.

---

6. Let f be a function from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = 3x - 5$. Find the values of a and b given that $(a, 4)$ and $(1, b)$ belong to f. **PTA - 6**

Solution :

$f(x) = 3x - 5$ can be written as $f = \{(x, 3x - 5) \mid x \in R\}$
$(a, 4)$ means the image of a is 4
That is, $f(a) = 4$
$3a - 5 = 4$
$\Rightarrow a = 3$
$(1, b)$ means the image of 1 is b.
That is, $f(1) = b \Rightarrow b = -2$
$3(1) - 5 = b \Rightarrow b = -2$

---

7. The distance S (in kms) travelled by a particle in time 't' hours is given by $S(t) = \frac{t^2 + t}{2}$. Find the distance travelled by the particle after
(i) three and half hours
(ii) eight hours and fifteen hours

Solution :

The distance travelled by the particle is given by $S(t) = \frac{t^2 + t}{2}$

(i) $t = 3.5$ hours. Therefore $S(3.5) = \frac{(3.5)^2 + 3.5}{2} = \frac{15.75}{2} = 7.875$

The distance travelled in 3.5 hours in 7.875 kms.

(ii) $t = 8.25$ hours. Therfore $S(8.25) = \frac{(8.25)^2 + 8.25}{2} = \frac{76.3125}{2} = 38.15625$

The distance travelled in 8.25 hours is 38.16 kms, approximately.

---

8. Find $f \circ g$ and $g \circ f$ when $f(x) = 2x + 1$ and $g(x) = x^2 - 2$

Solution :

$f(x) = 2x + 1, g(x) = x^2 - 2$
$f \circ g(x) = f(g(x)) = f(x^2 - 2) = 2(x^2 - 2) + 1 = 2x^2 - 3$
$g \circ f(x) = g(f(x)) = g(2x + 1) = (2x + 1)^2 - 2 = 4x^2 + 4x - 1$
Thus, $f \circ g = 2x - 3$, $g \circ f = 4x^2 + 4x - 1$. From the above, we see that $f \circ g \neq g \circ f$

9. If $f(x) = 3x - 2$, $g(x) = 2x + k$ and $f \circ g = g \circ f$, then find the value of k.

Solution : $f(x) = 3x - 2$, $g(x) = 2x + k$

$f \circ g(x) = f(g(x)) = f(2x + k) = 3(2x + k) - 2 = 6x + 3k - 2$

Thus, $f \circ g(x) = 6x + 3k - 2$

$g \circ f(x) = g(3x - 2) = 2(3x - 2) + k$

Thus, $g \circ f(x) = 6x - 4 + k$.

Given that $f \circ g = g \circ f$

Therefore, $6x + 3k - 2 = 6x - 4 + k$

$6x - 6x + 3k - k = -4 + 2 \Rightarrow k = -1$

10. Find k if $f \circ f(k) = 5$ where $f(k) = 2k - 1$ **PTA - 4**

Solution : $f \circ f(k) = f(f(k))$

$= 2(2k - 1) - 1 = 4k - 3$

Thus, $f \circ f(k) = 4k - 3$

But, it is given that $f \circ f(k) = 5$

Therefore $4k - 3 = 5 \Rightarrow k = 2$

GMQ & PTA - ADDITIONAL QUESTIONS

1. Let $A = \{x \in \mathbb{N} \mid 1 < x < 4\}$, $B = \{x \in \mid 0 \leq x < 2\}$ and $C = \{x \in \mathbb{N} \mid x < 3\}$.
Then verify that
(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Solution :

$A = \{x \in \mathbb{N} \mid 1 < x < 4\} = \{2, 3\}$, $B = \{x \in \mid 0 \leq x < 2\} = \{0, 1\}$,

$C = \{x \in \mathbb{N} \mid x < 3\} = \{1, 2\}$

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

$B \cup C = \{0, 1\} \cup \{1, 2\} = \{0, 1, 2\}$

$A \times (B \cup C) = \{2, 3\} \times \{0, 1, 2\} = \{(2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2)\}$ ----(1)

