

**First Mid-Term Test - 2023**  
**MATHEMATICS**

Time : 1.30 Hrs.

Marks : 50

**PART - I****Choose the correct answer****10 x 1 = 10**

- Let A and B be subsets of the universal set  $N$ , the set of natural numbers. Then  $A \cup [(A \cap B) \cup B^c]$  is  
a) A b)  $A^c$  c) B d)  $N$
- If  $A = \{(x, y) : y = e^x, x \in R\}$ ,  $B = \{(x, y) : y = e^{-x}, x \in R\}$  then  $n(A \cap B)$  is  
a) infinity b) 0 c) 1 d) 2
- If  $n(A) = 2$  and  $n(B \cup C) = 3$ , then  $n[(A \times B) \cup (A \times C)]$  is  
a)  $2^3$  b)  $3^2$  c) 6 d) 5
- The range of the function  $\frac{1}{1-2\sin x}$  is  
a)  $(-\infty, -1) \cup \left(\frac{1}{3}, \infty\right)$  b)  $\left(-1, \frac{1}{3}\right)$  c)  $\left[-1, \frac{1}{3}\right]$  d)  $(-\infty, -1] \cup \left[\frac{1}{3}, \infty\right)$
- The range of the function  $f(x) = ||x| - x|$ ,  $x \in R$  is  
a)  $[0, 1]$  b)  $[0, \infty)$  c)  $(0, 1)$  d)  $(0, 1)$
- The value of  $\log_{\sqrt{2}} 512$  is  
a) 16 b) 18 c) 9 d) 12
- The value of  $\log_3 11 \cdot \log_{11} 13 \cdot \log_{13} 15 \cdot \log_{15} 27 \cdot \log_{27} 81$  is  
a) 1 b) 2 c) 3 d) 4
- The solution set of the following inequality  $|x - 1| \geq |x - 3|$  is  
a)  $[0, 2]$  b)  $[2, \infty)$  c)  $(0, 2)$  d)  $(-\infty, 2)$
- If 3 is the logarithm of 343, then the base is  
a) 5 b) 7 c) 6 d) 9
- The number of solutions of  $x^2 + |x - 1| = 1$  is  
a) 1 b) 0 c) 2 d) 3

**PART - II****II. Answer the four questions. 16th question compulsory.****4 x 2 = 8**

- If (A) denotes the power set of A then find  $n(p(p(p(\phi))))$
- Find the number of subsets of A if  $A = \{x : x = 4n + 1, 2 \leq n \leq 5, n \in N\}$
- Solve for x :  $|3 - x| < 7$
- If the logarithm of 324 to base a is 4, then find a.
- Find the complete set of values of a for which the quadratic  $x^2 - ax + a + 2 = 0$  has equal roots.
- If  $n(A \cap B) = 3$  and  $n(A \cup B) = 10$ , then find  $n(P(A \Delta B))$

**PART - III****III. Answer the four questions. 22nd question is compulsory.****4 x 2 = 8**

- By taking suitable set A, B, C verify the following result  
 $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

18. Let  $A = \{a, b, c\}$   $R = \{(a, a) (b, b) (a, c)\}$ . Write down the minimum number of ordered pairs to be included to  $R$  to make it i) reflexive ii) symmetric iii) transitive iv) equivalence
19. Let  $f$  and  $g$  be the two functions from  $R$  to  $R$  defined by  $f(x) = 3x - 4$  and  $g(x) = x^2 + 3$  find  $g \circ f$  and  $f \circ g$ .
20. Solve  $\frac{1}{5} |10x - 2| < 1$
21. Find the real roots of  $x^4 = 16$ .
22. Prove  $\log \frac{a^2}{bc} + \log \frac{b^2}{ca} + \log \frac{c^2}{ab} = 0$

### PART - IV

IV. Answer the following questions.

4 x 5 = 20

23. If  $A$  and  $B$  are two sets so that  $n(B-A) = 2n(A-B) = 4n(A \cap B)$  and if  $n(A \cup B) = 14$ , then find  $n(P(A))$ .  
(OR)  
b) In the set  $Z$  of integers, define  $mRn$  if  $m - n$  is divisible by 7. Prove that  $R$  is an equivalence relation.
24. Let  $f, g : R \rightarrow R$  be defined as  $f(x) = 2x - |x|$ , and  $g(x) = 2x + |x|$ , find  $f \circ g$ .  
(OR)

b) Write the values of  $f$  at  $-4, 1, -2, 7, 0$  if  $f(x) =$

$$\begin{cases} -x + 4 & \text{if } -\infty < x \leq -3 \\ x + 4 & \text{if } -3 < x < -2 \\ x^2 - x & \text{if } -2 \leq x < 1 \\ x - x^2 & \text{if } 1 \leq x < 7 \\ 0 & \text{Otherwise} \end{cases}$$

25. If  $f : R \rightarrow R$  defined by  $f(x) = 3x - 5$ , prove that  $f$  is a bijection and find its inverse.  
(OR)

b) Find all values of  $x$  for which  $\frac{x^3(x-1)}{(x-2)} > 0$

26. a) Resolve into partial fractions.  $\frac{2x}{(x^2 + 1)(x - 1)}$

(OR)

b) If  $x = \sqrt{2} + \sqrt{3}$  find  $\frac{x^2 + 1}{x^2 - 2}$

Priteducation

M. SriAditya