1. If $A=\begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{bmatrix}$, verify that $A(adj\ A)=(adj\ A)A=|A|I_3$. (Example 1.1)

2. If $A=\begin{bmatrix}4 & 3\\ 2 & 5\end{bmatrix}$, find x and y such that $A^2+xA+yI_2=0_2$. Hence, find A^{-1}.

(Example 1.10)

3. If $A=\frac{1}{7}\begin{bmatrix}6 & -3 & a\\ b & -2 & 6\\ 2 & c & 3\end{bmatrix}$ is orthogonal, find a, b and c, and hence A^{-1}.

(Example 1.12)

4. If $F(\alpha)=\begin{bmatrix}\cos\alpha & 0 & \sin\alpha\\ 0 & 1 & 0\\ -\sin\alpha & 0 & \cos\alpha\end{bmatrix}$, show that $[F(\alpha)]^{-1}=F(-\alpha)$.

(Exercise 1.1 Q.No. 3)

5. If $A=\frac{1}{9}\begin{bmatrix}-8 & 1 & 4\\ 4 & 4 & 7\\ 1 & -8 & 4\end{bmatrix}$, Prove that $A^{-1}=A^T$. (Exercise 1.1 Q.No. 5)

6. If $A=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$, show that $A^{-1}=\frac{1}{2}(A^2-3I)$. (Exercise 1.1 Q.No. 14)

7. Find the inverse of $A=\begin{bmatrix}2 & 1 & 1\\ 3 & 2 & 1\\ 2 & 1 & 2\end{bmatrix}$ by Gauss-Jordan method.

(Example 1.21)

8. Find the inverse of each of the following by Gauss-Jordan method

(ii) $\begin{bmatrix}1 & -1 & 0\\ 1 & 0 & -1\\ 6 & -2 & -3\end{bmatrix}$ (iii) $\begin{bmatrix}1 & 2 & 3\\ 2 & 5 & 3\\ 1 & 0 & 8\end{bmatrix}$ (Exercise 1.2 Q.No. 3)

9. Solve the following system of equations, using matrix inversion method.
$2x_1+3x_2+3x_3=5, x_1-2x_2+x_3=-4, 3x_1-x_2-2x_3=3$ (Example 1.23)

10. If $A=\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$, Find the products AB and BA and hence solve the system of equations
$x-y+z=4, x-2y-2z=9, 2x+y+3z=1$. (Example. 1.24)

11. Solve the following system of linear equations by matrix inversion method.

(iii) $2x+3y-z=9, x+y+z=9, 3x-y-z=-1$

(iv) $x+y+z-2=0, 6x-4y+5z-31=0, 5x+2y+2z=13$

(Exercise 1.3 Q.No. 1)

12. If $A=\begin{bmatrix}-5 & 1 & 3\\ 7 & 1 & -5\\ 1 & -1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 1 & 2\\ 3 & 2 & 1\\ 2 & 1 & 3\end{bmatrix}$, find the products AB and BA and hence solve the system of equations

$x+y+2z=1, 3x+2y+z=7, 2x+y+3z=2$. (Exercise 1.3 Q.No. 2)

13. A man is appointed in a job with a monthly salary of certain amount and a fixed amount of annual increment. If his salary was ₹19,800 per month at the end of the first month after 3 years of service and ₹23,400 per month at the end of the first month after 9 years of service, find his starting salary and his annual increment. (Use matrix inversion method to solve the problem). (Exercise 1.3 Q.No. 3)

14. Four men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method. (Exercise 1.3 Q.No. 4)

15.The prices of three commodities A, B and C are ₹$x, y$and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q purchases 2 units of iC and sells 3 units of A and one unit of B. Person R purchases one unit of A and sells 3 unit of B and one unit of C. In the process, P, Q and R earn ₹15,000, ₹1,000 and₹4000 respectively. Find the prices per unit of A, B and C. (Use matrix inversion method to solve the problem) . (Exercise 1.3 Q.No. 4)

