K.V.E MATHS TUITION, THANJAVUR

5 MARKS

CHAPTER 1

APPLICATIONS OF MATRICES AND DETERMINANTS

1. If $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, verify that $A(adj\ A) = (adj\ A)A = |A|I_3$. **(Eg. 1.1)**
2. If $= \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2 + xA + yI_2 = 0_2$. Hence, find A^{-1}. **(Eg. 1.10)**
3. If $A = \frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a, b and c, and hence A^{-1}. **(Eg. 1.12)**
4. If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$. **(EX 1.1 - 3)**
5. If $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2 - 3A - 7I_2 = 0_2$. Hence find A^{-1}. **(EX 1.1 - 4)**
6. Show that the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{bmatrix}$ is non-singular and reduce it to the identity matrix by elementary row transformations. **(Eg. 1.19)**
7. Find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method. **(Eg. 1.21)**
8. Find the inverse of $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$ by Gauss-Jordan method: . **(EX 1.2 – 3(ii))**
9. Find the inverse of $A \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$ by Gauss-Jordan method. **(EX 1.2 – 3(iii))**
10. Solve the following system of equations, using matrix inversion method:
    $2x_1 + 3x_2 + 3x_3 = 5, x_1 - 2x_2 + x_3 = -4, 3x_1 - x_2 - 2x_3 = 3.$ **(Eg. 1.23)**

11. If $A=\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & 2\\ 2 & 1 & 3\end{bmatrix}$ find the products AB and BA and hence solve the system of equations $x-y+z=4, x-2y-2z=9, 2x+y+3z=1$. **(Eg. 1.24)**

12. Solve the following system of linear equations by matrix inversion method:
$2x+3y-z=9,\ x+y+z=9,\ 3x-y-z=-1$. **(EX 1.3 –(iii))**

13. Solve the following system of linear equations by matrix inversion method:
$x+y+z-2=0, 6x-4y+5z-31=0, 5x+2y+2z=13$ **(EX 1.3 –(iv))**

14. If $A=\begin{bmatrix}-5 & 1 & 3\\ 7 & 1 & -5\\ 1 & -1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 1 & 2\\ 3 & 2 & 1\\ 2 & 1 & 3\end{bmatrix}$ find the products AB and BA and hence solve the system of equations $x+y+2z=1, 3x+2y+z=7, 2x+y+3z=2$. **(EX 1.3 - 2)**

15. The prices of three commodities A, B and C are ₹xy, and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C . Person Q purchases 2 units of C and sells 3 units of A and one unit of B. Person Rpurchases one unit of $A$and sells 3 unit of $B$and one unit of C . In the process, P, Q and R earn ₹15,000, ₹1,000 and ₹4,000 respectively. Find the prices per unit of A, B and C. (Use matrix inversion method to solve the problem.) **(EX 1.3 - 5)**

16. Solve, by Cramer's rule, the system of equations $x_1-x_2=3, 2x_1+3x_2+4x_3=17, x_2+2x_3=7$. **(Eg. 1.25)**

17. In a $T20$ match, Chennai Super Kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y=ax^2+bx+c$ with respect to a xy –coordinate system in the vertical plane and the ball traversed through the points (10,8), (20,16), (30,18) can you conclude that Chennai Super Kings won the match? Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is (70,0)). **(Eg. 1.26)**

18. Solve the following systems of linear equations by Cramer's rule:
$3x+3y-z=11, 2x-y+2z=9, 4x+3y+2z=25$. **(EX 1.4 – 1(iii))**

19. Solve the following systems of linear equations by Cramer's rule:
$\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0, \frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0, \frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$. **(EX 1.4 - 1(iv))**

20. The upward speed $v(t)$ of a rocket at time t is approximated by $v(t)=at^2+bt+c, 0\le t\le 100$,where a, b and c are constants. It has been found that the speed at times $t=3, t=6$ and $t=9$ seconds are respectively 64, 133, and 208 miles per second respectively. Find the speed at time $t = 15$ seconds. (Use Gaussian elimination method.) **(Eg. 1.28)**

