

Class : 11

Register  
Number**FIRST MID TERM TEST - 2023**

Time Allowed : 1.30 Hours]

**MATHEMATICS**

[Max. Marks : 45

## Part - A

1. Choose the best alternative for the following. 2. Each question carries one mark. 10x1=10
- Let  $X = \{1, 2, 3, 4\}$  and  $R = \{(1, 1), (1, 2), (1, 3), (2, 2), (3, 3), (2, 1), (3, 1), (1, 4), (4, 1)\}$ . Then R is  
(a) reflexive (b) symmetric (c) transitive (d) equivalence
  - The function  $f: [0, 2\pi] \rightarrow [-1, 1]$  defined by  $f(x) = \sin x$  is  
(a) one-to-one (b) on to (c) bijection (d) cannot be defined
  - The number of students who take both the subjects Mathematics and Chemistry is 70. This represents 10% of the enrollment in Mathematics and 14% of the enrollment in Chemistry. The number of students take at least one of these two subjects, is  
(a) 1120 (b) 1130 (c) 1100 (d) insufficient data
  - If  $|x+2| \leq 9$ , then x belongs to  
(a)  $(-\infty, -7)$  (b)  $[-11, 7]$  (c)  $(-\infty, -7) \cup (11, \infty)$  (d)  $(-11, 7)$
  - Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = 1 - |x|$ . Then the range of f is  
(a)  $\mathbb{R}$  (b)  $(1, \infty)$  (c)  $(-1, \infty)$  (d)  $(-\infty, 1]$
  - If  $\frac{kx}{(x+2)(x-1)} = \frac{2}{x+2} + \frac{1}{x-1}$ , then the value of k is  
(a) 1 (b) 2 (c) 3 (d) 4
  - The equation whose roots are numerically equal but opposite in sign to the roots  $3x^2 - 5x - 7 = 0$   
(a)  $3x^2 - 5x - 7 = 0$  (b)  $3x^2 + 5x - 7 = 0$  (c)  $3x^2 - 5x + 7 = 0$  (d)  $3x^2 + x - 7 = 0$
  - Find a so that the sum and product of the roots of the equation  $2x^2 + (a-3)x + 3a - 5 = 0$  are equal is  
(a) 1 (b) 2 (c) 0 (d) 4
  - The solution  $5x-1 < 24$  and  $5x+1 > -24$  is  
(a) (4,5) (b) (-5,-4) (c) (-5,5) (d) (-5,4)
  - $\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \dots + \cos 179^\circ =$   
(a) 0 (b) 1 (c) -1 (d) 89

## PART - B

1. Answer any four of the following 2. Each question carry 2 marks. 4x2=8

- Let f and g be the two functions from  $\mathbb{R}$  to  $\mathbb{R}$ -defined by  $f(x) = 3x-4$  and  $g(x) = x^2+3$ . Find  $g \circ f$  and  $f \circ g$ .
- Solve for x:  $|3 - x| < 7$ .
- Construct a quadratic equation with roots 7 and -3.
- Find the value of  $\cos 105^\circ$

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15.  $\log a + \log a^2 + \log a^3 + \dots + \log a^n = \frac{n(n+1)}{2}$
16. Discuss the following relations for reflexivity, symmetry and transitivity: On the set of natural numbers, the relation R is defined by "xRy if  $x + 2y = 1$ ".

## PART - C

1. Answer any four of the following. 2. Each question carry 3 marks.

4 x 3 = 12

17. Find the domain  $f(x) = \frac{1}{1-2\sin x}$
18. Prove that  $\sqrt{3}$  is an irrational number.
19. A model rocket is launched from the ground. The height h reached by the rocket after t seconds from lift off is given by  $h(t) = -5t^2 + 100t$ ,  $0 \leq t \leq 20$ . At what time the rocket is 495 feet above the ground?
20. If  $A+B = 45^\circ$  show that,  $(1+\tan A)(1+\tan B) = 2$ .
21. If  $\alpha$  and  $\beta$  are the roots of the quadratic equation  $x^2 + \sqrt{2}x + 3 = 0$ , form a quadratic polynomial with zeros  $1/\alpha, 1/\beta$ .
22. Resolve the following rational expressions into partial fractions.  $\frac{3x+1}{(x-2)(x+1)}$

## PART - D

1. Answer the following. 2. Each question carry 5 marks.

3x5=15

23. a) If  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by,  $f(x) = 2x - 3$  prove that f is a bijection and find its inverse.

