Class : 11

Register Number

FIRST MID TERM TEST - 2023

Time Allowed : 1.30 Hours] MATHEMATICS [Max. Marks : 45

Part - A

1. Choose the best alternative for the following. 2. Each question carries one mark. 10x1=10

1. Let X = {1, 2, 3, 4} and R = {(1, 1), (1, 2), (1, 3), (2, 2), (3, 3), (2, 1), (3, 1), (1, 4),(4, 1)}. Then R is

(a) reflexive (b) symmetric (c) transitive (d) equivalence

2. The function $f : [0,2\pi] \to [-1,1]$ defined by $f(x) = \sin x$ is

(a) one-to-one (b) on to (c) bijection (d) cannot be defined

3. The number of students who take both the subjects Mathematics and Chemistry is 70. This represents 10% of the enrollment in Mathematics and 14% of the enrollment in Chemistry. The number of students take at least one of these two subjects, is

(a) 1120 (b) 1130 (c) 1100 (d) insufficient data

4. If $|x+2| \leq 9$, then x belongs to

(a) $(-\infty, -7)$ (b) $[-11, 7]$ (c) $(-\infty, -7) \cup (11, \infty)$ (d) $(-11, 7)$

5. Let $f : R \to R$ be defined by $f(x) = 1 - |x|$. Then the range of f is

(a) R (b) $(1,\infty)$ (c) $(-1,\infty)$ (d) $(-\infty,1]$

6. If $\frac{kx}{(x+2)(x-1)} = \frac{2}{x+2} + \frac{1}{x-1}$, then the value of k is

(a) 1 (b) 2 (c) 3 (d) 4

7. The equation whose roots are numerically equal but opposite in sign to the roots $3x^2 - 5x - 7 = 0$

(a) $3x^2 - 5x - 7 = 0$ (b) $3x^2 + 5x - 7 = 0$ (c) $3x^2 - 5x + 7 = 0$ (d) $3x^2 + x - 7 = 0$

8. Find a so that the sum and product of the roots of the equation $2x^2 + (a-3)x + 3a - 5 = 0$ are equal is

(a) 1 (b) 2 (c) 0 (d) 4

9. The solution $5x-1 < 24$ and $5x+1 > -24$ is

(a) (4,5) (b) (-5,-4) (c) (-5,5) (d) (-5,4)

10. $\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \ldots\ldots\ldots\ldots + \cos 179^\circ =$

(a) 0 (b) 1 (c) -1 (d) 89

PART - B

1. Answer any four of the following 2. Each question carry 2 marks. 4x2=8

11. Let f and g be the two functions from R to R-defined by $f(x) = 3x-4$ and $g(x) = x^2+3$. Find gof and fog.

12. Solve for x: $|3 - x| < 7$.

13. Construct a quadratic equation with roots 7 and -3.

14. Find the value of $\cos 105^\circ$

15. $\log a + \log a^2 + \log a^3 + \ldots\ldots + \log a^n = \frac{n(n+1)}{2}$

16. Discuss the following relations for reflexivity, symmetricity and transitivity: On the set of natural numbers, the relation R is defined by "xRy if $x + 2y = 1$".

PART - C

1. Answer any four of the following. 2. Each question carry 3 marks. **4 x 3 = 12**

17. Find the domain $f(x) = \frac{1}{1-2\sin x}$

18. Prove that $\sqrt{3}$ is an irrational number.

19. A model rocket is launched from the ground. The height h reached by the rocket after t sectons from lift off is given by $h(t) = -5t^2 + 100t$, $0 \leq t \leq 20$. At what time the rocket is 495 feet above the ground?

20. If $A+B = 45^\circ$ show that, $(1+\tan A)(1+\tan B) = 2$.

21. If α and β are the roots of the quadratic equation $x^2 + \sqrt{2}x + 3 = 0$, form a quadratic polynomial with zeros $1/\alpha, 1/\beta$.

22. Resolve the following rational expressions into partial fractions. $\frac{3x+1}{(x-2)(x+1)}$

PART – D

1. Answer the following. 2. Each question carry 5 marks. **3x5=15**

23. a) If $f: R \to R$ defined by , $f(x) = 2x - 3$ prove that f is a bijection and find its inverse.

(OR)

b) Write the values of f at -3,5,2,-1,0 if

$$f(x) = \begin{cases} x^2 + x - 5 & \text{if } x \in (-\infty, 0) \\ x^2 + 3x - 2 & \text{if } x \in (3, \infty) \\ x^2 & \text{if } x \in (0,2) \\ x^2 - 3 & \text{if other wise} \end{cases}$$

24. a) Resolve the following rational expressions into partial fractions. $\frac{x}{(x^2+1)(x-1)(x+2)}$

(OR)

b) Solve : $\frac{x+1}{x+3} < 3$

25. a) Write and Prove Napiers formula.

