SPECTRUM MHSS KATHIRKANTHAM

QUARTERLY EXAM - 2023

MATHS ANSWER - KEY

PART - A

1) b) $\frac{1}{9}$

2) b) $(A^T)^2$

3) b) consistent

4) d) $\sqrt{5}+2$

5) a) $\text{cis}\,\frac{2\pi}{3}$

6) b) $Re(z) = \frac{z-\bar{z}}{2}$

7) c) n imaginary roots

8) b) n

9) a) $|\alpha| \le \frac{1}{\sqrt{2}}$

10) c) 0

11) b) $\pi - \cos^{-1}x$

12) b) $2(a^2+b^2)$

13) b) $\frac{1}{3}$

14) b) $y^2 = -4\sqrt{2}\,x$

15) b) $\vec{b}$

16) b) parallel

17) a) $2\sqrt{3}$

18) c) $\frac{\sqrt{3}}{12}$

19) c) $\frac{\pi}{2}$

20) c) $\frac{1}{e^2}$

PART - B

21) sol: $|adj\,A| = \begin{vmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$

$= 9$

$A^{-1} = \pm \frac{1}{\sqrt{|adj\,A|}}\, adj\,A$

$= \pm \frac{1}{\sqrt{9}} \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$A^{-1} = \pm \frac{1}{3} \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

22) sol:

$2x+5y = 7$ — (1)

$6x+15y = 13$ — (2)

$A = \begin{pmatrix} 2 & 5 \\ 6 & 15 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$, $B = \begin{pmatrix} 7 \\ 13 \end{pmatrix}$

$[A|B] = \begin{bmatrix} 2 & 5 & 7 \\ 6 & 15 & 13 \end{bmatrix}$

$\pm$

22) $2x+5y+7=0$ — (1)

$6x+15y-13=0$ — (2)

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{6} = \frac{5}{15} = \frac{7}{-13}$

$\Rightarrow \frac{1}{3} = \frac{1}{3} \neq \frac{7}{13}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

∴ The linear equation is inconsistent

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23) $z = (2+3i)(1-i)$

$= 2-2i+3i+3$

$z = 5+i$

$z^{-1} = \frac{1}{z}$

$z^{-1} = \frac{5}{26} - i\frac{1}{26}$

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24) one root $= 3+2i$

Another root $= 3-2i$

sum of root $= 6$

Product of root $= 13$

Equation of polynomial

$x^2-6x+13=0$

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25) $y = 4\sin(-2x)$

$y = A\sin Bx$

Amplitude $= |A| = |4| = 4$

period $= \frac{2\pi}{|B|} = \frac{2\pi}{2} = \pi$

26) $= \tan\left(\tan^{-1}\frac{3x}{5}\right)$

$= \frac{3x}{5}$

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27) $x^2+y^2=16$ — (1)

$a^2=16$

$y = 2\sqrt{2}x + c$ — (2)

$m = 2\sqrt{2}$ $c = c$

$c^2 = a^2(1+m^2)$

$c^2 = 16(1+8)$

$c^2 = 16 \cdot 9$

$c^2 = 144$

$\boxed{c = \pm 12}$

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28) $\vec{r} = (2\vec{i}-\vec{j}+\vec{k}) + t(\vec{i}+2\vec{j}-2\vec{k})$

$\vec{b} = \vec{i}+2\vec{j}-2\vec{k}$

$\vec{n} = 6\vec{i}+3\vec{j}+2\vec{k}$

$\sin\theta = \frac{|\vec{b}\cdot\vec{n}|}{|\vec{b}|\,|\vec{n}|}$

$= \frac{6+6-4}{\sqrt{9}\cdot\sqrt{49}}$

$\sin\theta = \frac{8}{21}$

$\theta = \sin^{-1}\left(\frac{8}{21}\right)$

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29) $f(x) = \left|\frac{1}{x}\right|$ $x \in [-1,1]$

Rolle's theorem is not applicable

Because $f(x)$ is not continuous at $x=0$ and not differentiable.

