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Register No. _____

Time : 3.00 Hrs.

Quarterly Examination - 2023
MATHEMATICS

Marks : 90

PART - I

Answer all the questions. Choose the more appropriate answer from the given four alternatives **20 x 1 = 20**

1. Let X = {1, 2, 3, 4} and R = { (1, 1), (1, 2), (1, 3), (2, 2), (3, 3), (2, 1), (3, 1), (1, 4), (4, 1) }. Then R is
   a) reflexive b) symmetric c) transitive d) equivalence
2. If the function f : [–3, 3] → s defined by f(x) = x^2 is onto, then s is a) [–9, 9] b) R c) [–3, 3[d) [0, 9]
3. The value of $\log_a b \log_b c \log_c a$ is a) 2 b) 1 c) 3 d) 4
4. If tanα and tanβ are the roots of $x^2 + ax + b = 0$ then $\frac{\sin(\alpha+\beta)}{\sin\alpha \sin\beta}$ is equal to
   a) b/a b) a/b c) –a/b d) –b/a
5. If $nC_{10} > nC_r$ for all possible r, then a value of n is a) 10 b) 21 c) 19 d) 20
6. The number of five digit telephone numbers having alteast one of their digits repeated is
   a) 90000 b) 10000 c) 30240 d) 69760
7. The number of roots of $(x + 3)^4 + (x + 5)^4 = 16$ is a) 4 b) 2 c) 3 d) 0
8. $\cos1^0 + \cos2^0 + \cos3^0 + ... + \cos 179^0$ a) 0 b) 1 c) –1 d) 89
9. The principal value of cosθ lies in
   a) I & II quadrant b) II quadrant only c) III & IV quadrant d) None of these
10. The n^{th} term of the sequence $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{5}{16}$ .....is a) $2^n - n - 1$ b) $1 - 2^{-n}$ c) $2^{-n} + n - 1$ d) 2^{n-1}
11. Given that x, y and b are real numbers x < y, b > 0 then a) xb < yb b) xb > yb c) xb ≤ yb d) $\frac{x}{a} \geq \frac{y}{6}$
12. The equation of the line with slope 2 and the length of the perpendicular from the origin equal to $\sqrt{5}$ is
    a) $x - 2y = \sqrt{5}$ b) $2x - y = \sqrt{5}$ c) $2x - y = 5$ d) $x - 2y - 5 = 0$
13. The number of real roots of the equation $x^2 - 6x + 10 = 0$ is a) 2 b) 1 c) 0 d) none
14. In $2nC_3 : nC_3 = 11 : 1$ then n is a) 5 b) 6 c) 11 d) 7
15. The co-efficient of x^5 in the series e^{-2x} is a) $\frac{2}{3}$ b) $\frac{3}{2}$ c) $\frac{-4}{15}$ d) $\frac{4}{15}$
16. 1 + 3 + 5 + 7 + ... + 17 is equal to a) 101 b) 81 c) 71 d) 61
17. The y-intercept of the straight line passing through (1, 3) and perpendicular to 2x – 3y + 1 = 0 is
    a) $\frac{3}{2}$ b) $\frac{9}{2}$ c) $\frac{2}{3}$ d) $\frac{2}{9}$
18. The number of rectangles that a chessboard has a) 81 b) 9^9 c) 1296 d) 6561
19. Which of the following is not true? a) $\sin\theta = \frac{-3}{4}$ b) cosθ = –1 c) tanθ = 25 d) $\sec\theta = \frac{1}{4}$
20. If $\log_{\sqrt{x}} 0.25 = 4$ then the value of x is a) 0.5 b) 2.5 c) 1.5 d) 1.25

