ALPHA MATHS ACADAMY

JEE, CBSE AND BOARD EXAMINATION COACHING CENTER
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CHAPTER 1 TO 4

STANDARD 12

TIME: 3.00 HOURS **MATHEMATICS** **MARKS: 90**

PART-A $\mathbf{20 \times 1 = 20}$

1. If $sin^{-1} x + sin^{-1} y = \frac{2\pi}{3}$; then $cos^{-1} x + cos^{-1} y$ is equal to

   $(a)\ \frac{2\pi}{3}$ $(b)\ \frac{\pi}{3}$ $(c)\ \frac{\pi}{6}$ $(d)\ \pi$

2. If $z = \frac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$, then $|z|$ is equal to

   $(a)\ 0$ $(b)\ 1$ $(c)\ 2$ $(d)\ 3$

3. If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$ then $B^{-1} =$

   $(a)\ \begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ $(b)\ \begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ $(c)\ \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ $(d)\ \begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

4. The solution of the equation $|z| - z = 1 + 2i$ is

   $(a)\ \frac{3}{2} - 2i$ $(b)\ -\frac{3}{2} + 2i$ $(c)\ 2 - \frac{3}{2}i$ $(d)\ 2 + \frac{3}{2}i$

5. If $cot^{-1} x = \frac{2\pi}{5}$ for some $x \in R$, the value of $tan^{-1} x$ is

   $(a)\ -\frac{\pi}{10}$ $(b)\ \frac{\pi}{5}$ $(c)\ \frac{\pi}{10}$ $(d)\ -\frac{\pi}{5}$

6. If $A = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$ and $A(adjA) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then $k =$

   $(a)\ 0$ $(b)\ \sin\theta$ $(c)\ \cos\theta$ $(d)\ 1$

7. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is

   $(a)\ 0$ $(b)\ 1$ $(c)\ -1$ $(d)\ i$

8. If $(1+i)(1+2i)(1+3i)\cdots(1+ni) = x + iy$, then $2.5.10 \ldots (1+n^2)$ is

   $(a)\ 1$ $(b)\ i$ $(c)\ x^2 + y^2$ $(d)\ 1 + n^2$

9. $sin^{-1}(2cos^2 x - 1) + cos^{-1}(1 - 2sin^2 x) =$

   $(a)\ \frac{\pi}{2}$ $(b)\ \frac{\pi}{3}$ $(c)\ \frac{\pi}{4}$ $(d)\ \frac{\pi}{6}$

10. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satisfies

    $(a)\ |k| \le 6$ $(b)\ k = 0$ $(c)\ |k| > 6$ $(d)\ |k| \ge 6$

11. If the order of a square matrix A is 4 and $|A| = 5$,then $|adj(adjA)|$ is

    $(a) 25$ $(b)\ 5^4$ $(c) 125$ $(d) 5^9$

12. If A, B and C are invertible matrices of some order, then which one of the following is not true?

    $(a)\ adj\ A = |A|\ A^{-1}$ $(b)\ adj(AB) = (adj\ A)(adj\ B)$

$(c)\ detA^{-1} = (detA)^{-1}$ $(d)\ (ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

13. If α, β and γ are the zeros of $x^3 + px^2 + qx + r$, then $\sum \frac{1}{\alpha}$ is

$(a)\ -\frac{q}{r}$ $(b)\ -\frac{p}{r}$ $(c)\ \frac{q}{r}$ $(d)\ -\frac{q}{p}$

14. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ and $A^2 + xI = yA$, then the values of x and y are respectively

(a) 6 , 4 (b) 8 , 6 (c) 8 , 8 (d) 5 , 8

15. The conjugate of a complex number is $\frac{1}{i-2}$,Then the complex number is

$(a)\ \frac{1}{i+2}$ $(b)\ \frac{-1}{i+2}$ $(c)\ \frac{-1}{i-2}$ $(d)\ \frac{1}{i-2}$

16. If $\left|z - \frac{3}{z}\right| = 2$, then the least value of $|z|$ is

(a) 1 (b) 2 (c) 3 (d) 5

17. Multiplication of a complex number Z by $(-i)$ is the rotation about the origin by

(a) 90° counter clockwise direction (b) 90° clockwise direction

(c) 180° counter clockwise direction (d) 180° clockwise direction

18. If $sin^{-1} x + cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is equal to

$(a)\ \frac{1}{2}$ $(b)\ \frac{1}{\sqrt{5}}$ $(c)\ \frac{2}{\sqrt{5}}$ $(d)\ \frac{\sqrt{3}}{2}$

19. If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive zero, if and only if

$(a)\ a \geq 0$ $(b)\ a > 0$ $(c)\ a < 0$ $(d)\ a \leq 0$

20. A zero of $x^3 + 64$ is

(a) 0 (b) 4 (c) $4i$ (d) – 4

PART-B **$7 \times 2 = 14$**

Note: i) Answer any seven questions.

ii) Question No.30 is compulsory.