30) $z = 7-24i$

$|z| = \sqrt{49+576}$

$= \sqrt{625}$

$|z| = 25$

$\sqrt{7-24i} = \pm\left(\sqrt{\frac{25+7}{2}} + i\frac{-24}{|-24|}\sqrt{\frac{25-7}{2}}\right)$

$= \pm\left(\sqrt{\frac{32}{2}} - i\sqrt{\frac{18}{2}}\right)$

$= \pm\left(\sqrt{16} - i\sqrt{9}\right)$

$= \pm(4-3i)$

roots are $4-3i, -4+3i$

PART - C

31) $2x - y = 8$ — ①

$3x+2y = -2$ — ②

$A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

$|A| = 7 \neq 0$ A^{-1} is exist

$\text{adj } A = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$ $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

$X = A^{-1}B$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 8 \\ -2 \end{bmatrix}$

$= \frac{1}{7}\begin{bmatrix} 16-2 \\ -24-4 \end{bmatrix}$

$= \begin{bmatrix} 2 \\ -4 \end{bmatrix}$

$x = 2$ and $\boxed{y = -4}$

32) $A = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & -6 & 2 & -4 \end{bmatrix}$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 5R_1$

$\sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \to R_3 - 2R_2$

$\rho(A) = 2$

33) $= \left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)^{18}$

$= i^{18}\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)^{18}$

$= -1\left(\cos\frac{18\pi}{6} - i\sin\frac{18\pi}{6}\right)$

$= -(\cos 3\pi - i\sin 3\pi)$

$= 1+0i$

$= 1$

34) $|z_1| = 1$

$z_1\bar{z}_1 = 1$

$z_1 = \frac{1}{\bar{z}_1}$

$|||ly$ $z_2 = \frac{1}{\bar{z}_2}$

$\frac{z_1+z_2}{1+z_1 z_2} = \frac{\frac{1}{\bar{z}_1} + \frac{1}{\bar{z}_2}}{1+\frac{1}{\bar{z}_1}\cdot\frac{1}{\bar{z}_2}}$

$= \frac{\bar{z}_1+\bar{z}_2}{1+\bar{z}_1\bar{z}_2}$

$\frac{z_1+z_2}{1+z_1 z_2} = \overline{\left(\frac{z_1+z_2}{1+z_1 z_2}\right)}$ is real

$z = \bar{z}$ is real.

35) $P(x) = x^3 - 5x^2 - 4x + 20$

$P(2) = 8 - 20 - 8 + 20$

$P(2) = 0$

$x = 2$

$x-2$ is a factor

$$\begin{array}{c|cccc} 2 & 1 & -5 & -4 & 20 \\ & 0 & 2 & -6 & -20 \\ \hline & 1 & -3 & -10 & \boxed{0} \end{array}$$

$\therefore P(x) = (x-2)(x^2 - 3x - 10)$

$P(x) = (x-2)(x-5)(x+2)$

$\therefore \boxed{x=2}\ \boxed{x=5}\ \boxed{x=-2}$

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36) $= \sin^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right)$

$= -\sin^{-1}(1) + \cos^{-1} 1/2$

$= -\frac{\pi}{2} + \frac{\pi}{3}$

$= \frac{-\pi}{6}$

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37) $y^2 = 4ax$

Length of latus rectum

$= 4 \cancel{\times}\ 4a$

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38) $x^2 + 3y^2 = 12$ — (1)

$\frac{x^2}{12} + \frac{3y^2}{12} = 1$

$\frac{x^2}{12} + \frac{y^2}{4} = 1$

$a^2 = 12 \quad b^2 = 4$

$x - y + 4 = 0$ — (2)

$\cancel{mx} = -2$

$-y = 4 - x$

$y = x - 4$

$m = 1, \quad c = -4$

$c^2 = a^2m^2 + b^2$

$16 = 12 \cdot 1 + 4$

$\boxed{16 = 16}$

The line tangent to the ellipse.

39) $\begin{vmatrix} 7 & \lambda & -3 \\ 1 & 2 & -1 \\ -3 & 7 & 5 \end{vmatrix} = 90$

$7(17) - \lambda(2) - 3(13) = 90$

$119 - 2\lambda - 39 = 90$

$80 - 2\lambda = 90$

$-2\lambda = 90 - 80$

$-2\lambda = 10$

$\boxed{\lambda = -5}$

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40) $s = 13.8t - 4.9t^2$

$s'(t) = 13.8 - 9.8t$

$s''(t) = -9.8$

$t = 1 \Rightarrow s'(1) = 13.8 - 9.8$

$s'(1) = 4$

$t = 1 \Rightarrow s''(1) = -9.8$

$v = 0 \Rightarrow 13.8 - 9.8t = 0$

$\cancel{-}9.8t = \cancel{-}13.8$

$t = \frac{13.8}{9.8}$

$t = 1.4$

$\therefore s(1.4) = 13.8(1.4) - 4.9(1.4)^2$

$= 19.32 - 9.604$

$= 9.72$

PART - D

41) a)