$A \times B = \{2, 3\} \times \{0, 1\} = \{(2, 0), (2, 1), (3, 0), (3, 1)\}$

$A \times C = \{2, 3\} \times \{1, 2\} = \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$(A \times B) \cup (A \times C) = \{(2, 0), (2, 1), (3, 0), (3, 1) \cup (2, 1), (2, 2), (3, 1), (3, 2)\}$

$= \{(2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2)\}$ ------------- (2)

From (1) and (2), $A \times (B \cup C) = (A \times B) \cup (A \times C)$ is verified

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$B \cap C = \{0, 1\} \cap \{1, 2\} = \{1\}$

$A \times (B \cap C) = \{2, 3\} \times \{1\} = \{(2, 1), (3, 1)\}$ ----------- (3)

$A \times B = \{2, 3\} \times \{0, 1\} = \{(2, 0), (2, 1), (3, 0), (3, 1)\}$

$A \times C = \{2, 3\} \times \{1, 2\} = \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$(A \times B) \cap (A \times C) = \{(2, 0), (2, 1), (3, 0), (3, 1)\} \cap \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$= \{(2, 1), (3, 1)\}$ ----------- (4)

From (3) and (4), $A \times (B \cap C) = (A \times B) \cap (A \times C)$ is verified.

2. Given $f(x) = 2x - x^2$,

Solution :

(i) Replacing x with 1, we get $f(1) = 2(1) - (1)^2 = 2 - 1 = 1$

(ii) Replacing x with $x + 1$, we get

$f(x + 1) = 2(x + 1) - (x + 1^2) = 2x + 2 - (x^2 + 2x + 1) = -x^2 + 1$

(iii) $f(x) + f(1) = (2x - x^2) + 1 = - x^2 + 2x + 1$

[Note that $f(x) + f(1) \neq f(x + 1)$. In general $f(a + b)$ is not equal to $f(a) + f(b)$]

3. Forensic scientists can determine the height (in cms) of a person based on the length of their thigh bone. They usually do so using the function $h(b) = 2.47\,b + 54.10$ where b is the length of the thigh bone.

(i) Check if the function h is one – one

(ii) Also find the height of a person if the length of his thigh bone is 50 cms.

(iii) Find the length of the thigh bone if the height of a person is 147. 96 cms.

Solution :

(i) To check if h is one – one, we assume that $h(b_1) = h(b_2)$

Then we get,

$2.47\,b_1 + 54.10 = 2.47b_2 + 54.10$

$2.47b_1 = 2.47b_2 \Rightarrow b_1 = b_2$

Thus, $h(b_1) = h(b_2)$

$\Rightarrow b_1 = b_2$. So, the function h is one – one.

(ii) If the length of the thigh bone $b = 50$, then the height is

$h(50) = (2.47 \times 50) + 54.10 = 177.6$ cms

(iii) If the height of a person is 147.96 cms, then $h(b) = 147.96$ and so the length of the thigh bone is given by $2.47\,b + 54.10 = 147.96$

$$b = \frac{93.86}{2.47} = 38$$

Therefore, the length of the thigh bone is 38 cm.

4. If the function $f : R \to R$ defined by

$$f(x) = \begin{cases} 2x + 7, & x < -2 \\ x^2 - 2, & -2 \leq x < 3 \\ 3x - 2, & x \geq 3 \end{cases}$$

Then find the values of

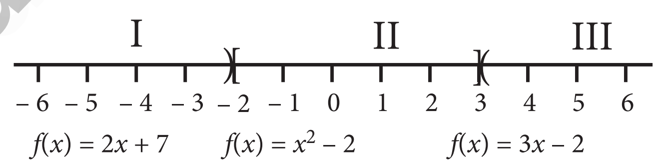
(i) $f(4)$ **(ii) $f(-2)$**

(iii) $f(4) + 2f(1)$ **(iv) $\frac{f(1) - 3f(4)}{f(-3)}$**

Solution :

The function f is defined by three values in intervals I, II, III as shown below

For a given value of $x = a$, find out the interval at which the point a is located, there after find $f(a)$ using the particular value defined in that interval



