Tirunelveli District

First Mid Term Exam - 11th 2023

Maths

PART - I

I

1) d. N

2) d. 16

3) c. [0, 1)

4) c n

5) a 4

6) c 3

7) b. 7

8) b [2, ∞)

9) a 10⁸ seconds

10 b $\frac{1}{12}$

Part - 2

$3 \times 2 = 6$

II) Answer Three questions. Qn.No. 15 is compulsory:

11.) If $A \times A$ has 16 elements, $S = \{(a, b) \in A \times A : a < b\}$; $(-1, 2)$ and $(0, 1)$ are two elements of S, then find the remaining elements of S

Solution:

$n(A \times A) = 16 \Rightarrow n(A) = 4$

$S = \{(-1, 0), (-1, 1), (0, 2), (1, 2)\}$

12.) Solve $-2x \geq 9$ when x is a natural number

Solution:

$x = 1, 2, 3, 4$

13.) Construct a quadratic equation with Roots 7 and -3

Solution

$\alpha = 7 \qquad \beta = -3$

$\alpha\beta = (7)(-3)$

$\alpha\beta = -21$

$x^2 - x + = 0$

$x^2 - 4x - 21 = 0$

Convert $\frac{\pi}{5}$ radian to degree

Ans:

π radian $= 180^\circ$

$\frac{\pi}{5}$ radian $= \frac{180^\circ}{5} = 36^\circ$

Find the number of subsets of A if $A = \{x : x = 4n+1, 2 \le n \le 5, n \in N\}$

solution.

$A = \{x : x = 4n+1, n = 2, 3, 4, 5\}$

$= \{9, 13, 17, 21\}$

$n(A) = 4$

$n(P(A)) = 2^4 = 16$

Part-3 $3 \times 3 =$

Answer three questions. Q.no 22 is compulsory.

Find the range of function $f(x) = \frac{1}{1 - 3\cos x}$

solution.

$-1 \le \cos x \le 1$

$\Rightarrow 3 \ge -3\cos x \ge -3$

$\Rightarrow -3 \le -3\cos x \le 3$

$\Rightarrow 1 - 3 \le 1 - 3\cos x \le 1 + 3$

$= -2 \le 1 - 3\cos x$ and $1 - 3\cos x \le 4$

~~Solve $\sqrt{x+4}$~~

Solve the equation $\sqrt{6-4x-x^2} = x+4$

Solution

$(x+4) \geq 0 \quad 6-4x-x^2 = (x+4)^2$

$= x \geq -4$

$= x^2+6x+5=0$

$x=-1, -5$

$\boxed{x=-1}$

3) Resolve into partial fraction: $\frac{x}{(x+3)(x-4)}$

Sln:

$$\frac{x}{(x+3)(x-4)} = \frac{A}{x+3} + \frac{B}{x-4}$$

$$\frac{x}{(x+3)(x-4)} = \frac{A(x-4)+B(x+3)}{(x+3)(x-4)}$$

$$x = A(x-4) + B(x+3)$$

$x=4$

$B = \frac{4}{7}$

$x=-3$

$A = \frac{3}{7}$

$$\frac{x}{(x+3)(x-4)} = \frac{3}{7(x+3)} + \frac{4}{7(x-4)}$$

1) Show that the relation $xy = -2$ is a function for a suitable domain. Find the domain and the range of the function.

Solution:

Let $f : R \to R$ defined as $f : x \to \frac{2}{x}$ the

$f(x) = -\frac{2}{x}$

$y = -\frac{2}{x}$

$\Rightarrow xy = -2$

$x = 0$

Let $f : R - \{0\} \to R$ defined as

$f(x) = -\frac{2}{x}$

$\Rightarrow y = -\frac{2}{x} \quad = xy = -2$

$f : R - \{0\} \to R \{0\}$

Domain $= R - \{0\}$

Range $= R - \{0\}$

Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15°

Solution:

Let s be the length of the arc of a circle of radius r subtending a central angle θ

$s = r\theta$

$\theta = 15^\circ$

$= 15 \times \frac{\pi}{180} = \frac{\pi}{12}$ radian

$s = r\theta$

$s = 5 \times \frac{\pi}{12} = \frac{5\pi}{12}$ cm

PART - 4

$4 \times 5 = 20$

Answer all the question

In the set Z of integers, define mRn if m-n is divisible by 7. Prove that R is an equivalent relation.