21. Solve the following systems of linear equations by Gaussian elimination method:
$2x - 2y + 3z = 2, x + 2y - z = 3, 3x - y + 2z = 1$. **(EX 1.5 – 1(i))**

22. Solve the following systems of linear equations by Gaussian elimination method:
$2x + 4y + 6z = 22, 3x + 8y + 5z = 27, -x + y + 2z = 2$ **(EX 1.5 – 1(ii))**

23. If $ax^2 + bx + c$ is divided by $x + 3, x - 5$ and $x - 1$, the remainders are 21,61 and 9 respectively. Find a, b and c. (Use Gaussian elimination method.) **(EX 1.5 - 2)**

24. An amount of ₹65,000 is invested in three bonds at the rates of 6%, 8% and 10% per annum respectively. The total annual income is ₹4,800. The income from the third bond is ₹600 more than that from the second bond. Determine the price of each bond. (Use Gaussian elimination method.) **(EX 1.5 - 3)**

25. A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6,8), (-2,-12)$ and $(3,8)$. He wants to meet his friend at $P(7,60)$. Will he meet his friend? (Use Gaussian elimination method.) **(EX 1.5 - 4)**

26. Test for consistency of the following system of linear equations and if possible solve: $x + 2y - z = 3, 3x - y + 2z = 1, x - 2y + 3z = 3, x - y + z + 1 = 0$. **(Eg. 1.29)**

27. Test for consistency of the following system of linear equations and if possible solve: $4x - 2y + 6z = 8, x + y - 3z = -1, 15x - 3y + 9z = 21$. **(Eg. 1.30)**

28. Test the consistency of the following system of linear equations $x - y + z = -9, 2x - y + z = 4, 3x - y + z = 6, 4x - y + 2z = 7$. **(Eg. 1.32)**

29. Find the condition on a, b and c so that the following system of linear equations has one parameter family of solutions: $x + y + z = a, x + 2y + 3z = b, 3x + 5y + 7z = c$. **(Eg. 1.33)**

30. Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7, x + y + \lambda z = \mu, x + 3y - 5z = 5$ has (i) no solution (ii) a unique solution (iii) an infinite number of solutions. **(Eg. 1.34)**

31. Test for consistency and if possible, solve the following systems of equations by rank method. $x - y + 2z = 2, 2x + y + 4z = 7, 4x - y + z = 4$. **(EX 1.6 – 1(i))**

32. Test for consistency and if possible, solve the following systems of equations by rank method. $3x + y + z = 2, x - 3y + 2z = 1, 7x - y + 4z = 5$. **(EX 1.6 – 1(ii))**

33. Test for consistency and if possible, solve the following systems of equations by rank method. $2x + 2y + z = 5, x - y + z = 1, 3x + y + 2z = 4$. **(EX 1.6 – 1(iii))**

34. Test for consistency and if possible, solve the following systems of equations by rank method. $2x - y + z = 2, 6x - 3y + 3z = 6, 4x - 2y + 2z = 4$. **(EX 1.6 – 1(iv))**

35. Find the value of k for which the equations $kx - 2y + z = 1, x - 2ky + z = -2, x - 2y + kz = 1$ have (i) no solution (ii) unique solution (iii) infinitely many solution **(EX 1.6 - 2)**

36. Investigate the values of λ and μ the system of linear equations $2x + 3y + 5z = 9, 7x + 3y - 5z = 8, 2x + 3y + \lambda z = \mu$,have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. **(EX 1.6 - 3)**

37. Solve the system: $x + 3y - 2z = 0, 2x - y + 4z = 0, x - 11y + 14z = 0$. **(Eg. 1.36)**

38. Solve the system: $x + y - 2z = 0, 2x - 3y + z = 0, 3x - 7y + 10z = 0, 6x - 9y + 10z = 0$. **(Eg. 1.37)**

39. Determine the values of λ for which the following system of equations $(3\lambda - 8)x + 3y + 3z = 0, 3x + (3\lambda - 8)y + 3z = 0, 3x + 3y + (3\lambda - 8)z = 0$. has a non-trivial solution. **(Eg. 1.38)**