$$9a+3b+c=64$$
$$36a+6b+c=133$$
$$81a+9b+c=208$$

$$[AB] = \begin{bmatrix} 9 & 3 & 1 & 64 \\ 36 & 1 & 1 & 133 \\ 81 & 9 & 1 & 208 \end{bmatrix}$$

$$\sim \begin{bmatrix} 9 & 3 & 1 & 64 \\ 0 & -6 & -3 & -123 \\ 0 & -18 & -8 & -368 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 4R_1 \\ R_3 \to R_3 - 9R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 9 & 3 & 1 & 64 \\ 0 & 2 & 1 & 41 \\ 0 & 9 & 4 & 184 \end{bmatrix} \begin{matrix} R_2 \to \frac{R_2}{-3} \\ R_3 \to \frac{R_3}{-2} \end{matrix}$$

$$\sim \begin{bmatrix} 9 & 3 & 1 & 64 \\ 0 & 2 & 1 & 41 \\ 0 & 18 & 8 & 368 \end{bmatrix} R_3 \to 2R_3$$

$$\sim \begin{bmatrix} 9 & 3 & 1 & 64 \\ 0 & 2 & 1 & 41 \\ 0 & 0 & -1 & -1 \end{bmatrix} R_3 \to R_3 - 9R_2$$

$$\sim \begin{bmatrix} 9 & 3 & 1 & 64 \\ 0 & 2 & 1 & 41 \\ 0 & 0 & 1 & 1 \end{bmatrix} R_3 \to R_3 - R_2$$

$$9a+3b+c=64$$
$$2b+c=41$$
$$\boxed{c=1}$$
$$\boxed{b=20}$$
$$\boxed{a=\cancel{B}}$$

$$v(t) = \frac{1}{3}t^2+20t+1$$

$$v(15) = 376$$

b)

$$\frac{1+z}{1-z} = e^{i2\theta}$$
$$1+z = e^{i2\theta}(1-z)$$
$$1+z = e^{i2\theta} - ze^{i2\theta}$$
$$ze^{i2\theta}+z = e^{i2\theta}-1$$
$$z(e^{i2\theta}+1) = e^{i2\theta}-1$$
$$z = \frac{e^{i2\theta}-1}{e^{i2\theta}+1}$$
$$z = \frac{e^{i\theta}-e^{-i\theta}}{e^{i\theta}+e^{-i\theta}}$$
$$z = \frac{\cancel{2}i\sin\theta}{\cancel{2}\cos\theta}$$
$$\boxed{z = i\tan\theta}$$

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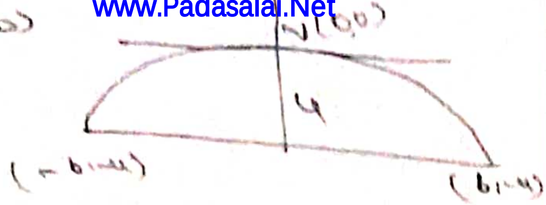
42) a)

$$\cos^{-1}x = \alpha \Rightarrow x = \cos\alpha$$
$$\cos^{-1}y = \beta \Rightarrow y = \cos\beta$$

$$\alpha+\beta = \pi - \cos^{-1}z$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$= xy - \sqrt{1-x^2}\cdot\sqrt{1-y^2}$$
$$\cos(\pi-\cos^{-1}z) = xy - \sqrt{1-x^2}\cdot\sqrt{1-y^2}$$
$$-\cos(\cos^{-1}z) = xy - \sqrt{1-x^2}\cdot\sqrt{1-y^2}$$
$$-z = xy - \sqrt{1-x^2}\cdot\sqrt{1-y^2}$$
$$-z-xy = -\sqrt{1-x^2}\sqrt{1-y^2}$$
$$z+xy = \sqrt{1-x^2}\cdot\sqrt{1-y^2}$$
$$\boxed{z^2+x^2+y^2+2xyz=1}$$

42) b)



Equation : $x^2 = -4ay$ — ①

passing through (6,-4)

$36 = -4a(-4)$

$\frac{36}{4} = 4a$

$\boxed{4a = 9}$

$\therefore x^2 = -9y$ — ②

diff w.r.to x

$2x = -9 \cdot \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{-2x}{9}$

$\left[\frac{dy}{dx}\right]_{(6,-4)} = \frac{-2(-6)}{9}$

$= \frac{\cancel{12}\,4}{\cancel{9}\,3}$

$m = \frac{4}{3}$

$\therefore \tan\theta = \frac{4}{3}$

$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

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43) (a) $z^3 = (\sqrt{3} + i)$

