Class : 11

Register Number

COMMOM QUARETERLY EXAMINATION 2023 - 24

MATHEMATICS

Time Allowed : 3.00 Hours] [Max. Marks : 90

PART – I

20 x 1 = 20

1. Answer all the questions by choosing the correct answer from the given 4 alternatives
2. Write question number, correct option and corresponding answer
3. Each question carries 1 mark

1. If $n((A \times B) \cap (A \times C)) = 8$ and $n(B \cap C) = 2$, then $n(A)$ is
   (1) 6 (2) 4 (3) 8 (4) 16
2. Let $f : R \to R$ be defined by $f(x) = 1 - |x|$. Then the range of f is
   (1) R (2) $(1,\infty)$ (3) $(-1,\infty)$ (4) $(-\infty, 1]$
3. If $f(x) = 2x - 3$ and $g(x) = x^2 + x - 2$, then g o f(x) is
   (1) $2(2x^2 - 5x + 2)$ (2) $2x^2 - 5x - 2$ (3) $2(2x^2 + 5x + 2)$ (4) $2x^2 + 5x - 2$
4. The logarithmic form of $5^2 = 25$ is
   (1) $\log_5 2 = 25$ (2) $\log_2 5 = 25$ (3) $\log_5 25 = 2$ (4) $\log_{25} 5 = 2$
5. Given that x, y and b are real numbers $x < y, b > 0$, then
   (1) $xb < yb$ (2) $xb > yb$ (3) $xb \leq yb$ (4) $\frac{x}{b} \geq \frac{y}{b}$
6. If 3 is the logarithm of 343, then the base is
   (1) 5 (2) 7 (3) 6 (4) 9
7. The maximum value of $4\sin^2 x + 3\cos^2 x + \sin\frac{x}{2} + \cos\frac{x}{2}$ is
   (1) $4 + \sqrt{2}$ (2) $3 + \sqrt{2}$ (3) 9 (4) 4
8. Which of the following is not true?
   (1) $\sin\theta = -\frac{3}{4}$ (2) $\cos\theta = -1$ (3) $\tan\theta = 25$ (4) $\sec\theta = \frac{1}{4}$
9. A wheel is spinning at 2 radians/second. How many seconds will it take to make 10 complete rotations?
   (1) 10π seconds (2) 20π seconds (3) 5π seconds (4) 15π seconds
10. $2\sin 5x\cos x$ in expressed as.
    (1) $\sin 6x + \cos 4x$ (2) $\sin 6x + \sin 4x$ (3) $\cos 6x + \sin 4x$ (4) $\cos 6x + \cos 4x$
11. $5c_5 + 5c_4 + 5c_3 + 5c_2 + 5c_1$ is equal to.
    (1) 30 (2) 31 (3) 32 (4) 33
12. The number of 5 digit numbers all digits of which are odd is
    (1) 25 (2) 5^5 (3) 5^6 (4) 625.
13. The number of rectangles that a chessboard has ...
    (1) 81 (2) 9^9 (3) 1296 (4) 6561
14. The coefficient of x^8y^{12} in the expansion of $(2x + 3y)^{20}$ is
    (1) 0 (2) $2^8 3^{12}$ (3) $2^8 3^{12} + 2^{12} 3^8$ (4) $^{20}C_8 2^8 3^{12}$
15. The coefficient of x^5 in the series e^{-2x} is
    (1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) $\frac{-4}{15}$ (4) $\frac{4}{15}$.
16. If $\Sigma n = 210$ Then $\Sigma n^2 =$
    (1) 2870 (2) 2160 (3) 2970 (4) 34410

17. The angle between the lines $2x - y + 3 = 0$ and $x + 2y + 3 = 0$ is

(1) 90° (2) 60° (3) 45° (4) 30°

18. Which of the following point lie on the locus of $3x^2 + 3y^2 - 8x - 12y + 17 = 0$

(1) (0,0) (2) (−2,3) (3) (1,2) (4) (0, −1)

19. The image of the point (2,3) in the line $y = -x$ is

(1) (−3, −2) (2) (−3,2) (3) (−2, −3) (4) (3,2)

20. If a vertex of a square is at the origin and its one side lies along the line $4x + 3y - 20 = 0$, then the area of the square is

(1) 20 sq. units (2) 16 sq. units (3) 25 sq. units (4) 4 sq.units

PART – II

1. Answer any 7 questions
2. Each question carries 2 marks
3. Question number 30 is compulsory

7x2=14

21. For a set A, $A \times A$ contains 16 elements and two of its elements are (1, 3) and (0, 2). Find the elements of A.