(i) First, we see that, $x = 4$ lie in the third interval. Therefore,

$f(x) = 3x - 2$; $f(4) = 3(4) - 2 = 10$

(ii) $x = -2$ lies in the second interval. Therefore,

$f(x) = x^2 - 2$; $f(-2) = (-2)^2 - 2 = 2$

(iii) From (i), $f(4) = 10$

To find $f(1)$, first we see that $x = 1$ lies in the second interval

Therefore, $f(x) = x^2 - 2 \Rightarrow f(1) = 1^2 - 2 = -1$

Therefore, $f(4) + 2f(1) = 10 + 2(-1) = 8$

(iv) We know that $f(1) = -1$ and $f(4) = 8$

For finding $f(-3)$, we see that $x = -3$ lies in the first interval

Therefore,

$f(x) = 2x + 7$; thus, $f(-3) = 2(-3) + 7 = 1$

Therefore,

$$\frac{f(1)-3f(4)}{f(-3)} = \frac{-1-3(10)}{1} = -31$$

5. If $f(x) = 2x + 3$, $g(x) = 1 - 2x$ and $h(x) = 3x$. Prove that $f \circ (g \circ h) = (f \circ g) \circ h$ PTA - 5

Solution :

$f(x) = 2(x) + 3$, $g(x) = 1 - 2x$, $h(x) = 3x$

Now, $(f \circ g)(x) = f(g(x))$

$= f(1 - 2x) = 2(1-2x) + 3 = 5 - 4x$

Since, $(f \circ g) \circ h(x) = (f \circ g)(h(x))$

$= (f \circ g)(3x) = 5 - 4(3x) = 5 - 12x$ --------(1)

$(g \circ h)(x) = g(h)(x))$

$= g(3x) = 1 - 2(3x) = 1 - 6x$

Since, $f \circ (g \circ h)(x) = f(1 - 6x)$

$= 2(1 - 6x) + 3 = 5 - 12x$ ----- (2)

From (1) and (2),

we get $(f \circ g) \circ h = f \circ (g \circ h)$

6. Find x if $gff(x) = fgg(x)$, given $f(x) = 3x + 1$ and $g(x) = x + 3$

Solution:

$gff(x) = g[f\{f(x)\}$

(This means "g of f of f of x")

$= g[f(3x + 1)] = g[3(3x + 1) + 1]$

$= [3(3x + 1)] + 3 = 9x + 7$

$fgg(x) = f[g\{g(x)\}]$

(This means "f of g of g of x")

$= f[g(x + 3)] = f[(x + 3) + 3]$

$f(x + 6) = [3(x + 6) + 1] = 3x + 19$

These two quantities being equal,

we get $9x + 7 = 3x + 19$.

Solving this equation we obtain $x = 2$.

7. Let A = {1,2, 3, 4} and B = {2,5, 8,11,14} be two sets. Let $f : A \to B$ be a function given by $f(x) = 3x - 1$. Represent this function

(i) by arrow diagram SEP 20

(ii) in a table form

(iii) as a set of ordered pairs

(iv) in a graphical form PTA - 3

Solution :

$A = \{1,2, 3, 4\}$;

$B = \{2,5, 8,11,14\}$; $f(x) = 3x - 1$

$f(1) = 3(1) - 1 = 3 - 1 = 2$;

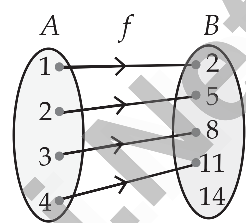
$f(2) = 3(2) - 1 = 6 - 1 = 5$

$f(3) = 3(3) - 1 = 9 - 1 = 8$;

$f(4) = 4(3) - 1 = 12 - 1 = 11$

(i) Arrow diagram

Let us represent the function $f : A \to B$ by an arrow diagram.



(ii) Table form

The given function f can be represented in a tabular form as given below

x	1	2	3	4
$f(x)$	2	5	8	11

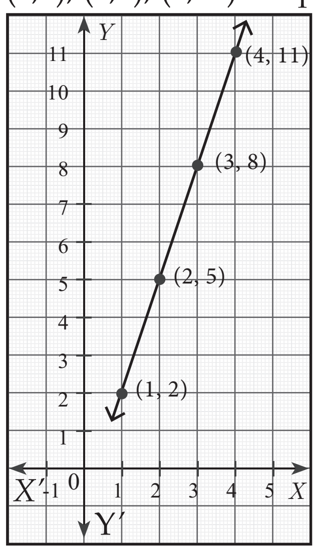
(iii) Set of ordered pairs

The function f can be represented as a set of ordered pairs as

$f = \{(1,2),(2,5),(3,8),(4,11)\}$

(iv) Graphical form

In the adjacent XY -plane the points (1,2), (2,5), (3,8), (4,11) are plotted



8. R = {(x, -2), (-5,y)} represents the identity function, find the values of x and y. PTA - 6

Solution :