40. By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8 + O_2 \rightarrow CO_2 + H_2O$. (The above is the reaction that is taking place in the burning of organic compound called isoprene.) **(Eg. 1.39)**

41. If the system of equations $px + by + cz = 0,\ ax + qy + cz = 0, ax + by + rz = 0$, has a non-trivial solution and $p \neq a, q \neq b, r \neq c$, prove that $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$. **(Eg. 1.40)**

42. Solve the following system of homogenous equations. $3x + 2y + 7z = 0, 4x - 3y - 2z = 0, 5x + 9y + 23z = 0$. **(EX 1.7 – 1(i))**

43. Solve the following system of homogenous equations. $2x + 3y - z = 0, x - y - 2z = 0, 3x + y + 3z = 0$. **(EX 1.7 – 1(ii))**

44. Determine the values of λ for which the following system of equations $x + y + 3z = 0, 4x + 3y + \lambda z = 0, 2x + y + 2z = 0$, has (i)a unique solution (ii) a non-trivial solution. **(EX 1.7 - 2)**

45. By using Gaussian elimination method, balance the chemical reaction equation: $C_2H_6 + O_2 \rightarrow H_2O + CO_2$. **(EX 1.7 - 3)**

CHAPTER 2

COMPLEX NUMBERS

1. Show that (i) $(2+i\sqrt{3})^{10}+(2-i\sqrt{3})^{10}$ is real and
(ii) $\left(\frac{19+9i}{5-3i}\right)^{15}-\left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary. **(Eg. 2.8)**
2. Let z_1, z_2 and z_3 be complex numbers such that $|z_1|=|z_2|=|z_3|=r>0$ and $z_1+z_2+z_3\neq 0$, P.T. $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right|=r$. **(Eg. 2.15)**
3. If z_1, z_2 and z_3 are three complex numbers such that $|z_1|=1, |z_2|=2, |z_3|=3$ and $|z_1+z_2+z_3|=1$, Prove that $|9z_1z_2+4z_1z_3+z_2z_3|=6$. **(EX 2.5 - 7)**
4. Given the complex number $z=\ 3+2i$, represent the complex numbers z, iz and $z+iz$ in one Argand diagram. Show that these complex numbers form the vertices of an isosceles right triangle. **(Eg. 2.18)**
5. If $z=x+iy$ is a complex number such that $Im\left(\frac{2z+1}{iz+1}\right)=0$. Show that the locus of z is $2x^2+2y^2+x-2y=0$. **(EX 2.6 - 2)**
6. If $z=x+iy$ and $\arg\left(\frac{z-i}{z+2}\right)=\frac{\pi}{4}$, then show that $x^2+y^2+3x-3y+2=0$. **(EX 2.7 - 6)**
7. Simplify (i) $(1+i)^n$ (ii) $(-\sqrt{3}+3i)^{31}$. **(Eg. 2.31)**
8. Solve the equation $z^3+8i=0$, where $z\in C$. **(Eg. 2.34)**
9. Find all cube roots of $\sqrt{3}+i$. **(Eg. 2.35)**
10. Suppose z_1, z_2 and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z|=2$. If $z_1=1+i\sqrt{3}$, then find z_2 and z_3. **(Eg. 2.36)**
11. Show that $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5=-\sqrt{3}$. **(EX 2.8 - 2)**
12. If $2\cos\alpha=x+\frac{1}{x}$ and $2\cos\beta=y+\frac{1}{y}$ show that
(i) $\frac{x}{y}+\frac{y}{x}=2\cos(\alpha-\beta)$ (ii) $xy-\frac{1}{xy}=2i\sin(\alpha+\beta)$
(iii) $\frac{x^m}{y^n}-\frac{y^n}{x^m}=2i\sin(m\alpha-n\beta)$ (iv) $x^my^n+\frac{1}{x^my^n}=2\cos(m\alpha+n\beta)$. **(EX 2.8 - 4)**
13. Solve the equation $z^3+27=0$. **(EX 2.8 - 5)**
14. Prove that the values of $\sqrt[4]{-1}$ are $\pm\frac{1}{\sqrt{2}}(1\pm i)$. **(EX 2.8 - 10)**