16. Solve, by Cramer's rule, the system of equations.

$x_1 - x_2 = 3, 2x_1 + 3x_2 + 4x_3 = 17, x_2 + 2x_3 = 7$(Example 1.25)

17. In a T20 Match, Chennai Super Kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. the ball traversed along a path in a vertical plane and the equation of the path is $y = ax^2 + bx + c$ with respect to a xy-cordinate system in the vertical plane and the ball traversed through the points $(10,8), (20,16), (40,22)$ can you conclude that Chennai Super Kings won the match?
Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is $(70,0)$.)
(Example 1.26)

18. Solve the following systems of linear equations by Cramer's rule.

(iii)$3x + 3y - z = 11, 2x - y + 2z = 9, 4x + 3y + 2z = 25$

(iv) $\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0, \frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0, \frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$ (Exercise 1.4 Q.No. 1)

19. In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly? (Use Cramer's rule to solve the problem). . (Exercise 1.4 Q.No. 2)

20. A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution? (Use Cramer's rule to solve the problem). . (Exercise 1.4 Q.No. 3)

21. A fish tank can be filled in 10 minutes using both pumps A and B simultaneously. However, pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes. How long would it take each pump to fill the tank by itself? (Use Cramer's rule to solve the problem). (Exercise 1.4 Q.No. 4)

22. A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is `150. The cost of the two dosai, two idlies and four vadais is `200. The cost of five dosai, four idelies and two vadais is `250. The family has `350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had? (Exercise 1.4 Q.No. 5)

23. Solve the following system of linear equations, by Gaussian elimination method

$4x + 3y + 6z = 25, x + 5y + 7z = 13, 2x + 9y + z = 1$.(Example.1.27)

24. The upward speed $v(t)$ of a rocket at time t is approximated by $v(t) = at^2 + bt + c$, $0 \le t \le 100$ where a, b and c are constant. It has been found that the speed at times $t = 3, t = 6$ and $t = 9$ seconds are respectively, 64, 133 and 208 miles per second respectively. Find the speed at time $t = 15$ seconds.
(Use Gaussian elimination method) (Example. 1.28)

25. Solve the following systems of linear equations by Gaussian elimination method:

(i)$2x - 2y + 3z = 2,\ x + 2y - z = 3,\ 3x - y + 2z = 1$

(ii)$2x + 4y + 6z = 22,\ 3x + 8y + 5z, -x + y + 2z = 2$ (Exercise 1.5 Q.No. 1)

26. If $ax^2 + bx + c$ is divided by $x + 3, x - 5$ and $x - 1$, the remainders are 21,61 and 9 respectively. Find a, b and c. (use Gaussian elimination method.)

(Exercise 1.5 Q.No. 2)

27. An amount of `65,000 is invested in three bonds at the rates of 6%, 8% and 10% per annum respectively. The total annual income is `5,000. The income from the third bond is `800 more than that from the second bond. Determine the price of each bond. (Use Gaussian elimination method.) (Exercise 1.5 Q.No. 3)

28. A body is walking along the path $y = ax^2 + bx + c$ through the points $(-6,8), (-2,-12)$ and $(3,8)$. He wants to meet his friend at $P(7,60)$. will he meet his friend? (Use Gaussian elimination method) (Exercise 1.5 Q.No. 4)

29. Test for consistency of the following system of linear equations and if Possible solve.

x + 2y – z =3, 3x – y + 2z = 1 , x – 2y + 3z = 3 , x – y + z + 1 = 0 (Example 1.29)

30. Test for consistency of the following system of linear equations and if possible solve. $4x-2y+6z=8, x+y-3z=-1, 15x-3y+9z=21$ (Example. 1.30)

31. Test for consistency of the following system of linear equations and if possible solve $x-y+z=-9, 2x-2y+2z=-18, 3x-3y+3z+27=0$ (Example. 1.31)