22. Prove that the relation "friendship" is not an equivalence relation on the set of all people in Chennai.

23. Solve $(2x + 1)^2 - (3x + 2)^2 = 0$.

24. Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15°.

25. If ${}^{10}P_r = {}^{7}P_{r+2}$ find r.

26. How many different selections of 5 books can be made from 12 different books if,

(i) Two particular books are always selected ?

(ii) Two particular books are never selected ?

27. Find $\sqrt[3]{1001}$ approximately (two decimal places.

28. Compute the sum of first n terms of the series: $8 + 88 + 888 + 8888 + \cdots$

29. Find the coefficient of x^4 in the expansion of $\frac{3-4x+x^2}{e^{2x}}$.

30. Find the value of $\tan\frac{\pi}{12}$.

PART – II

1. Answer any 7 questions
2. Each question carries 3 marks
3. Question number 40 is compulsory

7x3=21

31. Find the range of the function $f(x) = \frac{1}{1-3\cos x}$.

32. Find all values of x for which $\frac{x^3(x-1)}{(x-2)} > 0$.

33. If $\left(x^{1/2} + x^{-1/2}\right)^2 = 9/2$, then find the value of $(x^{1/2} - x^{-1/2})$ for $x > 1$.

34. Show that $\tan 75° + \cot 75° = 4$.

35. If the three angles in a triangle are in the ratio 1 : 2 : 3, then prove that the corresponding sides are in the ratio $1 : \sqrt{3} : 2$.

36. Prove that $\frac{(2n)!}{n!} = 2^n(1.3.5 \cdots (2n-1))$.

37. Find the constant term of $\left(2x^3 - \frac{1}{3x^2}\right)^5$.

38. The length of the perpendicular drawn from the origin to a line is 12 and makes an angle 150° with positive direction of the x-axis. Find the equation of the line.

39. Show that the sum of $(m+n)^{th}$ and $(m-n)^{th}$ term of an AP. is equal to twice the m^{th} term.

40. Prove that $n! + (n+1)! = n!(n+2)$.

PART – IV

1. Answer all the questions
2. Each question carries 5 marks

$7 \times 5 = 35$

41. a) In a survey of 5000 persons in a town, it was found that 45% of the persons know Language A, 25% know Language B, 10% know Language C, 5% know Languages A and B, 4% know Languages B and C, and 4% know Languages A and C. If 3% of the persons know all the three Languages, find the number of persons who knows only Language A.

(OR)

b) Prove that the sum of the first n non-zero even numbers is $n^2 + n$.

42. a) A quadratic polynomial has one of its zeros $1 + \sqrt{5}$ and it satisfies $p(1) = 2$. Find the quadratic polynomial.

(OR)

b) In the binomial expansion of $(a+b)^n$, the coefficients of the 4^{th} and 13^{th} terms are equal to each other, find n.

43. a) If p_1 and p_2 are the lengths of the perpendiculars from the origin to the straight lines $x \sec\theta + y \operatorname{cosec}\theta = 2a$ and $x\cos\theta - y\sin\theta = a\cos 2\theta$, then prove that $p_1^2 + p_2^2 = a^2$.

(OR)

b) Find the values of (i) sin 18° (ii) cos 18°

44. a) Find the equations of lines parallel to $3x - 4y - 5 = 0$ at a unit distance from it.

(OR)

b) A committee of 7 peoples has to be formed from 8 men and 4 women. In how many ways can this be done when the committee consists of (i) exactly 3 women ? (ii) at least 3 women ? (iii) at most 3 women ?