21. If A is symmetric, prove that then $Adj\ A$ is also symmetric

22. Show that $\left(2 + i\sqrt{3}\right)^{10} + \left(2 - i\sqrt{3}\right)^{10}$ is real

23. Show that, if p, q, r are rational, the roots of the equation $x^2 - 2px + p^2 - q^2 + 2qr - r^2 = 0$ are rational.

24. Find the period and amplitude of $y = \sin 7x$

25. Find non –zero integral solution of $|1 - i|^x = 2^x$

26. If $Adj\ A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$. Find A^{-1}

27. Write in polar form of the complex numbers $3 - i\sqrt{3}$

28. Prove that a line cannot intersect a circle at more than two points.

29. For the value of x, the inequality $\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$ holds?

30. Solve the equation $\cos^2 x - 9\cos x + 20 = 0$.

PARTC-C $\mathbf{7 \times 3 = 21}$

Note: ***i***) ***Answer any seven questions.***

ii) ***Question No.40 is compulsory.***

31. Given $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$; $B = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, Find a matrix X such that $AXB = C$

32. If z_1 , z_2 and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = |z_1 + z_2 + z_3| = 1$,

Find the value of $\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$

33. Find the condition that the roots of cubic equation $x^3 + ax^2 + bx + c = 0$ are in the ratio $p:q:r$.

34. Find the value of $tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \frac{1}{2}cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$

35. Find the rank of the matrices $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it to a row-echelon form.

36. If $\tan^{-1} x \tan^{-1} y \tan^{-1} z = \pi$, show that $x + y + z = xyz$.

37. If the system of equations $px + by + cz = 0$, $ax + qy + cz = 0$, $ax + by + rz = 0$ has a non-trivial solution and $p \neq a$, $q \neq b$, $r \neq c$ prove that $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$

38. Find the argument of $\frac{1-i}{1+i}$

39. Determine the possible number of positive real zeros and negative real zeros of

$x^4 - 6x^3 + 8x^2 + 2x - 1$.

40. If $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\frac{1}{\sqrt{3}}$, find the value of x.

PART-D $7 \times 5 = 35$

Note: Answer all the questions.

41. (a) The prices of three commodities A, B and C are rupees x, y and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q purchase 2 unit of C and sells 3 units of A and one unit of B. Person R purchases one unit of A and sells 3 units of B and 1 unit of C. In the process P, Q and R earn ₹ 15,000, ₹ 1,000 and ₹ 4,000 respectively. Find the prices per unit of A, B and C. (Use matrix inversion method to solve the problem) (***or***)

    (b) Find the number of solution of the equation $\tan^{-1}(x-1) + \tan^{-1} x + \tan^{-1}(x+1) = \tan^{-1}(3x)$

42. (a) $cos\ \alpha + cos\ \beta + cos\ \gamma = sin\ \alpha + sin\ \beta + sin\ \gamma = 0,$

    Show that (i) $cos\ 3\alpha + cos\ 3\beta + cos\ 3\gamma = 3cos(\alpha+\beta+\gamma)$

    (ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\ sin\ (\alpha+\beta+\gamma)$ (***or***)

    (b) Solve the system of linear equations by matrix inversion method

    $2x + 3y - z = 9, x + y + z = 9, 3x - y - z = -1$

43. (a) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution. (***or***)

    (b) If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ and $0 < x, y, z, < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$

44. (a) Determine the values of λ for which the following system of equations $x + y + 3z = 0$, $4x + 3y + \lambda z = 0, 2x + y + 2z = 0$ has (i) a unique solution (ii) a non- trivial solution

    (***or***)

    (b) If the system of equations $ax + y + z = 0,\ x + by + z = 0,\ x + y + cz = 0,$

    (where $a \neq 1,\ b \neq 1,\ c \neq 1$) has a non-trivial solution, then show that $\frac{a}{1-a} + \frac{b}{1-c} + \frac{c}{1-c} = 1.$

45. (a) If $z = x + iy$ is a complex number such that $Im\left(\frac{2z+1}{iz+1}\right) = 0$. Show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0.$ (***or***)

    (b) Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$

46. (a) Suppose z_1, z_2, and z_3, are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$.

If $z_1 = 1 + i\sqrt{3}$, then find z_2 *and* z_3. **(or)**

(b) Let P and Q be rational numbers such that $\sqrt{q}$ is irrational. If $p + \sqrt{q}$ is a root of a quadratic equation with rational coefficients, then $p - \sqrt{q}$ is also a root of the same equation.

47. (a) If $a = \cos 2\alpha + i \sin 2\alpha, b = \cos 2\beta + i \sin 2\beta$ *and* $c = \cos 2\gamma + i \sin 2\gamma$

Prove that $i)\ \sqrt{abc} + \frac{1}{\sqrt{abc}} = 2\cos(\alpha + \beta + \gamma)$ $\quad ii)\ \frac{a^2b^2+c^2}{abc} = 2\cos 2(\alpha + \beta - \gamma)$

(or)

(b) Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$, if $6x^2 < 1$

****************** ALL THE BEST *****************

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