32. Test the consistency of the following system of linear equations. $x-y+z=-9, 2x-y+z=4, 3x-y+z=6, 4x-y+2z=7$ (Example. 1.32)

33. Find the condition on a, b and c so that the following system of linear equations has one parameter family of solutions: $x+y+z=a, x+2y+3z=b, 3x+5y+7z=c$ (Example. 1.33)

34. Investigate for what values of λ and μ the system of linear equations. $x+2y+z=7, x+y+\lambda z=\mu, x+3y-5z=5$ has
(i) no solution (ii) a unique solution
(iii)an infinite number of solutions. (Example. 1.34)

35.Test for consistency and if possible, solve the following systems of equations by rank method).

(i)$x-y+2z=2,\ 2x+y+4z=7,\ 4x-y+z=4$

(ii)$3x+y+z=2,\ x-3y+2z=1,\ 7x-y+4z=5$

(iv)$2x-y+z=2,\ 6x-3y+3z=6,\ 4x-2y+2z=4$ (Exercise 1.6 Q.No. 1)

36. Find the value of k for which the equations

$kx-2y+z=1,\ x-2ky+z=-2, x-2y+kz=1$ have

(i)no solution (ii)unique solution (iii)infinitely many solution

(Exercise 1.6 Q.No. 2)

37. Investigate the values of λ and μ the system of linear equations:

$2x+3y+5z=9,\ 7x+3y-5z=8,\ 2x+3y+\lambda z=\mu$, have (i)no solution

(ii)a unique solution (iii)an infinite number of solutions) . (Exercise 1.6 Q.No. 3)

38. Solve the following system. $x+2y+3z=0, 3x+4y+4z=0, 7x+10y+12z=0$. (Example. 1.35)

39. Solve the system. $x+3y-2z=0, 2x-y+4z=0, x-11y+14z=0$. (Example. 1.36)

40. Solve the system. $x+y-2z=0, 2x-3y+z=0, 3x-7y+10z=0, 6x-9y+10z=0$. (Example. 1.37)

41. Determine the values of λ for which the following system of equations $(3\lambda-8)x+3y+3z=0, 3x+(3\lambda-8)y+3z=0, 3x+3y+(3\lambda-8)z=0$ has a non-trivial solution. . (Example. 1.38)

42. By using Gaussian elimination method, balance the chemical reaction equation.
$C_5H_8+O_2 \rightarrow CO_2+H_2O$.
(The above is the reaction that is taking place in the burning of organic compound called isoprene.) . (Example. 1.39)

43. If the system of equations $px+by+cz=0, ax+qy+cz=0, ax+by+z=0$ has a non-trivial solution and $p\neq a, q\neq b, r\neq c$, prove that $\frac{p}{p-a}+\frac{q}{q-b}+\frac{q}{q-c}=2$. (Example. 1.40)

44. Solve the following system of homogenous equations.) (Exercise 1.7 Q.No. 1)

(i) $3x+2y+7z=0, 4x-3y-2z=0, 5x+9y+23z=0$

(ii) $2x+3y-z=0, x-y-2z=0,\ 3x+y+3z=0$

45. Determine the values of λ for which the following system of equations

$x+y+3z=0, 4x+3y+\lambda z=0, 2x+y+2z=0$ has

(i)a unique solution (ii)a non –trivial solution) (Exercise 1.7 Q.No. 2)

46. By using Gaussian elimination method, balance the chemical reaction equation:

$C_2H_6+O_2 \rightarrow H_2O+CO_2$) (Exercise 1.7 Q.No. 3)

1. Show that (i) $\left(2+i\sqrt{3}\right)^{10}+\left(2-i\sqrt{3}\right)^{10}$ is real and

(ii) $\left(\frac{19+9i}{5-3i}\right)^{15}-\left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary. (Example. 2.8 Pg. No.64)

2. The complex numbers u, v and w are related by $\frac{1}{u}=\frac{1}{v}+\frac{1}{w}$

If $v=3-4i$ and $w=4+3i$ find u in rectangular form.

