ST. ANNE'S ACADEMY
(MATHS & PHYSICS TUITION CENTRE)
I FLOOR, JAFRO DENTAL CLINIC, HOLY CROSS COLLEGE ROAD, PUNNAI NAGAR, NAGERCOIL – 629004

Question Difficulty Level
★★★☆☆

Common Quarterly Exam Sep 2023 – Model Question Paper
CLASS – XII - MATHEMATICS

Time Allowed : 3 Hrs | Maximum Marks : 90

PART – I

I. Answer ALL questions. **20x1 = 20**

1) If $A=\begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$, then $|A^{-1}|$ =

(1) A^{-1} (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) $2A^{-1}$

2) The eccentricity of the circle, $(x-h)^2+(y-k)^2=a^2$ where, centre (h,k) and radius a *is*:

(1) 0 (2) -1 (3) 1 (4) >1

3) If $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$, then the value of $[\vec{a},\vec{b},\vec{c}]$ is

(1) $|\vec{a}|\,|\vec{b}|\,|\vec{c}|$ (2) $\frac{1}{3}|\vec{a}|\,|\vec{b}|\,|\vec{c}|$ (3) 1 (4) -1

4) If $|\text{adj}(\text{adj } A)|=|A|^{16}$, then the order of the square matrix A is

(1) 3 (2) 4 (3) 2 (4) 5

5) The equation of the Auxiliary circle to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ is

(1) $x^2+y^2=16$ (2) $x^2+y^2-6y+7=0$

(3) $x^2+y^2=9$ (4) $x^2+y^2-6y+5=0$

6) If $\omega\neq 1$ is a cubic root of unity then, $(1+\omega-\omega^2)^7$ equals

(1) $-\omega^2$ (2) $-128\,\omega^2$ (3) -128 (4) 128

7) If α,β,γ are the angles made by a straight line with the coordinate axes then the value of $\cos 2\alpha+\cos 2\beta+\cos 2\gamma$ equals to.

(1) -2 (2) $\frac{\pi}{3}$ (3) -1 (4) $\frac{\pi}{4}$

8) The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2}, z=2$ and $\frac{x-1}{1}=\frac{2y+3}{3}=\frac{z+5}{2}$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

9) If $x+y=k$ is a normal to the parabola $y^2=12x$, then the value of k is

(1) 3 (2) -1 (3) 1 (4) 9

10) If $\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3}$; then $\cos^{-1}x+\cos^{-1}y$ is equal to

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{6}$ (4) π

11) The equation whose roots are opposite in sign to those of $x^2-3x-4=0$ is

(1) $3x^2-3x-4=0$ (2) $x^2+3x-4=0$

(3) $3x^2+3x-4=0$ (4) $x^2+3x+4=0$

12) The polynomial x^3-kx^2+9x has three real zeros if and only if, k satisfies

(1) $|k|\le 6$ (2) $k=0$ (3) $|k|>6$ (4) $|k|\ge 6$

13) If $(1+i)(1+2i)(1+3i)\cdots(1+ni)=x+iy$, then $2\cdot 5\cdot 10\cdots(1+n^2)$ is

(1) 1 (2) i (3) x^2+y^2 (4) $1+n^2$

14) If $\rho(A)=\rho([A\,|\,B])$, then the system $AX=B$ of linear equations is

(1) consistent and has a unique solution (2) consistent

(3) consistent and has infinitely many solution (4) inconsistent

15) The equation $\tan^{-1}x-\cot^{-1}x=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has

(1) no solution (2) unique solution

(3) two solutions (4) infinite number of solutions

16) If $z=x+iy$ is a complex number such that 3x+(3x-y)i =4-6i then the value of z is

(1) $\frac{4}{3}$+10i (2) 10i (3) 6i (4) 4+6i

17) The radius of the circle $3x^2+by^2+4bx-6by+b^2=0$ is

(1) 1 (2) 3 (3) $\sqrt{10}$ (4) $\sqrt{11}$

18) If $A=\begin{bmatrix}3&-3&4\\2&-3&4\\0&-1&1\end{bmatrix}$, then $A(\text{adj}\,A)$ is

(1) $\begin{bmatrix}3&-3&4\\2&-3&4\\0&-1&1\end{bmatrix}$ (2) $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ (3) $\begin{bmatrix}-3&3&-4\\-2&3&-4\\0&1&-1\end{bmatrix}$ (4) $\begin{bmatrix}3&-3&4\\0&-1&1\\2&-3&4\end{bmatrix}$

ST. ANNE'S ACADEMY
(MATHS & PHYSICS TUITION CENTRE)
I FLOOR, JAFRO DENTAL CLINIC, HOLY CROSS COLLEGE ROAD, PUNNAI NAGAR, NAGERCOIL – 629004

19) The domain of the function defined by $f(x) = \sin^{-1} 2x$ is

(1) $[\frac{1}{2}, \frac{1}{2}]$ (2) $[-\frac{1}{2}, \frac{1}{2}]$ (3) $[0, 1]$ (4) $[-1, 0]$

20) The circle passing through $(1,-2)$ and touching the axis of x at $(3,0)$ passing through the point

(1) $(-5,2)$ (2) $(2,-5)$ (3) $(5,-2)$ (4) $(-2,5)$

PART – II

II. Answer any SEVEN questions. [Question 30 is compulsory] **7x2 = 14**

21) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it to a row-echelon form.