$z = (\sqrt{3} + i)^{1/3}$

$r = 2 \qquad \theta = \frac{\pi}{6}$

$z^3 = r(\cos\theta + i\sin\theta)$

$z = (2)^{1/3}\left(\cos\frac{\pi + 12k\pi}{18} + i\sin\frac{\pi + 12k\pi}{18}\right)$

$k = 0, 1, 2.$

$k = 0 \quad z = 2^{1/3} \operatorname{cis} \frac{\pi}{18}$

$k = 1 \quad z = 2^{1/3} \operatorname{cis} \frac{13\pi}{18}$

$k = 2 \quad z = 2^{1/3} \operatorname{cis} \frac{25\pi}{18}$

(b) $y^2 + 8x - 6y + 1 = 0$

$y^2 - 6y = -8x - 1$

$(y-3)^2 - 9 = -8x - 1$

$(y-3)^2 = -8x - 1 + 9$

$(y-3)^2 = -8x + 8$

$(y-3)^2 = -8(x-1)$

$y - 3 = -4(2)(x-1)$

The parabola open L.W

$Y = y - 3 \quad X = x - 1 \quad a = 2$

Details	Refer to X, Y	Refer to $X = x-1$ $Y = y-3$
Vertex	(0,0)	$x-1=0$, $x=1$ / $y-3=0$, $y=3$ V(1,3)
Focus	(-a,0)	$x-1=-2$, $x=-1$ / $y-3=0$, $y=3$ F(-1,3)
~~Vertex~~		

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44) a) $2x + 3y + 5z = 9$ — ①

$7x + 3y - 5z = 8$ — ②

$2x + 3y + \lambda z = \mu$ — ③

$A = \begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -5 \\ 2 & 3 & \lambda \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 9 \\ 8 \\ \mu \end{bmatrix}$

$AB = \begin{bmatrix} 2 & 3 & 5 & 9 \\ 7 & 3 & -5 & 8 \\ 2 & 3 & \lambda & \mu \end{bmatrix}$

$\sim \begin{bmatrix} 7 & 3 & -5 & 8 \\ 2 & 3 & 5 & 9 \\ 2 & 3 & \lambda & \mu \end{bmatrix} \quad R_1 \leftrightarrow R_2$

$$\sim \left[\begin{array}{cccc|c} 7 & 3 & -5 & -8 \\ 0 & \frac{15}{7} & \frac{45}{7} & \frac{47}{7} \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right] \begin{array}{l} R_2 \to R_2 - \frac{2}{7}R_1 \\ R_3 \to R_3 - R_2 \end{array}$$

$$\sim \left[\begin{array}{cccc} 7 & 3 & -5 & -8 \\ 0 & 15 & 45 & 47 \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right] R_2 \to R_2 \times 7$$

case (i): $\lambda = 5$

$\rho(A) = 2 \quad \rho[A|B] = 3$

$\rho(A) \neq \rho[A|B]$

Inconsistent no solution

case (ii): $\lambda \neq 5, \mu \neq 9$

$\rho(A) = 3 \quad \rho[A|B] = 3$

$\rho(A) = \rho[A|B] = n$

consistent unique solution

case (iii): $\lambda = 5 \quad \mu = 9$

$\rho(A) = 2 \quad \rho[A|B] = 2$

$\rho(A) = \rho[A|B] = 2 < n$

consistent, Infinitely many solution

(b) $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

$6(x^2 + 1/x^2) - 35(x + 1/x) + 62 = 0$

$6(y^2 - 2) - 35y + 62 = 0$

$6y^2 - 35y + 50 = 0$

$(3y - 10)(2y - 5) = 0$

case (i): $3y - 10 = 0$

$\frac{x^2+1}{x} = \frac{10}{3}$

$3x^2 - 10x + 3 = 0$

$(x-3)(3x-1) = 0$

$x = 3, \quad x = 1/3$

case (ii) $2y - 5 = 0$

$x + \frac{1}{x} = \frac{5}{2}$

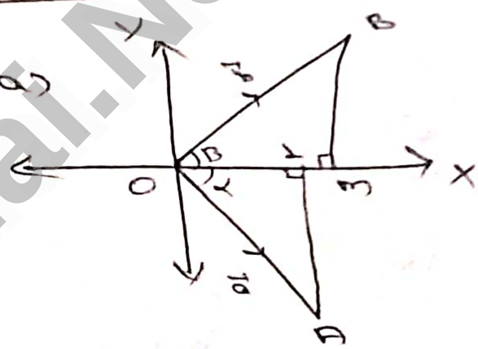
$2x^2 - 5x + 2 = 0$

$(x-2)(2x-1) = 0$

$x = 2, \quad x = 1/2$

Hence roots are $2, 3, 1/2, 1/3$.