(Exercise 2.4 Q.No.4)

3. Prove the following properties: (i) z is real if and only if $z=\bar{z}$

(ii) $Re(z)=\frac{z+\bar{z}}{2}$ and $Im(z)=\frac{z-\bar{z}}{2i}$ (Exercise 2.4 Q.No.5)

4. Find the least value of the positive integer n for which $\left(\sqrt{3}+i\right)^n$

(i)real (ii)Purely imaginary . (Exercise 2.4 Q.No.6)

5. Show that (i) $\left(2+i\sqrt{3}\right)^{10}-\left(2-i\sqrt{3}\right)^{10}$ is purely imaginary

(ii) $\left(\frac{19-7i}{9+i}\right)^{12}+\left(\frac{20-5i}{7-6i}\right)^{12}$ is real . (Exercise 2.4 Q.No.7)

6. Show that the points $1, \frac{-1}{2}+i\frac{\sqrt{3}}{2}$ and $\frac{-1}{2}-i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle. (Example 2.14 Pg.No.70)

7. Let z_1, z_2 and z_3 be complex numbers such that $|z_1|=|z_2|=|z_3|=r>0$ and $z_1+z_2+z_3\neq 0$. Prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right|=r$. (Example 2.15 Pg. No.70)

8. For any two complex numbers z_1 and z_2, such that $|z_1|=|z_2|=1$ and $z_1z_2\neq -1$, then show that $\frac{z_1+z_2}{1+z_1z_2}$ is a real number. (Exercise 2.5 Q.No.2)

9. If z_1, z_2 and z_3 are three complex numbers such that $|z_1|=1, |z_2|=2, |z_3|=3$ and $|z_1+z_2+z_3|=1$, show that $|9z_1z_2+4z_1z_3+z_2z_3|=6$. (Exercise 2.5 Q.No.7)

10. Show that the equation $z^3+2\bar{z}=0$ has five solutions. (Exercise 2.5 Q.No.9)

11. Given the complex number $z=3+2i$, represent the complex numbers z, iz and $z+iz$ in one Argand diagram. show that these complex numbers form the vertices of an isosceles right triangle. (Example 2.18 Pg. No.73)

12. Obtain the Cartesian form of the locus of z in each of the following cases.

(i) $|z|=|z-i|$ (ii) $|2z-3-i|=3$ (Example. 2.21 Pg.No.74)

13. If $z=x+iy$ is a complex number such that $\left|\frac{z-4i}{z+4i}\right|=1$. Show that the locus of z is real axis. (Exercise 2.6 Q.No.1)

14. If $z=x+iy$ is a complex number such that $Im\left(\frac{2z+1}{iz+1}\right)=0$, show that the locus of z is $2x^2+2y^2+x-2y=0$. (Exercise 2.6 Q.No.2)

15. Obtain the Cartesian form of the locus of $z=x+iy$ in each of the following Case (i) $[Re(iz)]^2=3$ (ii) $Im[(1-i)z+1]=0$ (iii) $|z+i|=|z-1|$. (Exercise 2.6 Q.No.3)

16. Obtain the Cartesian equation for the locus of $z=x+iy$ in each of the following cases.

(i) $|z-4|=16$ (ii) $|z-4|^2-|z-1|^2=16$. (Exercise 2.6 Q.No.5)

17. Find the principal argument Arg z, when $z=\frac{-2}{1+i\sqrt{3}}$.

(Example. 2.24 Pg.No. 80)

18. Find the product $\frac{3}{2}\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{3}\right).6\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$ in rectangular form. (Example. 2.24 Pg.No. 81)

19. Find the quotient $\frac{2\left(\cos\frac{9\pi}{4}+i\sin\frac{9\pi}{4}\right)}{4\left(\cos\left(\frac{-3\pi}{2}\right)+i\sin\left(\frac{-3\pi}{2}\right)\right)}$ in rectangular form.

