12 **R**

Register No. ________

Time : 3.00 Hrs.

Quarterly Examination - 2023
MATHEMATICS

Marks : 90

PART - A

Answer all the following questions.
Choose the most appropriate answer from the given four alternatives. **20 x 1 = 20**

1. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ then $9I_2 - A =$ a) A^{-1} b) $\frac{A^{-1}}{2}$ c) $3A^{-1}$ d) $2A^{-1}$
2. If A and B are orthogonal matrix then $(AB)^T(AB)$ is a) A b) B c) I d) AB
3. If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ and $A(\text{adj } A) = \begin{pmatrix} K & O \\ O & K \end{pmatrix}$ then K = a) 0 b) $\sin\theta$ c) $\cos\theta$ d) 1
4. If A, B and C are invertible matrices of some order, then which one of the following is not true?
   a) $\text{adj}A = |A| A^{-1}$ b) $\text{adj}(AB) = (\text{adj}A)(\text{adj}B)$ c) $\det A^{-1} = (\det A)^{-1}$ d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$
5. If $z = x + iy$ then $\text{Re}(1/z)$ is a) x b) y c) $\frac{x}{x^2+y^2}$ d) $\frac{y}{x^2+y^2}$
6. The value of $|i|$ is a) ± 1 b) $\pm i$ d) -1 d) 1
7. If $|z| = 1$ then the value of $\frac{1+z}{1+\bar{z}}$ is a) z b) $\bar{z}$ c) $1/z$ d) 1
8. The zero of $x^3 + 64$ is a) 0 b) 4 c) 4i d) -4
9. If the sum of the co-efficient of the polynomial is zero then one of its root is a) -1 b) 1 c) 0 d) none of the above
10. The number of positive zeros of the polynomial $\sum_{r=0}^{n} nC_r(-1)^r x^r$ is a) 0 b) n c) $< n$ d) r
11. The value of $\tan^{-1}x + \cot^{-1}x$ is a) $\pi/4$ b) $\pi/2$ c) $\pi/6$ d) ∞
12. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is a) [1, 2] b) [−1, 1] c) [0, 1] d) [−1, 0]
13. If $x = 1/5$ the value of $\cos(\cos^{-1}x + 2\sin^{-1}x)$ is a) $-\sqrt{\frac{24}{25}}$ b) $\sqrt{\frac{24}{25}}$ c) $\frac{1}{5}$ d) $\frac{-1}{5}$
14. The radius of the circle passing through the point (6, 2) two of whose diameter are $x + y = 6$ and $x + 2y = 4$
    a) 10 b) $2\sqrt{5}$ c) 6 d) 4
15. The condition for the line $y = mx + c$ to be a tangent to the parabola $y^2 = 4ax$ is
    a) $c^2 = a^2(1 + m^2)$ b) $c = \frac{a}{m}$ c) $c^2 = a^2$ d) $c = \frac{m}{a}$
16. The eccentricity of an ellipse $(x-3)^2 + (y-4)^2 = \frac{y^2}{9}$ is a) $\frac{\sqrt{3}}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{3\sqrt{2}}$ d) $\frac{1}{\sqrt{3}}$
17. The distance of the plane $3x - 6y + 2z + 7 = 0$ from the origin is a) 0 b) 1 c) 2 d) 3
18. If $\vec{a}$ and $\vec{b}$ are parallel vectors then the value of $[\vec{a}\ \vec{b}\ \vec{c}]$ is a) 2 b) -1 c) 1 d) 0
19. If $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ $\vec{b} = \hat{i}+\hat{j}$ $\vec{c} = \hat{i}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda\vec{a} + \mu\vec{b}$ then the value of $\lambda + \mu$ is a) 0 b) 1 c) 6 d) 3
20. The angle between the line $\vec{r} = (\hat{i}+2\hat{j}-3\hat{k}) + t(2\hat{i}+\hat{j}-2\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}+\hat{j}) + 4 = 0$ is
    a) 0^0 b) 30^0 c) 45^0 d) 90^0

