H

COMMON FIRST MID-TERM TEST – 2023

Standard XII

Reg.No. : | | | | | | |

MATHEMATICS

Time: 1.30 hrs. Part - I Marks: 50

I. Choose the correct answer: 10 x 1 = 10

1. If a is a 3 x 3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then BB^T =
   a) I_3 b) B^T c) B d) A

2. If $A = \begin{bmatrix} 3/5 & 4/5 \\ x & 3/5 \end{bmatrix}$ and $A^T = A^{-1}$ then the value of x is
   a) $-3/5$ b) $4/5$ c) $-4/5$ d) $3/5$

3. If $A = \begin{bmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{bmatrix}$ and $AB = I_2$ then B =
   a) $(\sin^2\theta/2)A$ b) $(\cos^2\theta)\ I$ c) $(\cos^2\theta/2)A$ d) $(\cos^2\theta/2)A^T$

4. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A|^3 = 125$ then the value of α is
   a) ±5 b) ±3 c) ±2 d) ±1

5. The area of the triangle formed by the complex numbers z, iz and z + iz in the Argand's diagram is a) $|z|^2$ b) $\frac{3}{2}|z|^2$ c) $2|z|^2$ d) $\frac{1}{2}|z|^2$

6. The solution of the equation $|z| - z = 1 + 2i$ is
   a) $\frac{-3}{2} + 2i$ b) $\frac{3}{2} - 2i$ c) $2 + \frac{3}{2}i$ d) $2 - \frac{3}{2}i$

7. If $\omega \neq 1$ is a cube root of unity and $(1 + \omega)^7 = A + B\omega$ then (A,B) equals
   a) (–1,1) b) (1,1) c) (1,0) d) (0,1)

8. The roots of the equation $z^4 + 1 = 0$ are
   a) $(\pm 2 \pm 2i)$ b) $\frac{1}{\sqrt{2}}(\pm 1 \pm i)$ c) $(\pm 1 \pm i)$ d) none of these

9. According to the rational root theorem which number is not possible rational zero of $4x^7 + 2x^4 - 10x^3 - 5$? a) $4/5$ b) –1 c) $5/4$ d) 5

10. The number of positive zeros of the polynomial $\sum_{r=0}^{n} nCr(-1)^r x^r$ is
    a) 0 b) n c) r d) <n

Part - II

II. Answer any 4 questions. (Q.No.16 is compulsory) 4 x 2 = 8

11. Find the rank of the following matrix by minor method $\begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$

12. Show that $|z + 2 - i| < 2$ represents interior points of a circle. Find its centre and radius.

13. Find the following $\left|(1+i)(2+3i)(4i-3)\right|$

14. If α and β are the roots of the equation $2x^2 - 7x + 13 = 0$ then find $\alpha^2 + \beta^2$.

15. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

16. If A is a non-singular matrix of odd order, prove that |adj A| is positive.

Part - III

III. Answer any 4 questions. (Q.No.22 is compulsory) 4 x 3 = 12

17. If $A = \begin{bmatrix} 1 & 3 \\ 2 & -5 \end{bmatrix}$, verify that A(adj A) = (adj A) A = $|A|I_2$.

18. Solve the following systems of linear equations by Cramer's rule :

$\frac{3}{x} + 2y = 12$; $\frac{2}{x} + 3y = 13$

19. Find the fourth roots of Unity.

20. Solve the equation : $x^4 - 14x^2 + 45 = 0$

21. Show that $(2 + i\sqrt{3})^{10} + (2 - i\sqrt{3})^{10}$ is real.

22. if k is a real, discuss the nature of the roots of the polynomial $2x^2 + kx + k = 0$, in terms of k.

Part - IV

IV. Answer all the questions. 4 x 5 = 20

23. a) If $A = \frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ 6 & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a, b and c and hence find A^{-1}. *(OR)*

b) Solve the following system of homogeneous equations.
$2x + 3y - z = 0$; $x - y - 2z = 0$; $3x + y + 3z = 0$

24. a) Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has (i) no solution (ii) a unique solution (iii) an infinite number of solutions. *(OR)*

b) Show that the cube root of unity 1, ω and ω^2 points are the vertices of an equilateral triangle.

25. a) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$ *(OR)*

b) Find all the cube roots of $\sqrt{3} + i$

26. a) If $2 + i$ and $3 - \sqrt{2}$ are roots of the equation
$x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$ *(OR)*

b) Solve the equation : $3x^4 - 16x^3 + 26x^2 - 16x + 3 = 0$

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