CHAPTER 3

THEORY OF EQUATIONS

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1. Form the equation whose roots are the squares of the roots of the cubic equation $x^3 + ax^2 + bx + c = 0$. **(Eg. 3.6)**
2. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$,if the product of two roots is 1. **(EX 3.1 -)**
3. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$,if it is given that two of its roots are in the ratio $3:2$. **(EX 3.1 - 6)**
4. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5} - \sqrt{3}$as a root. **(EX 3.2 - 4)**
5. If $2 + i$and $3 - \sqrt{2}$are roots of the equation $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$, find all roots. **(Eg. 3.15)**
6. Determine $k$and solve the equation $2x^3 - 6x^2 + 3x + k = 0$if one of its roots is twice the sum of the other two roots. **(EX 3.3 - 4)**
7. Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$and $\sqrt{3}$ are two of its zeros. **(EX 3.3 - 5)**
8. Solve the equation $(x-2)(x-7)(x-3)(x+2) + 19 = 0$. **(Eg. 3.23)**
9. Solve the equation $(2x-3)(6x-1)(3x-2)(x-12) - 7 = 0$. **(Eg. 3.24)**
10. Solve the equation $(x-5)(x-7)(x+6)(x+4) = 504$. **(EX 3.4 – 1(i))**
11. Solve the equation $(x-4)(x-7)(x-2)(x+1) = 16$. **(EX 3.4 – 1(ii))**
12. Solve :$(2x-1)(x+3)(x-2)(2x+3) + 20 = 0$. **(EX 3.4 - 2)**
13. Solve the following equation: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$. **(Eg. 3.28)**
14. Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$if it is known that $\frac{1}{3}$ is a solution. **(EX 3.5 - 7)**
15. Discuss the maximum possible number of positive and negative roots of the polynomial equations $x^2 - 5x + 6$and $x^2 - 5x + 16$.Also draw rough sketch of the graphs. **(EX 3.6 - 2)**

CHAPTER 4

INVERSE TRIGONOMETRIC FUNCTIONS

1. If $a_1, a_2, a_3, \ldots, a_n$ is an arithmetic progression with common difference d, P.T. $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+..+\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right]=\frac{a_n-a_1}{1+a_1a_n}$. **(Eg. 4.23)**
2. Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right)+tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$. **(Eg. 4.28)**
3. Solve $\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right)=\sin\left(\cot^{-1}\left(\frac{3}{4}\right)\right)$. **(Eg. 4.29)**