45. a) Write the values of f at −4, 1, −2, 7, 0 if $f(x) = \begin{cases} -x+4 & \text{if } -\infty < x \le -3 \\ x+4 & \text{if } -3 < x < -2 \\ x^2 - x & \text{if } -2 \le x < 1 \\ x - x^2 & \text{if } 1 \le x < 7 \\ 0 & \text{otherwise} \end{cases}$

(OR)

b) Resolve into partial fractions : $\frac{2x}{(x^2+1)(x-1)}$

46. a) If $A + B + C = \pi$, prove that $\cos A + \cos B + \cos C = 1 + 4\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$

(OR)

b) The AM of two numbers exceeds their GM by 10 and HM by 16. Find the numbers.

47. a) If $\frac{a}{x} + \frac{y}{b} = 1$ and $\frac{b}{y} + \frac{z}{c} = 1$ Then show that $\frac{x}{a} + \frac{c}{z} = 1$.

(OR)

b) Show that the equation $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$ represents a pair of parallel lines. Find the distance between them.

ST. JOSEPH'S HR. SEC. SCHOOL - CHENGALPATTU

COMMON QUARETERLY EXAMINATION - 2023 - 24

STD: 11 MATHEMATICS - KEY

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27.09.23

I 1	2	4
2	4	$(-\infty, 1]$
3	1	$2(2x^2-5x+2)$
4	3	$\log_5 25 = 2$
5	1	$xb < yb$
6	2	7
7	1	$4+\sqrt{2}$
8	4	$\sec\theta = \frac{1}{4}$
9	1	10π seconds
10	2	$\sin 6x + \sin 4x$
11	2	31
12	2	5^5
13	3	1296
14	4	$20C_8\, 2^8 3^{12}$
15	3	$-4/15$
16	1	2870
17	1	90
18	3	(1,2)
19	1	(-3,-2)
20	2	16 sq. units

II 21. $A = \{0,1,2,3\}$

22 R is not transitive

23 $4x^2+4x+1-9x^2-12x-4=0$

$-5x^2-8x-3=0 \Rightarrow 5x^2+8x+3=0$

$x = -1, -3/5$

24 $S = r\theta$, $r=5$, $\theta = 15^\circ \Rightarrow \theta = 15\times\frac{\pi}{180}$

$S = 5\times\frac{\pi}{12} = \frac{5\pi}{12}$

25. $\frac{10!}{(10-r)!} = \frac{7!}{(5-r)!} \Rightarrow r = 4$

26 (i) $10C_3 = 120$

(ii) $10C_5 = 252$

27 $(1000+1)^{1/3} \simeq 10.003$

28. $8+88+888+\cdots = \frac{8}{9}\left[\frac{10(10^n-1)}{9} - n\right]$

29. $(3-4x+x^2)e^{-2x} = (3-4x+x^2)\left(1-2x+2x^2-\frac{4x^3}{3}+\frac{2x^4}{3}-\cdots\right)$

coefficient of $x^4 = \frac{28}{3}$

30. $\tan\frac{\pi}{12} = \tan 15^\circ = \tan(45^\circ-30^\circ)$

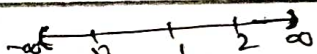
$= \frac{\tan 45 - \tan 30}{1+\tan 45\cdot\tan 30} = \frac{1-\frac{1}{\sqrt3}}{1+1\cdot\frac{1}{\sqrt3}} = \frac{\sqrt3-1}{\sqrt3+1}$

III 31. $-1 \le \cos x \le 1$

$-\frac{1}{2} \ge \frac{1}{1-3\cos x} \ge \frac{1}{4}$

Range of $f = (-\infty, -1/2] \cup [1/4, \infty)$

32 $x = 0, 1, 2$



Interval	Sign of $\frac{x^3(x-1)}{(x-2)}$
$(-\infty, 0)$	−
$(0,1)$	+
$(1,2)$	−
$(2,\infty)$	+

Soln set $(0,1)\cup(2,\infty)$

33 $x+\frac{1}{x}+2 = \frac{9}{2} \Rightarrow x+\frac{1}{x} = \frac{5}{2}$

$\left(x^{1/2}-x^{-1/2}\right)^2 = x+\frac{1}{x}-2$

$= \frac{5}{2}-2 = \frac{1}{2}$

$x^{1/2}-x^{-1/2} = \frac{1}{\sqrt2}$

34 $\tan 75^\circ = \tan(45^\circ+30^\circ) = \frac{\sqrt3+1}{\sqrt3-1}$

$\cot 75^\circ = \cot(45^\circ+30^\circ) = \frac{\sqrt3-1}{\sqrt3+1}$

$\tan 75^\circ + \cot 75^\circ = \frac{(\sqrt3+1)^2+(\sqrt3-1)^2}{(\sqrt3-1)(\sqrt3+1)} = \frac{8}{2} = 4$