(OR)

- b) Write the values of f at -3,5,2,-1,0 if

$$f(x) = \begin{cases} x^2 + x - 5 & \text{if } x \in (-\infty, 0) \\ x^2 + 3x - 2 & \text{if } x \in (3, \infty) \\ x^2 & \text{if } x \in (0, 2) \\ x^2 - 3 & \text{if otherwise} \end{cases}$$

24. a) Resolve the following rational expressions into partial fractions.  $\frac{x}{(x^2+1)(x-1)(x+2)}$

(OR)

b) Solve:  $\frac{x+1}{x+3} < 3$

25. a) Write and Prove Napier's formula.

(OR)

b) Prove  $\log 2 + 16 \log \frac{16}{15} + 12 \log \frac{25}{24} + 7 \log \frac{81}{80} = 1$

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STD: 11

MATHEMATICS - KEY

PART-A

- 1 b symmetric  
 2 b onto  
 3 b 1130  
 4 b  $[-11, 7]$   
 5 d  $(-\infty, 1]$   
 6 c 3  
 7 b  $3x^2 + 5x - 7 = 0$   
 8 b 2  
 9 c  $(-5, 5)$   
 10 a 0

PART-B

- 11  $(g \circ f)(x) = 9x^2 - 24x + 19$   
 $(f \circ g)(x) = 3x^2 + 5$
12.  $-7 < 3 - x < 7$   
 $\Rightarrow -4 < x < 10$
13.  $x^2 - 4x - 21 = 0$
14.  $\cos(60 + 45) = \frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$   
 $= \frac{1 - \sqrt{3}}{2\sqrt{2}}$
15. LHS  
 $= \log a + 2 \log a + 3 \log a + \dots + n \log a$   
 $= \log a (1 + 2 + 3 + \dots + n)$   
 $= \frac{n(n+1)}{2} \log a$

16. R not reflexive  
 R not symmetric  
 R not transitive

PART-C

17.  $\sin x = \frac{1}{2}$        $\sin x = \sin \pi/6$   
 $x = n\pi + (-1)^n \pi/6, n \in \mathbb{Z}$   
 Domain of  $f(x)$  is  $\mathbb{R} - (n\pi + (-1)^n \pi/6), n \in \mathbb{Z}$

18.  $\sqrt{3} = \frac{m}{n} \Rightarrow 3n^2 = m^2, m = 2k$   
 $3n^2 = 4k^2$  (even)  
 $\sqrt{3}$  is an irrational number

19.  $0 < h(t) < 495$   
 $0 < -5t^2 + 100t < 495$   
 $-5t^2 + 100t - 495 = 0 \Rightarrow t^2 - 20t + 99 = 0$   
 $(t-11)(t-9) = 0$   
 $t = 11 \text{ sec}, t = 9 \text{ sec}$

20.  $\tan(A+B) = \tan 45^\circ$   
 $\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1 \Rightarrow \tan A + \tan B = 1 - \tan A \tan B$   
 $(1 + \tan A)(1 + \tan B) = 2$

21.  $\alpha + \beta = -\sqrt{2}$        $\alpha\beta = 3$   
 Sum =  $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{-\sqrt{2}}{3}$   
 Prod =  $\frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{3}$   
 $x^2 - x\left(\frac{-\sqrt{2}}{3}\right) + \frac{1}{3} = 0 \Rightarrow 3x^2 + \sqrt{2}x + 1 = 0$