CHAPTER 5

TWO DIMENSIONAL ANALYTICAL GEOMETRY-II

1. Find the equation of the circle passing through the points $(1,1), (2,-1)$ and $(3,2)$. **(Eg. 5.10)**
2. A road bridge over an irrigation canal have two semi circular vents each with a span of $20m$ and the supporting pillars of width $2m$. Use $Fig.$ 5.16 to write the equations that model the arches. **(Eg. 5.13)**
3. Find the equation of the circle through the points $(1,0), (-1,0), (0,1)$. **(EX 5.1 - 6)**
4. Find the vertex, focus, directrix, and length of the latus rectum of the parabola $x^2 - 4x - 5y - 1 = 0$. **(Eg. 5.17)**
5. For the ellipse $4x^2 + y^2 + 24x - 2y + 21 = 0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2. **(Eg. 5.21)**
6. Find the centre, foci, and eccentricity of the hyperbola $11x^2 - 25y^2 - 44x + 50y - 256 = 0$. **(Eg. 5.24)**
7. Find the vertex, focus, equation of directrix and length of the latus rectum of the following: $x^2 - 2x + 8 + 17 = 0$. **(EX 5.2 – 4(iv))**
8. Find the vertex, focus, equation of directrix and length of the latus rectum of the following: $y^2 - 4y - 8x + 12 = 0$ **(EX 5.2 – 4(v))**
9. Identify the type of conic and find centre, foci, vertices, and directrices of each of the following: $18x^2 + 12y^2 - 144x + 48y + 120 = 0$. **(EX 5.2 – 8(v))**
10. Identify the type of conic and find centre, foci, vertices, and directrices of each of the following: $9x^2 - y^2 - 36x - 6y + 18 = 0$ **(EX 5.2 – 8(vi))**
11. Find the equations of the two tangents that can be drawn from $(5,2)$ to the ellipse $2x^2 + 7y^2 = 14$. **(EX 5.4 - 1)**
12. Find the equations of tangents to the hyperbola $\frac{x^2}{16} - \frac{y^2}{64} = 1$ which are parallel to$10x - 3y + 9 = 0$. **(EX 5.4 - 2)**
13. Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact. **(EX 5.4 - 3)**
14. A semielliptical archway over a one-way road has a height of $3m$ and a width of $12m$. The truck has a width of $3m$ and a height of $27.m$. Will the truck clear the opening of the archway? ($Fig.$ 5.6) **(Eg. 5.30)**
15. The maximum and minimum distances of the Earth from the Sun respectively are $152 \times 10^6 km$ and $94.5 \times 10^6 km$. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus. **(Eg. 5.31)**
16. The parabolic communication antenna has a focus at $2m$ distance from the vertex of the antenna. Find the width of the antenna $3m$ from the vertex. **(Eg. 5.33)**
17. Two coast guard stations are located 600 km apart at points $A(0,0)$ and $B(0,600)$. A distress signal from a ship at P is received at slightly

different times by two stations. It is determined that the ship is $200\ km$ farther from station A than it is from station B . Determine the equation of hyperbola that passes through the location of the ship. **(Eg. 5.39)**

18. Certain telescopes contain both parabolic mirror and a hyperbolic mirror. In the telescope shown in figure the parabola and hyperbola share focus F_1 which is $14m$above the vertex of the parabola. The hyperbola's second focus F_2 is $2m$ above the parabola's vertex. The vertex of the hyperbolic mirror is $1m$ below F_1. Position a coordinate system with the origin at the centre of the hyperbola and with the foci on the y –axis. Then find the equation of the hyperbola. **(Eg. 5.40)**
19. A bridge has a parabolic arch that is $10m$ high in the centre and $30m$ wide at the bottom. Find the height of the arch $6m$ from the centre, on either sides. **(EX 5.5 - 1)**
20. A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be $16m$, and the height at the edge of the road must be sufficient for a truck $4m$ high to clear if the highest point of the opening is to be $5m$ approximately . How wide must the opening be? **(EX 5.5 - 2)**
21. At a water fountain, water attains a maximum height of $4m$ at horizontal distance of $0.5\ m$ from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of $0.75\ m$ from the point of origin. **(EX 5.5 - 3)**
22. An engineer designs a satellite dish with a parabolic cross section. The dish is $5m$ wide at the opening, and the focus is placed $1.2\ m$ from the vertex (a) Position a coordinate system with the origin at the vertex and the x –axis on the parabola's axis of symmetry and find an equation of the parabola. (b) Find the depth of the satellite dish at the vertex. **(EX 5.5 - 4)**
23. Parabolic cable of a $60m$ portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every $6m$ along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex. **(EX 5.5 - 5)**
24. Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is $150\ m$ tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower. **(EX 5.5 - 6)**
25. A rod of length $1.2\ m$ moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is $0.3\ m$ from the end in contact with x –axis is an ellipse. Find the eccentricity. **(EX 5.5 - 7)**

26. Assume that water issuing from the end of a horizontal pipe, 7.5 $m$above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5 $m$below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground? **(EX 5.5 - 8)**

27. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4 m when it is 6 m away from the point of projection. Finally it reaches the ground 12 m away from the starting point. Find the angle of projection. **(EX 5.5 – 9)**

28. Points A and B are 10 km apart and it is determined from the sound of an explosion heard at those points at different times that the location of the explosion is 6 km closer to A than B . Show that the location of the explosion is restricted to a particular curve and find an equation of it. **(EX 5.5 - 10)**

CHAPTER 6

APPLICATIONS OF VECTOR ALGEBRA

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1. By vector method, prove that $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$. **(Eg. 6.3)**
2. With usual notations, in any triangle ABC, prove by vector method that $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$. **(Eg. 6.4)**
3. Prove by vector method that $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$. **(Eg. 6.5)**
4. If D is the midpoint of the side BC of a triangle ABC, then show by vector method that $|\overrightarrow{AB}|^2+|\overrightarrow{AC}|^2=2\left(|\overrightarrow{AD}|^2+|\overrightarrow{BD}|^2\right)$. **(Eg. 6.6)**
5. Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent. **(Eg. 6.7)**
6. Using vector method, prove that $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$. **(EX 6.1 - 9)**
7. Prove by vector method that $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$. **(EX 6.1 - 10)**
8. If $\vec{a}=-2\vec{i}+3\vec{j}-2\vec{k},\ \vec{b}=3\vec{i}-\vec{j}+3\vec{k},\ \vec{c}=2\vec{i}-5\vec{j}+\vec{k}$, find $(\vec{a}\times\vec{b})\times\vec{c}$ and $\vec{a}\times(\vec{b}\times\vec{c})$ State whether they are equal. **(Eg. 6.22)**
9. If $\vec{a}=2\vec{i}-\vec{j},\ \vec{b}=\vec{i}-\vec{j}-4\vec{k},\ \vec{c}=3\vec{j}-\vec{k}$ and $\vec{d}=2\vec{i}+5\vec{j}+\vec{k}$, verify that (i) $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{b},\vec{d}]\vec{c}-[\vec{a},\vec{b},\vec{c}]\vec{d}$ **(Eg. 6.23)**
10. If $\vec{a}=2\vec{i}-\vec{j},\ \vec{b}=\vec{i}-\vec{j}-4\vec{k},\ \vec{c}=3\vec{j}-\vec{k}$ and $\vec{d}=2\vec{i}+5\vec{j}+\vec{k}$, verify that (ii) $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{c},\vec{d}]\vec{b}-[\vec{b},\vec{c},\vec{d}]\vec{a}$ **(Eg. 6.23)**
11. If $\vec{a}=2\vec{i}+3\vec{j}-\vec{k},\ \vec{b}=3\vec{i}+5\vec{j}+2\vec{k},\ \vec{c}=-\vec{i}-2\vec{j}+3\vec{k}$, verify that $(\vec{a}\times\vec{b})\times\vec{c}=(\vec{a}.\vec{c})\vec{b}-(\vec{b}.\vec{c})\vec{a}$. **(EX 6.3 – 4(i))**
12. If $\vec{a}=2\vec{i}+3\vec{j}-\vec{k},\ \vec{b}=3\vec{i}+5\vec{j}+2\vec{k},\ \vec{c}=-\vec{i}-2\vec{j}+3\vec{k}$, verify that $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$. **(EX 6.3 – 4(ii))**
13. Find the vector equation in parametric form and Cartesian equations of a straight passing through the points $(-5,7,-4)$ and $(13,-5,2)$. Find the point where the straight line crosses the xy –plane. **(Eg. 6.27)**
14. Find the points where the straight line passes through $(6,7,4)$ and $(8,4,9)$ cuts the xz and yz planes. **(EX 6.4 - 3)**
15. Find the point of intersection of the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$. **(Eg. 6.33)**
16. Find the equation of a straight line passing through the point of intersection of the straight lines $\vec{r}=(\vec{i}+3\vec{j}-\vec{k})+t(2\vec{i}+3\vec{j}+2\vec{k})$ and $\frac{x-2}{1}=\frac{y-4}{2}=\frac{z+3}{4}$, and perpendicular to both straight lines. **(Eg. 6.34)**
17. Determine whether the pair of straight lines $\vec{r}=(2\vec{i}+6\vec{j}+3\vec{k})+t(2\vec{i}+3\vec{j}+4\vec{k}),\vec{r}=(2\vec{j}-3\vec{k})+s(\vec{i}+2\vec{j}+3\vec{k})$are parallel. Find the shortest distance between them. **(Eg. 6.35)**