35. $a=\theta$, $b=2\theta$, $c=3\theta$

$a+b+c = 180 \Rightarrow \theta = 30^\circ$

$a:b:c = \sin 30^\circ : \sin 60^\circ : \sin 90^\circ$

$= \frac{1}{2} : \frac{\sqrt3}{2} : 1 \Rightarrow 1 : \sqrt3 : 2$

36 $\frac{(2n)!}{n!} = \frac{1\cdot2\cdot\cdots(2n-2)(2n-1)2n}{n!}$

$= \frac{1\cdot3\cdot5\cdots(2n-1)\times 2^n\times(1\cdot2\cdot3\cdots(n-1)n)}{n!}$

37. $T_{r+1} = {}^nC_r x^{n-r} a^r = {}^5C_r (2x^3)^{5-r} \left(-\frac{1}{3x^2}\right)^r$

$15-5r = 0 \Rightarrow r = 3$

$T_{3+1} = \frac{-40}{27}$

38. $x\cos 150^\circ + y \sin 150^\circ = 12$

$\sqrt{3}x - y + 24 = 0$

39. $T_{m+n} = a + (m+n-1)d$

$T_{m-n} = a + (m-n-1)d$

$T_{m+n} + T_{m-n} = 2a + (2m-2)d = 2[a+(m-1)d]$

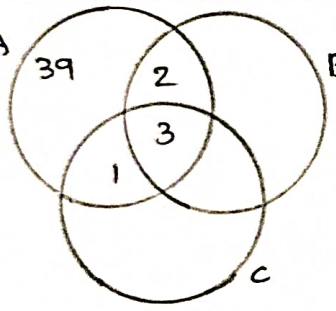
$= 2T_m$

40 $n! + (n+1)! = n! + (n+1)n!$

$= n!(1+n+1)$

$= n!(n+2)$ RHS

IV/41. a.



$= 5000 \times \frac{39}{100}$

$= 1950$

b. $P(n) = 2+4+6+\cdots+2n = n^2+n$

$n=1$ $P(1) =$ LHS : 2 , RHS : 2

$P(1)$ is true

$n=k$ $P(k) = 2+4+6+\cdots+2k = k^2+k$

$n=k+1$ $P(k+1) = 2+4+6+\cdots+2k+2(k+1)$

$= (k+1)(k+2)$

$P(k)$ is true, $P(k+1)$ is true

42. a. $1+\sqrt{5}$ is a root, $1-\sqrt{5}$ is also root $\alpha+\beta = 2, \alpha\beta = -4$

$P(x) = k(x^2 - 2x - 4)$

$P(1) = 2$

$k(1^2 - 2 - 4) = 2 \Rightarrow k = -2/5$

$P(x) = -\frac{2}{5}(x^2 - 2x - 4)$

b. $T_{3+1} = {}^nC_3 a^{n-3} b^3$

$T_{12+1} = {}^nC_{12} a^{n-12} b^{12}$

${}^nC_3 = {}^nC_{12} \Rightarrow n = 3+12 \Rightarrow \boxed{n=15}$

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43 a. $P_1 = \pm\left(\frac{x(\theta)+y(\theta)-2a}{\sqrt{\sec^2\theta+\tan^2\theta}}\right) = 2a\sin\theta\cos\theta$

$P_2 = \pm\frac{x(\theta)y(\theta) - a\cos 2\theta}{\sqrt{\sin^2\theta+(-\cos\theta)^2}} = a\cos 2\theta$

$P_1^2 + P_2^2 = (2a\sin\theta\cos\theta)^2 + (a\cos 2\theta)^2$

$= a^2(\sin^2 2\theta + \cos^2 2\theta)$

$P_1^2 + P_2^2 = a^2$

b. $\theta = 18^\circ \Rightarrow 5\theta = 90^\circ \Rightarrow 3\theta + 2\theta = 90^\circ$