Solution:

mRn if m-n is divisible by 7

a) $mRm = m - m = 0$

0 is divisible by 7

∴ It is reflexive

5) mRn = $(m-n)$ is divisible by 7

$nRm = (n-m) = -\{m-n\}$ is also divisible by 7

It is symmetric

c) $mRn \Rightarrow (m-n)$ is by 7 $= \frac{(m-n)}{7} = \frac{k}{7}$

$mRr \Rightarrow (n-r)$ is divisible $7 = \frac{(n-r)}{7} = \frac{l}{7}$

$\Rightarrow mRr = m-r = \left(\frac{k}{7}+n\right)-\left(n-\frac{l}{7}\right)$

$m-r = \frac{k}{7} + n - n + \frac{l}{7}$

$(m-r) = \frac{1}{7}(k+l)$ is divisible by 7

It is transitive

mRn if $m-n$ is divisible by 7

$\therefore R$ is an equivalence relation

If the arcs of same lengths in two circles subtend central angles 30° and 80° find the ratio of their radii.

<u>Solution</u>

Let r_1 and r_2 be the radii of the two given circles and l be length of the arc.

$\theta_1 = 30° = \frac{\pi}{6}$ radian

$\theta_2 = 80° = \frac{4\pi}{9}$ radian

Given that $l = r_1\theta = r_2\theta$

$$\frac{\pi}{6} r_1 = \frac{4\pi}{9} r_2$$

$$\frac{r_1}{r_2} = \frac{8}{3}$$

$$r_1 : r_2 = 8:3$$

Let $f, g : R \to R$ be defined as $f(x) = 2x - |x|$ and $g(x) = 2x + |x|$ Find $f \circ g$.

solution:

$$|x| = \begin{cases} -x & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$$

$$f(x) = \begin{cases} 2x - (-x) & \text{if } x \leq 0 \\ 2x - x & \text{if } x > 0 \end{cases}$$

$$f(x) = \begin{cases} 3x & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$$

$$g(x) = \begin{cases} 2x + (-x) & \text{if } x \leq 0 \\ 2x + x & \text{if } x > 0 \end{cases}$$

$$g(x) = \begin{cases} x & \text{if } x \leq 0 \\ 3x & \text{if } x > 0 \end{cases}$$

$$(f \circ g)(x) = f(g(x)) = f(x) = 3x$$

$3x \leq 0$ whenever $x \leq 0$

let $x > 0$

$$(f \circ g)(x) = f(g(x)) = f(g(x)) = f(3x) = 3x$$

$(f \circ g)(x) = 3x$ for all x

rite the values of the $-4, 1, -2, 7, 0$ if $f(x) = \begin{cases} -x+4 & \text{if } -\infty < x \le -3 \\ x+4 & \text{if } -3 < x < -2 \\ x^2 - x & \text{if } -2 \le x < 1 \\ x - x^2 & \text{if } 1 \le x < 7 \\ 0 & \text{if otherwise} \end{cases}$

Solution:

$$f(-4) = x + 4 = -(-4) + 4 = 8$$

$$f(1) = x - x^2 = 1 - 1 = 0$$

$$f(-2) = x^2 - x = (-2)^2 - (-2) = 4 + 2 = 6$$

$$f(7) = \qquad 0$$

$$f(0) = x^2 - x$$

$$= 0 - 0$$

$$= 0$$

23.) If $a^2 + b^2 = 7ab$, show that $\log \frac{a+b}{3} = \frac{1}{2}(\log a + \log b)$