R = {(x, -2), (-5,y)}

represents the identity function

$\therefore x = -2$

$y = -5$

9. *Let A = {1, 2, 3, ..., 100}* and *R* be the relation defined as "is cube of " on *A*. Find the domain and range of R. PTA - 4

Solution :

A = {1, 2, 3, ..., 100}

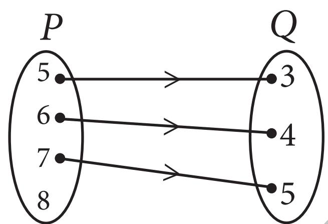
The relation is defined as 'is cube of'

R = {1,1), (2,8), (3,27), (4,64)}

∴ Domain of R = {1,2,3,4}

Range of R = {1,8,27,64}

***10.* The arrow diagram shows a relationship between the sets, P and Q. Write the relatio in (i) Set builder form (ii) Roster form (iii) What is the domain and range of R.** Sep - 2020 Aug - 2022



Solution :

(i) Set builder form of

R = $\{(x, y) \mid y = x - 2, x \in P, y \in Q\}$

(ii) Roster form R = *{(5, 3), (6, 4), (7, 5)}*

(iii) Domain of R = *{(5, 6, 7)* and range of *R = {3, 4, 5}*

11. A = {1, 3, 5} and B = {2, 3} then show that n(A × B) = n (A) × n (B). Sep - 2021

Solution :

A×B = {1,3,5} × {2,3}

= {(1,2), (1,3), (3,2), (3,3), (5,2),(5,3)}

n (A×B) = 6

n (A) = 3

n (B) = 2

∴n (A×B) = n(A) × n (B)

⇒6 = 3 × 2

= 6

12. If *A×B* = {(3,2), (3,4), (5,2), (5,4)} then find A and B. May 2022

Solution :

A×B = {(3,2), (3,4), (5,2), (5,4)}

We have A = {set of all first coordinates of elements of A×B}. ∴ A = {3,5}

B= {set of all second coordinates of elements of A × B} ∴ *B* = {2,4}

Thus A = {3,5} and *B* = {2,4}

13. Find k if $f \circ f(k) = 5$ where $f(k) = 2k - 1$ April - 2023

Solution :

$f \circ f(k) = f(f(k))$

$= 2(2k - 1) - 1 = 4k - 3.$

$f \circ f(k) = 4k - 3$

But $f \circ f(k) = 5$

$\therefore 4k - 3 = 5 \Rightarrow k = 2$

14. Let A = {x ∈ W/ x<3}, B = {x ∈ N/1<x ≤ 5} and C = {3, 5, 7} verify that A×(B∪C) = (A×B)∪(A×C) Apr - 2023

Solution :

A = {0, 1, 2}

B = {2, 3, 4, 5}

C = {3, 5, 7}

B∪C = {2, 3, 4, 5, 7}

A×(B∪C) = {(0, 2), (0, 3), (0, 4), (0, 5), (0, 7), (1, 2), (1, 3), (1, 4), (1, 5), (1, 7), (2, 2), (2, 3), (2, 4), (2, 5), (2, 7)} ———(1)

A×B = {(0, 2), (0, 3), (0, 4), (0, 5), (1, 2) (1, 3) (1, 4) (1, 5) (2, 2) (2, 3), (2, 4), (2, 5)}

A×C = {(0, 3), (0, 5), (0, 7), (1, 3), (1, 5), (0, 7) (1, 3) (1, 5) (1, 7) (2, 3), (2, 5), (2, 7)}

(A×B)∪(A×C)

= {(0, 2), (0, 3), (0, 4), (0, 5), (0, 7) (1, 2) (1, 3) (1, 4) (1, 5) (1, 7) (2, 2) (2, 3), (2, 4), (2, 5), (2, 7)} ———(2)

∴ From (1) and (2)

A×(B∪C) = (A×B)∪(A×C)

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