18. Show that the lines $\frac{x-3}{3}=\frac{y-3}{-1}, z-1=0$ and $\frac{x-6}{2}=\frac{z-1}{3}, y-2=0$ intersect. Also find the point of intersection. **(EX 6.5 - 4)**
19. Find the vector parametric, vector non-parametric and Cartesian formof the equation of the plane passing through the points $(-1,2,0),(2,2,-1)$ and parallel to the straight line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$. **(Eg. 6.44)**
20. Find the non-parametric form of vector equation, and cartesian equation of the plane passing through the point (2,3,6) and parallel to the straight lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-3}{1}$ and $\frac{x+3}{2}=\frac{y-3}{-5}=\frac{z+1}{-3}$. **(EX 6.7 -)**
21. Find the parametric form of vector equation, and Cartesian equations of the plane passing through the points $(2,2,1),(9,3,6)$ and perpendicular to the plane $2x+6y+6z=9$. **(EX 6.7 - 2)**
22. Find parametric form of vector equation and Cartesian equations of the plane passing through the points $(2,2,1),(1,-2,3)$ and parallel to the straight line passing through the points $(2,1,-3)$ and $(-1,5,-8)$. **(EX 6.7 - 3)**
23. Find the non-parametric form of vector equation of the plane passing through the point $(1,-2,4)$ and perpendicular to the plane $x+2y-3z=11$ and parallel to the line $\frac{x+7}{3}=\frac{y+3}{-1}=\frac{z}{1}$. **(EX 6.7 - 4)**
24. Find the parametric form of vector equation, and Cartesian equations of the plane containing the line $\vec{r}=(\vec{i}-\vec{j}+3\vec{k})+t(2\vec{i}-\vec{j}+4\vec{k})$ and perpendicular to plane $\vec{r}.(\vec{i}+2\vec{j}+\vec{k})=8$. **(EX 6.7 - 5)**
25. Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the points $(3,6,-2),(-1,-2,6)$ and $(6,-4,-2)$. **(EX 6.7 - 6)**
26. Show that the lines $\vec{r}=(-\vec{i}-3\vec{j}-5\vec{k})+s(3\vec{i}+5\vec{j}+7\vec{k})$ and $\vec{r}=(2\vec{i}+4\vec{j}+6\vec{k})+t(\vec{i}+4\vec{j}+7\vec{k})$ are coplanar. Also, find the non-parametric form of vector equation of the plane containing these lines. **(Eg. 6.46)**
27. Show that the straight lines $\vec{r}=(5\vec{i}+7\vec{j}-3\vec{k})+s(4\vec{i}+4\vec{j}-5\vec{k})$ and $\vec{r}=(8\vec{i}+4\vec{j}+5\vec{k})+t(7\vec{i}+\vec{j}+3\vec{k})$ are coplanar. Find the vector equation of the plane in which they lie. **(EX 6.8 - 1)**
28. Show that the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{3}$ and $\frac{x-1}{-3}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar. Also, find the plane containing these lines. **(EX 6.8 - 2)**
29. If the straight lines $\frac{x-1}{2}=\frac{y+1}{\lambda}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{\lambda}$ are coplanar, find λ and equations of the planes containing these two lines. **(EX 6.8 - 4)**
30. Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point (4,3,2) to the plane $x+2y+3z=2$. **(EX 6.9 - 8)**

Every Successful Person Has A Painful Story, Accept Pain

And Ready To Get Success.

Wish you all the Best.

By

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