$\sin 2\theta = \sin(90^\circ - 3\theta) = \cos 3\theta$

$2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$

$4\sin^2\theta + 2\sin\theta - 1 = 0$

$\sin\theta = \frac{-2 \pm \sqrt{4-4(4)(-1)}}{2(4)} = \frac{-1\pm\sqrt{5}}{4}$

$\sin 18^\circ = \frac{\sqrt{5}-1}{4}$, $\cos 18^\circ = \frac{1}{4}\sqrt{10+2\sqrt{5}}$

44 a. $\frac{3x-4y+k}{\sqrt{3^2+(-4)^2}} = \pm 1 \Rightarrow \frac{3x-4y+k}{5} = \pm 1$

$3x - 4y + k \pm 5 = 0$

b. (i) ${}^8C_4 \times {}^4C_3 = \boxed{280}$

(ii) ${}^8C_4 \times {}^4C_3 + {}^8C_3 \times {}^4C_4 = 280 + 56 = \boxed{336}$

(iii) ${}^8C_4 \times {}^4C_1 + {}^8C_3 \times {}^4C_2 + {}^8C_2 \times {}^4C_1 + {}^8C_1 \times {}^4C_0$

$= 280 + 336 + 112 + 8$

$= \boxed{736}$

45 a. $f(-1) = 8$, $f(1) = 0$, $f(-2) = 6$

$f(7) = 0$, $f(0) = 0$

b. $= \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$

$2x = A(x^2+1) + (Bx+C)(x-1)$

$x = 1 \Rightarrow \boxed{A = 1}$

$x = 0 \Rightarrow A = \boxed{C = 1}$

$x = -1 \Rightarrow \boxed{B = -1}$

$= \frac{1}{x-1} + \frac{1-x}{x^2+1}$

46. a. $\cos A + \cos B + \cos C = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) + \cos C$

$= 2\cos(\pi/2 - C/2)\cos\left(\frac{A}{2} - \frac{B}{2}\right) + \cos C$

$= 1 + 2\sin\left(\frac{C}{2}\right)\left[\cos\left(\frac{A}{2} - \frac{B}{2}\right) + 1 - 2\sin^2\left(\frac{C}{2}\right)\right]$

$= 1 + 2\sin\left(\frac{C}{2}\right)\left[\cos\left(\frac{A}{2} - \frac{B}{2}\right) - \cos\left(\frac{A}{2} + \frac{B}{2}\right)\right]$

$= 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

46 b. $AM = \frac{a+b}{2}$, $GM = \sqrt{ab}$, $HM = \frac{2ab}{a+b}$

$G = A-10$, $H = A-16$

$(A-10)^2 = A(A-16)$

$A^2 + 100 - 20A = A^2 - 16A \Rightarrow a+b = 50$ — ①

$a = A-10 = 25-10 = 15 \Rightarrow b = \frac{225}{a}$ — ②

② in ① $\Rightarrow$

$a^2 - 50a + 225 = 0$

$a = 5, \quad b = 45$

47. a. $\frac{a}{x} + \frac{y}{b} = 1 \Rightarrow \frac{x}{a} = \frac{b}{b-y}$ — ①

$\frac{b}{y} + \frac{z}{c} = 1 \Rightarrow \frac{c}{z} = \frac{y}{y-b}$ — ②

① + ② $\Rightarrow \frac{x}{a} + \frac{c}{z} = \frac{b}{b-y} - \frac{y}{b-y}$

$$\boxed{\frac{x}{a} + \frac{c}{z} = 1}$$

b. $a = 9, \quad h = -12 \quad g = -16, \quad b = 16, \quad f = 8$

$c = -12$

$h^2 - ab = 0 \Rightarrow (-12)^2 - 9(16) = 0$

$144 - 144 = 0$ (11^{th} line)

Distance between the parallel line.

$= 2\sqrt{\frac{g^2 - ac}{a(a+b)}} = 2\sqrt{\frac{(-6)^2 - 9(-12)}{9(9+16)}}$

$= 2\sqrt{\frac{144}{9(25}} = \frac{2 \times 12}{3 \times 5} = \frac{8}{5}$ units.

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