45) a)



Let $\vec{a} = \vec{OA}$, $\vec{b} = \vec{OB}$ be unit vectors & make an angles α, β

$\vec{OL} = \vec{OA} \cos\alpha \hat{i}$

$\vec{LA} = \vec{OA} \sin\alpha \hat{j}$

$\hat{a} = \vec{OL} + \vec{LA} = \cos\alpha \hat{i} - \sin\alpha \hat{j}$

$\vec{b} = \vec{OM} + \vec{MB} = \cos\beta \hat{i} + \sin\beta \hat{j}$

$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin(\alpha+\beta) \hat{k}$

$= \sin(\alpha+\beta)\hat{k}$ — (1)

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\alpha & -\sin\alpha & 0 \\ \cos\beta & \sin\beta & 0 \end{vmatrix}$$

$= \hat{k}(\cos\alpha\sin\beta + \sin\alpha\cos\beta)$ — (2)

(1) = (2)

$\sin(\alpha+\beta) = \cos\alpha\sin\beta + \sin\alpha\cos\beta$

b) $y = (\sin x)^{\tan x}$

$\log y = \tan x \cdot \log \sin x$

$\log y = \dfrac{\log \sin x}{\cot x}$

$\lim_{x \to \frac{\pi}{2}} \log y = \lim_{x \to \frac{\pi}{2}} \dfrac{\log \sin x}{\cot x} = \dfrac{0}{0}$

$= \lim_{x \to \frac{\pi}{2}} \dfrac{\frac{1}{\sin x} \cdot \cos x}{-\operatorname{cosec}^2 x}$

$= \lim_{x \to \frac{\pi}{2}} -\cos x \cdot \sin x = 0$

$\therefore \lim_{x \to \frac{\pi}{2}} \log y = 0$

$\log \lim_{x \to \frac{\pi}{2}} y = e^0$

$\lim_{x \to \frac{\pi}{2}} (\sin x)^{\tan x} = 1$

x

46) (a) $\vec{a} = 2\vec{i} + 2\vec{j} + \vec{k}$

$\vec{b} = 2\vec{i} + 3\vec{j} + 3\vec{k}$

$\vec{c} = 3\vec{i} + 2\vec{j} + \vec{k}$

parametric form

$\vec{r} = \vec{a} + s\vec{b} + t\vec{c}$

$\vec{r} = 2\vec{i} + 2\vec{j} + \vec{k} + s(2\vec{i} + 3\vec{j} + 3\vec{k}) + t(3\vec{i} + 2\vec{j} + \vec{k})$

cartesian equation:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} x - 2 & y - 2 & z - 1 \\ 2 & 3 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0$$

$(x-2)(-3) - (y-2)(-7) + (z-1)(-5) = 0$

$-3x + 6 + 7y - 14 - 5z + 5 = 0$

$-3x + 7y - 5z - 3 = 0$

$3x - 7y + 5z + 3 = 0$

x

(b) $y = x^2$ — ①

$y = (x-3)^2$ — ②

① = ② $\quad x^2 = (x-3)^2$

$\cancel{x^2} = \cancel{x^2} + 9 - 6x$

$6x = 9$

$x = \frac{3}{2}$

$\therefore y = \frac{9}{4} \quad P(3/2, 9/4)$

from ① $y = x^2$

$\frac{dy}{dx} = 2x$

$\boxed{m_1 = 3}$

from ② $y = (x-3)^2$

$\frac{dy}{dx} = 2(x-3)$

$\boxed{m_2 = -3}$

$\tan\theta = \left|\dfrac{3-(-3)}{1-9}\right|$

$= \frac{3}{4}$

$\theta = \tan^{-1}\left(\frac{3}{4}\right)$

x

47 (a) $(x-4)(x-2)(x-7)(x+1) = 16$

$(x^2 - 6x + 8)(x^2 - 6x - 7) = 16$

put $y = x^2 - 6x$

$(y+8)(y-7) = 16$

$y^2 + y - 72 = 0$

$(y+9)(y-8) = 0$

case (i): $y + 9 = 0$

$y = -9$

$x^2 - 6x + 9 = 0$