(OR)

b) Prove $\log 2 + 16 \log \frac{16}{15} + 12 \log \frac{25}{24} + 7 \log \frac{81}{80} = 1$

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FIRST MID TERM TEST - 2023

STD : 11 MATHEMATICS - KEY MARKS : 45

PART-A

1	b	symmetric
2	b	onto
3	b	1130
4	b	$[-11, 7]$
5	d	$(-\infty, 1]$
6	c	3
7	b	$3x^2+5x-7=0$
8	b	2
9	c	$(-5, 5)$
10	a	0

PART-B

11. $(g \circ f)(x) = 9x^2 - 24x + 19$

$(f \circ g)(x) = 3x^2 + 5$

12. $-7 < 3 - x < 7$

$\Rightarrow -4 < x < 10$

13. $x^2 - 4x - 21 = 0$

14. $\cos(60+45) = \frac{1}{2}\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}\frac{1}{\sqrt{2}}$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$

15. LHS

$= \log a + 2\log a + 3\log a + \cdots + n\log a$

$= \log a (1+2+3+\cdots n)$

$= \frac{n(n+1)}{2}\log a$

16. R not reflexive

R not symmetric

R not transitive

PART-C

17. $\sin x = \frac{1}{2}$ $\sin x = \sin \pi/6$

$x = n\pi + (-1)^n \pi/6$, $n \in Z$

Domain of $f(x)$ is $R - \left(n\pi + (-1)^n \frac{\pi}{6}\right)$, $n \in Z$

18. $\sqrt{3} = \frac{m}{n} \Rightarrow 3n^2 = m^2$, $m = 2k$

$3n^2 = 4k^2$ (even)

$\sqrt{3}$ is an irrational number

19. $0 < h(t) < 495$

$0 < -5t^2 + 100t < 495$

$-5t^2 + 100t - 495 = 0 \Rightarrow t^2 - 20t + 99 = 0$

$(t-11)(t-9) = 0$

$t = 11$ sec, $t = 9$ sec

20. $\tan(A+B) = \tan 45^\circ$

$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1 \Rightarrow \tan A + \tan B = 1 - \tan A \tan B$

$(1+\tan A)(1+\tan B) = 2$

21. $\alpha + \beta = -\sqrt{2}$ $\alpha\beta = 3$

Sum $= \frac{1}{\alpha} + \frac{1}{\beta} = -\frac{\sqrt{2}}{3}$

Pro $= \frac{1}{\alpha}\cdot\frac{1}{\beta} = \frac{1}{3}$

$x^2 - x\left(\frac{-\sqrt{2}}{3}\right) + \frac{1}{3} = 0 \Rightarrow 3x^2 + \sqrt{2}x + 1 = 0$

22. $= \frac{A}{x-2} + \frac{B}{x+1}$

$3x+1 = A(x+1) + B(x-2)$

$x-2=-1$: $B = 7/3$ $x = 2$: $A = 7/3$

$\frac{3x+1}{(x-2)(x+1)} = \frac{7}{3(x-2)} + \frac{2}{3(x+1)}$

PART-D

23 a. one to one : $f(x) = f(y) \Rightarrow 2x-3 = 2y-3$

$\Rightarrow x = y$, f is one to one

onto : $x = \frac{y+3}{2}$, $f(x) = 2\left(\frac{y+3}{2}\right) - 3 = y$

f is onto

Inverse : $x = \frac{y+3}{2}$, $f^{-1}(y) = \frac{y+3}{2}$, $f^{-1}(x) = \frac{x+3}{2}$

b. $f(-3) = 1$ $f(-1) = -5$

$f(5) = 38$ $f(0) = -3$

$f(2) = 1$

24. a. $\frac{x}{(x^2+1)(x-1)(x+2)} = \frac{Ax+B}{(x^2+1)} + \frac{C}{x-1} + \frac{D}{x+2}$

$x = (Ax+B)(x-1)(x+2) + C(x^2+1)(x+2) + D(x^2+1)(x-1)$

$x=1 \Rightarrow C = 1/6$

$x=-2 \Rightarrow D = 2/15$

$x=0 \Rightarrow B = 1/10$

coefficient of $x^3 \Rightarrow A = -3/10$

$= \frac{1-3x}{10(x^2+1)} + \frac{1}{6(x-1)} + \frac{2}{15(x+2)}$

b. $\frac{x+1}{x+3} - 3 < 0 \Rightarrow \frac{x+1-3(x+3)}{x+3} < 0$

$\frac{x+4}{x+3} > 0$

x	$x+3$	$x+4$	$\frac{x+4}{x+3}$
$x<-4$	−	−	+
$-4<x<-3$	−	+	−
$x>-3$	+	+	+
$x=-4$	−	0	0

Solution : $(-\infty,-4) \cup (-3,\infty)$

25. a. $\triangle ABC$, (i) $\tan\frac{A+B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

$\frac{a-b}{a+b}\cot\frac{C}{2} = \frac{2R\sin A - 2R\sin B}{2R\sin A + 2R\sin B}\cot\frac{C}{2}$

$= \frac{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}\cot\frac{C}{2}$

$= \cot\left(\frac{A+B}{2}\right)\tan\frac{A-B}{2}\cot\frac{C}{2}$

$= \tan\frac{C}{2}\tan\frac{A+B}{2}\cot C/2$

$= \tan\frac{A-B}{2}$

II by

(ii). $\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$

(iii) $\tan\frac{C-A}{2} = \frac{c-a}{c+a}\cot\frac{B}{2}$

b. $= \log 2 + \log\left(\frac{16}{15}\right)^{16} + \log\left(\frac{25}{24}\right)^{12} + \log\left(\frac{81}{80}\right)^{7}$

$= \log 2 \times \frac{(2^4)^{16}}{(3\times5)^{16}} \times \frac{(5^2)^{12}}{(2^3\times3)^{12}} \times \frac{(3^4)^7}{(2^4\times5)^7}$

$= \log \frac{2^{64}\cdot5^{24}\cdot3^{28}}{3^{28}\cdot5^{23}\cdot2^{64}}$

$= \log 2^1\times5^1 = \log_{10}10 = 1$ RHS

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