(Example. 2.24 Pg.No. 82)

20. If $z = x + iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, show that $x^2 + y^2 = 1$.

(Example. 2.24 Pg.No. 82)

21. Write in polar form of the complex number.

(iv) $\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$. (Exercise 2.7 Q.No.1)

22. Find the rectangular form of the complex numbers

(i) $\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$ (ii) $\frac{\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}}{2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)}$.

(Exercise 2.7 Q.No.2)

23. If $(x_1 + iy_1)(x_2 + iy_2)(x_3 + iy_3) \ldots (x_n + iy_n) = a + ib$, show that

(i) $(x_1^2 + y_1^2)(x_2^2 + y_2^2)(x_3^2 + y_3^2) \ldots (x_n^2 + y_n^2) = a^2 + b^2$

(ii) $\sum_{r=1}^{n} \tan^{-1}\left(\frac{y_r}{x_r}\right) = \tan^{-1}\left(\frac{b}{a}\right) + 2k\pi, k \in \mathbb{Z}$.. (Exercise 2.7 Q.No.3)

24. If $\frac{1+z}{1-z} = \cos 2\theta + i\sin 2\theta$, show that $z = i\tan\theta$.. (Exercise 2.7 Q.No.4)

25. If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$, show that

(i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$ and

(ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$. (Exercise 2.7 Q.No.5)

26. If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$.

(Exercise 2.7 Q.No.6)

27. If $z = (\cos\theta + i\sin\theta)$, show that $z^n + \frac{1}{z^n} = 2\cos n\theta$

and $z^n - \frac{1}{z^n} = 2i\sin n\theta$. (Example. 2.28 Pg.No. 84)

28. Simplify (i) $(1 + i)^{18}$ (ii) $\left(-\sqrt{3} + 3i\right)^{31}$ (Example 2.31 Pg. No. 85)

29. Find the cube roots of unity. (Example 2.32 Pg. No. 89)

30. Find the fourth roots of unity. (Example 2.33 Pg. No. 89)

31. solve the equation $z^3 + 8i = 0$, where $z \in \mathbb{C}$. (Example 2.34 Pg. No. 90)

32. Find all cube roots of $\sqrt{3} + i$. (Example 2.35 Pg. No. 91)

33. Suppose z_1, z_2 and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$, then find z_2 and z_3.

(Example 2.36 Pg. No. 91)

34. If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$, show that

(i) $\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha - \beta)$ (ii) $xy - \frac{1}{xy} = 2i\sin(\alpha + \beta)$

(iii) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$ (iv) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$

(Exercise 2.8 Q.No.4)

34. Solve the equation $z^3 + 27 = 0$ (Exercise 2.8 Q.No.5)

35. If $\omega \neq 1$ is a cube root of unity, show that the roots of the equation $(z - 1)^3 + 8 = 0$ are $-1, 1 - 2\omega, 1 - 2\omega^2$ (Exercise 2.8 Q.No.6)

36. Find the value of $\sum_{k=1}^{8}\left(\cos\frac{2k\pi}{9} + i\sin\frac{2k\pi}{9}\right)$ (Exercise 2.8 Q.No.7)

37. If $\omega \neq 1$ is a cube root of unity, show that

(i) $(1 - \omega + \omega^2)^6 + (1 + \omega - \omega^2)^6 = 128$

(ii) $(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8) \ldots \left(1 + \omega^{2^{11}}\right) = 1$

(Exercise 2.8 Q.No.8)

38. If $z = 2 - 2i$, find the rotation of z by θ radians in the counter clockwise direction about the origin when

(i) $\theta = \frac{\pi}{3}$ (ii) $\theta = \frac{2\pi}{3}$ (iii) $\theta = \frac{3\pi}{2}$ (Exercise 2.8 Q.No.9)