PART - B

Answer any 7 questions. Question number 30 is compulsory. **7 x 2 = 14**

21. Is it possible to find the inverse of the matrix $A = \begin{pmatrix} 2 & 3 \\ 6 & 9 \end{pmatrix}$? Give reason
22. Find the rank of $A = \begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$ by minor method.
23. Simplify : $i^{59} + \frac{1}{i^{59}}$
24. Find the centre and radius of the circle $|3z-5+i| = 4$

25. If α, β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$. Find the value of $\sum \frac{1}{\beta\gamma}$

26. Find the principal value of $\sin^{-1}(2)$ if it exists

27. If $\cot^{-1}\left(\frac{1}{7}\right) = \theta$ find the value of $\cos\theta$

28. Find the vaue of c if $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$

29. If the vectors $2\hat{i} - \hat{j} + 3\hat{k}$, $3\hat{i} + 2\hat{j} + \hat{k}$ and $\hat{i} + m\hat{j} + 4\hat{k}$ are coplanar find the value of m.

30. Find the acute angle between the planes $\vec{r} \cdot (2\hat{i} + 2\hat{j} + 2\hat{k}) = 11$ and $4x - 2y + 2z = 15$

PART - C

Answer any 7 questions. Q.No.40 is compulsory. **7 x 3 = 21**

31. Test for consistency of the given system of equation $2x + 2y + z = 5$ $x - y + z = 1$ $3x + y + 2z = 4$.

32. If $adj(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$ find A^{-1}.

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33. If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$ find the complex number z in the rectangular form.

34. Show that $(2 + i\sqrt{3})^{10} - (2 - i\sqrt{3})^{10}$ is purely imaginary.

35. Find the sum of the squares of the roots of $ax^4 + bx^3 + cx^2 + dx + e = 0$, $a \neq 0$

36. Discuss the maximum possible number of positive and negative roots of the polynomial equation. $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$

37. Obtain the equation of the circle for which (3, 4) and (2, –7) are the end points of a diameter.

38. Find the equations of tangent and normal to the ellipse $x^2 + 4y^2 = 32$ when $\theta = \pi/4$

39. Find the vector equation in parametric form and cartesian equations of the line passing through (–4, 2, –3) and is parallel to the line $\frac{-x-2}{4} = \frac{y+3}{-2} = \frac{2z-6}{3}$

40. Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$

PART - IV

Answer all the following questions. **7 x 5 = 35**

41. a) Solve the system of equation $x + y + z = 2$ $6x - 4y + 5z - 31 = 0$, $5x + 2y + 2z = 13$ by matrix inversion method. **(OR)** b) Solve : $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

42. a) Solve the equation $z^3 + 8i = 0$ where $z \in C$ **(OR)** b) Solve the equation $(x - 2)(x - 7)(x - 3)(x + 2) + 19 = 0$

43. a) Solve the following system of equation $x + 2y + 3z = 0$ $3x + 4y + 4z = 0$ $7x + 10y + 12z = 0$ **(OR)**
b) Solve : $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

44. a) If $z = x + iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$ then show that $x^2 + y^2 = 1$ **(OR)**
b) Prove by Vector method $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$

45. a) Show that the lines $\frac{x-3}{3} = \frac{y-3}{-1}$, $z - 1 = 0$ and $\frac{x-6}{2} = \frac{z-1}{3}$, $y - 2 = 0$ are intersecting lines and find the point of intersection. **(OR)**
b) Find the parametric vector and cartesian equation of the plane through the points (2, 2, 1) (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$

46. a) Find the centre, foci, vertices and directrices of the conic $18x^2 + 12y^2 - 144x + 48y + 120 = 0$ **(OR)**
b) If the normal at the point t_1 on the parabola $y^2 = 4ax$ meets the parabola again at the point t_2 then prove that $t_2 = -(t_1 + 2/t_1)$

47. a) On lighting the rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection. **(OR)**
b) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $0 < x, y, z < 1$ then show that $x^2 + y^2 + z^2 + 2xyz = 1$

10 x 2 = 20

ANSWER ALL THE QUESTIONS

21) If A is symmetric, prove that then adj A is also symmetric.

22) Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

23) If adj(A) = $\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}.

24) Find adj(adj (A)) if adj A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$.

25) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it to a row-echelon form.

26) Find the rank of the following matrices by minor method:
$\begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$

27) For any 2 × 2 matrix, if A (adj A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ then find |A|.

28) If A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & c \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$ then find a and c.

29) If A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 2 & 5 \end{bmatrix}$, find the adj (AB)

30) State and prove reversal law

10 x 3 = 30

ANSWER ALL THE QUESTIONS

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31) If A = $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, verify that A(adj A) = (adj A)A = |A| I_3.

32) Verify the property $(A^T)^{-1} = (A^{-1})^T$ with A = $\begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$.

33) If F(α) = $\begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$.

34) Decrypt the received encoded message $\begin{bmatrix} 2 & -3 \end{bmatrix}\begin{bmatrix} 20 & 4 \end{bmatrix}$ with the encryption matrix $\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$ and the decryption matrix as its inverse, where the system of codes are described by the numbers 1 - 26 to the letters A - Z respectively, and the number 0 to a blank space.

35) Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form.

36) Find the inverse of A = $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method.

37) Find the condition on a, b and c so that the following system of linear equations has one parameter family of solutions: x + y + z = a, x + 2y + 3z = b, 3x + 5y + 7z = c.

38) Solve the following system of homogenous equations.
3x + 2y + 7z = 0, 4x − 3y − 2z = 0, 5x + 9y + 23z = 0

39) Solve the following systems of linear equations by Cramer's rule:
$\frac{3}{x} + 2y = 12$, $\frac{2}{x} + 3y = 13$

40) Use Cramer's Rule to solve : 4x - 7y = 16, 2x - 9y = -7

10 x 5 = 50

ANSWER ALL THE QUESTIONS

41) (a) If A = $\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x + y + 2z = 1$, $3x + 2y + z = 7$, $2x + y + 3z = 2$.

42) Four men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

43) In a T20 match, a team needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y = ax^2 + bx + c$ with respect to a xy-coordinate system in the vertical plane and the ball traversed through the points (10, 8), (20, 16) (40, 22) can you conclude that the team won the match?
Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is (70, 0).)

44) A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is Rs. 150. The cost of the two dosai, two idlies and four vadais is Rs. 200. The cost of five dosai, four idlies and two vadais is Rs. 250. The family has Rs. 350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had ?

45) The upward speed v(t)of a rocket at time t is approximated by $v(t) = at^2 + bt + c$, $0 \le t \le 100$ where a, b and c are constants. It has been found that the speed at times t = 3, t = 6, and t = 9 seconds are respectively, 64, 133, and 208 miles per second respectively. Find the speed at time t = 15 seconds. (Use Gaussian elimination method.)

46) If $ax^2 + bx + c$ is divided by $x + 3$, $x - 5$, and $x - 1$, the remainders are 21, 61 and 9 respectively. Find a, b and c. (Use Gaussian elimination method.)

47) A boy is walking along the path $y = ax^2 + bx + c$ through the points (−6, 8), (−2, −12) and (3, 8). He wants to meet his friend at P(7, 60). Will he meet his friend? (Use Gaussian elimination method.)

48) By using Gaussian elimination method, balance the chemical reaction equation :
$C_5H_8 + O_2 \longrightarrow CO_2 + H_2O$.

49) Determine the values of λ for which the following system of equations $x + y + 3z = 0$, $4x + 3y + \lambda z = 0$, $2x + y + 2z = 0$ has
(i) a unique solution
(ii) a non-trivial solution

50) Solve the following systems of linear equations by Cramer's rule:
$\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0$, $\frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0$, $\frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$