Solution

add $2ab$ $= 7ab + 2ab$

$$a + b^2 = 9ab$$

sin θ) = 4n , then prove that (m²

$$\frac{(a+b)^2}{9} = ab$$

$$\left(\frac{a+b}{3}\right)^2 = ab$$

taking log on both sides

$$\log\left(\frac{a+b}{3}\right)^2 = \log ab$$

$$= \log\left(\frac{a+b}{3}\right) = \log a + \log b$$

$$= \log\left(\frac{a+b}{3}\right) = \frac{1}{2}(\log a + \log b).$$

If the equations $x^2 - ax + b = 0$ and $x^2 - ex + f = 0$ have one root in common and if the second equation has equal roots that $ae = 2(b+f)$

solution:

let α be the common root

then $\alpha^2 - a\alpha + b = 0 \ldots (1)$

$\alpha + \beta = a$

$\alpha\beta + b \Rightarrow \beta = \frac{b}{\alpha} = \frac{2b}{e}$

$x^2 - ex + f = 0$

α, β

$\alpha \times \alpha = f$

$\alpha^2 = f$

α and α^2 value in (1)

$f - a\left(\frac{e}{2}\right) + b = 0$

$f - \frac{ae}{2} + b = 0$

$\frac{ae}{2} = b + f \Rightarrow ae = 2(b+f)$

sum of roots $= \alpha + \alpha = e \Rightarrow 2\alpha = e \Rightarrow \alpha = \frac{e}{2}$

Find a quadratic polynomial $f(x)$ such that, $f(0)=1$, $f(-2)=0$
$f(1)=0$

Solution:

$$f(x)=ax^2+bx+c$$

$$f(0)=a(0)^2+b(0)+c=1$$

$c=1$

$$f(-2)=0$$

$$f(1)=0$$

$$4a-2b+c=1 \text{ and } a+b+c=0$$

$c=1$

$$4a-2b=-1 \text{ and } a+b=-1$$

$$a=b=\frac{-1}{2}$$

$$f(x)=-\frac{1}{2}x^2-\frac{1}{2}x+1$$

If $\cot\theta(1+\sin\theta)=4m$ and $\cot\theta(1-\sin\theta)=4n$, then prove that $(m^2-n^2)^2=mn$.

Solution:

$\cot\theta(1+\sin\theta)=4m$

$\Rightarrow \frac{\cos\theta}{\sin\theta}(1+\sin\theta)=4m$

$\Rightarrow \cot\theta+\cos\theta=4m$

$\Rightarrow m=\frac{\cot\theta+\cos\theta}{4}$

$\cot\theta(1-\sin\theta)=4n$

$\Rightarrow \frac{\cos\theta}{\sin\theta}(1-\sin\theta)=4n$

$\Rightarrow \cot\theta-\cos\theta=4n$

$\Rightarrow n=\frac{\cot\theta-\cos\theta}{4}$

To prove $(m^2-n^2)^2=mn$

$RHS=mn=\left(\frac{\cot\theta+\cos\theta}{4}\right)\left(\frac{\cot\theta-\cos\theta}{4}\right)$

$=\frac{\cot^2\theta-\cos^2\theta}{16}$

$(m^2-n^2)=([m+n)(m-n)])$

$m+n=\frac{\cot\theta+\cos\theta+\cot\theta-\cos\theta}{4}=\frac{2\cot\theta}{4}=\frac{\cot\theta}{2}$

$m-n=\frac{\cot\theta+\cos\theta-\cot\theta+\cos\theta}{4}=\frac{2\cos\theta}{4}\ \frac{\cos\theta}{2}$

$\therefore (m+n)(m-n) = \frac{\cot\theta}{2}\frac{\cos\theta}{2} = \frac{\cot\theta\cos\theta}{4}$

so LHS

$= (m^2-n^2)^2 = [(m-n)(m+n)]^2$

$= \left(\frac{\cot\theta \quad \cos\theta}{4}\right)^2 = \frac{\cot^2\theta\cos^2\theta}{16}$

RHS

$= mn = \frac{\cot^2\theta - \cos^2\theta}{16}$

$= \frac{1}{16}\left[\frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta\right] = \frac{\cos^2}{16\sin^2\theta}[1-\sin^2\theta$

$= \frac{1}{16}\cot^2\theta\cos^2\theta = LHS$

$\therefore LHS = RHS \Rightarrow (m^2-n^2)^2 = mn$