22) Is $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ true? Justify your answer.

23) If $8iz^3 + 12z^2 - 18z + 27i = 0$ find the value of $|z|$

24) If $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$, find c.

25) If $2\hat{i} - \hat{j} + 3\hat{k}, 3\hat{i} + 2\hat{j} + \hat{k}, \hat{i} + m\hat{j} + 4\hat{k}$ are coplanar, find the value of m.

26) Find the angle between the straight line $\vec{r} = (2\hat{i} + 3\hat{j} + \hat{k}) + t(\hat{i} - \hat{j} + \hat{k})$ and the plane $2x - y + z = 5$.

27) Form the quadratic equation whose one of the roots is $\frac{1}{2+\sqrt{5}}$

28) State Rouché-Capelli theorem.

29) Find the value of $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$.

30) Find the point of intersection of the tangents to the parabola $y^2 = 8x$ at $t_1=t$ and $t_2=3t$.

PART – III

III. Answer any SEVEN questions. [Question 40 is compulsory] **7x3 = 21**

31) If $\cos^{-1}p + \cos^{-1}q + \cos^{-1}r = \pi$ then find the value of $p^2 + q^2 + r^2 + 2pqr$

32) If $\frac{1+z}{1-z} = \cos 2\theta + i \sin 2\theta$, show that $z = i \tan\theta$.

33) Determine the values of λ for which the following system of equations
$(3\lambda - 8)x + 3y + 3z = 0,\ 3x + (3\lambda - 8)y + 3z = 0,\ 3x + 3y + (3\lambda - 8)z = 0$
has a non-trivial solution.

34) With usual notations, in any triangle *ABC*, prove by vector method that
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

35) Find the equations of the two tangents that can be drawn at the ends of latus rectum line of the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$

36) Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$.

37) Solve : $2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}} = \frac{b}{a} + \frac{6a}{b}$.

38) The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

39) If $\omega \neq 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3 + 8 = 0$ are $-1, 1-2\omega, 1-2\omega^2$.

40) Find the image of the point whose position vector is $\hat{i} + 2\hat{j} + 3\hat{k}$ in the plane $\vec{r} \cdot (\hat{i} + 2\hat{j} + 4\hat{k}) = 38$. Find also the foot of the perpendicular from the point $\hat{i} + 2\hat{j} + 3\hat{k}$ in the given plane.

PART – IV

IV. Answer ALL questions. **7x5 = 35**

41) a) Find the domain of $\sin^{-1}(2 - 3x^2)$

OR

b) If $ax^2 + bx + c$ is divided by $x+3, x-5$, and $x-1$, the remainders are $21, 61$ and 9 respectively. Find a, b and c. (Use Gaussian elimination method.)

ST. ANNE'S ACADEMY
(MATHS & PHYSICS TUITION CENTRE)
I FLOOR, JAFRO DENTAL CLINIC, HOLY CROSS COLLEGE ROAD, PUNNAI NAGAR, NAGERCOIL – 629004

42) a) Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point $(4,3,2)$ to the plane $x+2y+3z=2$.

OR

b) Solve the equation $z^3+27i=0$

43) a) If the roots of $x^3+px^2+qx+r=0$ are in H.P., prove that $9pqr=27r^2+2q^3$. Assume $p, q, r\neq 0$

OR

b) Find the centre, foci, and eccentricity of the hyperbola $11x^2-25y^2-44x+50y-256=0$

44) a) By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8+O_2\rightarrow CO_2+H_2O.$

OR

b) Find the non-parametric form of vector equation and cartesian equation of the plane passing through the point $(1,-2,4)$ and perpendicular to the plane $x+2y-3z=11$ and parallel to the line $\frac{x+7}{3}=\frac{y+3}{-1}=\frac{z}{1}$.

45) a) If $a_1, a_2, a_3, \ldots a_n$ is an arithmetic progression with common difference d, prove that $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right]=\frac{a_n-a_1}{1+a_1a_n}$.

OR

b) If $z=x+iy$ and $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2}$, show that $x^2+y^2=1$.

46) a) Solve : $(x-5)(x-7)(x+6)(x+4)=504$

OR

b) Find the equation of the circle described on the chord $3x+y+5=0$ of the circle $x^2+y^2=16$ as diameter.

47) a) Two coast guard stations are located 600 km apart at points $A(0,0)$ and $B(0,600)$ A distress signal from a ship at P is received at slightly different times by two stations. It is determined that the ship is 200 km farther from station A than it is from station B. Determine the equation of hyperbola that passes through the location of the ship.

OR

b) For any three vectors $\vec{a},\vec{b},\vec{c}$ find the value of $[\vec{a}\times(\vec{b}\times\vec{c}),\ \vec{b}\times(\vec{c}\times\vec{a}),\ \vec{c}\times(\vec{b}\times\vec{a})]$ if $\vec{a}\cdot\vec{b}=1,\ \vec{b}\cdot\vec{c}=2,\ \vec{c}\cdot\vec{a}=3$.