PART - II

Answer any seven questions. Q.No. 30 is compulsory. **7 x 2 = 14**

21. Find the domain of $\frac{1}{1-2\sin x}$
22. Find the distance between the parallel lines 3x – 4y + 5 = 0 and 6x – 8y – 15 = 0
23. Simplify and hence find the value of n $\frac{3^{2n}9^2 3^{-n}}{3^{3n}} = 27$
24. Find the value of $\sin34^0 + \cos64^0 - \cos4^0$.
25. Find the distinct permutations of the letters of the word MISSISSIPPI?
26. Find the path traced out the point (ct, c/t) here t ≠ 0 is the parameter and c is a constant.
27. Find the middle term in the expansion of $(x + y)^7$.
28. Find the principal value of $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

29. Write the first 6 terms of the sequences whose n^{th} term a_n is $a_n = \begin{cases} n & \text{if } n \text{ is } 1, 2 \text{ or } 3 \\ a_{n-1} + a_{n-2} + a_{n-3} & \text{if } n > 3 \end{cases}$

30. If P(A) denotes the power set of A then find n(P(P(P(ϕ)))) **7 x 3 = 21**

PART - II

Answer any seven questions. Q.No. 40 is compulsory.

31. Find the values of p for which the difference between the roots of the equation $x^2 + px + 8 = 0$ is 2.

32. Show that the points $(0, \frac{-3}{2})$, (1, –1) and $(2, -\frac{-1}{2})$ are collinear.

33. Find the value of $\sin 18^0$.

34. If A and B are two sets so that n(B–A) = 2n(A – B) = 4n (A ∩ B) and if n(AUB) = 14 then find n(P(A)).

35. Find the rank of the word IITJEE.

36. Compute the sum of first n terms of the series 8 + 88 + 888 + 8888 + ...

37. In a ΔABC prove that (b + c) cosA + (c + a) cos B + (a + b) cosC = a + b + c.

38. Find $\sqrt[3]{1001}$ approximately (two decimal places)

39. Prove that $35C_5 + \sum_{r=0}^{4} (39 - r)\, C_4 = 40C_5$.

40. Compute $\log_{27} 27 - \log_{9} 9$

PART - IV **7 x 5 = 35**

Answer all the questions.

41. a) If f : R → R is defined by f(x) = 3x – 5. Prove that f is a bijection and find its inverse.

(OR)

b) If a, b, c are respectively the p^{th}, q^{th} and r^{th} terms of a GP, show that (q – r) log a + (r – p) log b + (p – q) log c = 0

42. a) Resolve the following rational expressions into partial fractions $\frac{2x^2 + 5x - 11}{x^2 + 2x - 3}$

(OR)

A ΔOPQ is formed by the pair of straight lines $x^2 - 4xy + y^2 = 0$ and the line PQ. The equation of PQ is x + y – 2 = 0. Find the equation of the median of the triangle ΔOPQ drawn from the origin O.

43. a) Prove that $\frac{\cot(180^0 + \theta)\sin(90 - \theta)\cos(-\theta)}{\sin(270^0 + \theta)\tan(-\theta)\operatorname{cosec}(360 + \theta)} = \cos^2\theta\cot\theta$

(OR)

b) How many strings are there using the letters of the word INTERMEDIATE if

i) The vowels and consonants are alternative

ii) All the vowels are together iii) vowels are never together

44. a) Using the mathematical induction, show that for any natural number n

$$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \ldots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

(OR)

b) From the curve y = |x|, draw i) y = |x – 1| + 1 ii) y = |x + 1| – 1 iii) y = |x + 2| – 3

45. a) Find the equation of a straight line parallel to 2x + 3y = 10 and which is such that the sum of its intercepts on the axes is 15.

(OR)

b) Prove that $\sqrt[3]{x^3 + 7} - \sqrt[3]{x^3 + 4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

46. a) If the difference of the roots of the equation $2x^2 - (a + 1)x + a - 1 = 0$ is equal to their product then prove that $a = 2$.

(OR)

b) If the product of the 4^{th}, 5^{th} and 6^{th} terms of a G.P is 4096 and if the product of 5th, 6th and 7th terms of it is 32768, find the sum of first 8 terms of the geometric progression.

47. a) State the prove Napier's formula.

(OR)

b) If A + B + C = $\pi/2$ prove that cos2A + cos2B + cos2C = 1 + 4sinA sinB sinC.