22.  $= \frac{A}{x-2} + \frac{B}{x+1}$   
 $3x + 1 = A(x+1) + B(x-2)$   
 $\frac{x-1}{B} = \frac{2}{3}$        $\frac{x=2}{A} = \frac{7}{3}$   
 $\frac{3x+1}{(x-2)(x+1)} = \frac{7}{3(x-2)} + \frac{2}{3(x+1)}$

PART-D

23. a. one to one:  $f(x) = f(y) \Rightarrow 2x - 3 = 2y - 3$   
 $\Rightarrow x = y$ ,  $f$  is one to one  
 onto:  $x = \frac{y+3}{2}$ ,  $f(x) = 2\left(\frac{y+3}{2}\right) - 3 = y$   
 $f$  is onto  
 Inverse:  $x = \frac{y+3}{2}$ ,  $f^{-1}(y) = \frac{y+3}{2}$ ,  $f^{-1}(x) = \frac{x+3}{2}$

b.  $f(-3) = 1$        $f(-1) = -5$   
 $f(1) = 38$        $f(0) = -3$   
 $f(2) = 1$

24. a.  $\frac{x}{(x^2+1)(x-1)(x+2)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1} + \frac{D}{x+2}$

$$x = (Ax+B)(x-1)(x+2) + C(x^2+1)(x-1) + D(x^2+1)(x+2)$$

$$x=1 \Rightarrow C = \frac{1}{6}$$

$$x=-2 \Rightarrow D = \frac{2}{15}$$

$$x=0 \Rightarrow B = \frac{1}{10}$$

$$\text{co-efficient of } x^3 \Rightarrow A = -\frac{3}{10}$$

$$= \frac{1-3x}{10(x^2+1)} + \frac{1}{6(x-1)} + \frac{2}{15(x+2)}$$

b.  $\frac{x+1}{x+3} - 3 < 0 \Rightarrow \frac{x+1-3(x+3)}{x+3} < 0$

$$\frac{x+4}{x+3} > 0$$

x	x+3	x+4	$\frac{x+4}{x+3}$
x < -4	-	-	+
-4 < x < -3	-	+	-
x > -3	+	+	+
x = -4	-	0	0

$$\text{solution: } (-\infty, -4) \cup (-3, \infty)$$

25. a.  $\Delta ABC$ , (i)  $\tan \frac{A+B}{2} = \frac{a-b}{a+b} \cot \frac{C}{2}$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

$$\frac{a-b}{a+b} \cot \frac{C}{2} = \frac{2R \sin A - 2R \sin B}{2R \sin A + 2R \sin B} \cot \frac{C}{2}$$

$$= \frac{2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}}{2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}} \cot \frac{C}{2}$$

$$= \cot \left( \frac{A+B}{2} \right) \tan \frac{A-B}{2} \cot \frac{C}{2}$$

$$= \tan \frac{C}{2} \tan \frac{A+B}{2} \cot \frac{C}{2}$$

$$= \tan \frac{A-B}{2}$$

ii)

$$(ii). \tan \frac{B-C}{2} = \frac{b-c}{b+c} \cot \frac{A}{2}$$

$$(iii) \tan \frac{C-A}{2} = \frac{c-a}{c+a} \cot \frac{B}{2}$$

b.  $= \log 2 + \log \left( \frac{16}{15} \right)^{16} + \log \left( \frac{25}{24} \right)^{12} + \log \left( \frac{81}{80} \right)^7$

$$= \log 2 + \log \frac{(2^4)^{16}}{(3 \times 5)^{16}} + \log \frac{(5^2)^{12}}{(2^3 \times 3)^{12}} + \log \frac{(3^4)^7}{(2^4 \times 5)^7}$$

$$= \log \frac{2^{65} \cdot 5^{24} \cdot 3^{28}}{3^{28} \cdot 5^{23} \cdot 2^{64}}$$

$$= \log 2^1 \times 5^1 = \log_{10} 10 = 1 \text{ RHS}$$

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