# ST. ANNE'S ACADEMY
(MATHS & PHYSICS TUITION CENTRE)

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Common Quarterly Exam Sep 2023 (Model Exam Question Paper)

CLASS – XII - MATHEMATICS

Time Allowed : 3 Hrs Maximum Marks : 90

## PART – I

**I. Answer ALL questions.** **20x1 = 20**

1) The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2}, z=2$ and $\frac{x-1}{1}=\frac{2y+3}{3}=\frac{z+5}{2}$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

2) $\sin^{-1}(\cos x)=\frac{\pi}{2}-x$ is valid for

(1) $-\pi\le x\le 0$ (2) $0\le x\le\pi$ (3) $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$ (4) $-\frac{\pi}{4}\le x\le\frac{3\pi}{4}$

3) The area of quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ is

(1) $4(a^2+b^2)$ (2) $2(a^2+b^2)$ (3) $a^2+b^2$ (4) $\frac{1}{2}(a^2+b^2)$

4) The principal argument of the complex number $\frac{\left(1+i\sqrt{3}\right)^2}{4i\left(1-i\sqrt{3}\right)}$ is

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\frac{5\pi}{6}$ (4) $\frac{\pi}{2}$

5) If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ and $A(\text{adj } A)=\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}$, then $k=$

(1) 0 (2) $\sin\theta$ (3) $\cos\theta$ (4) 1

6) The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ having centre at $(0,3)$ is

(1) $x^2+y^2-6y-7=0$ (2) $x^2+y^2-6y+7=0$

(3) $x^2+y^2-6y-5=0$ (4) $x^2+y^2-6y+5=0$

7) If $\vec{a},\vec{b},\vec{c}$ are three non-coplanar unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\vec{b}+\vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(1) $\frac{\pi}{2}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{4}$ (4) $\pi$

8) According to the rational root theorem, which number is not possible rational zero of $4x^7+2x^4-10x^3-5$?

(1) $-1$ (2) $\frac{5}{4}$ (3) $\frac{4}{5}$ (4) 5

9) The radius of the circle $3x^2+by^2+4bx-6by+b^2=0$ is

(1) 1 (2) 3 (3) $\sqrt{10}$ (4) $\sqrt{11}$

10) $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)$ is equal to

(1) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (2) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ (3) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ (4) $\tan^{-1}\left(\frac{1}{2}\right)$

11) If $[\vec{a},\vec{b},\vec{c}]=1$, then the value of $\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a})\cdot\vec{b}}+\frac{\vec{b}\cdot(\vec{c}\times\vec{a})}{(\vec{a}\times\vec{b})\cdot\vec{c}}+\frac{\vec{c}\cdot(\vec{a}\times\vec{b})}{(\vec{c}\times\vec{b})\cdot\vec{a}}$ is

(1) 1 (2) $-1$ (3) 2 (4) 3

12) If $\cot^{-1}\left(\sqrt{\sin\alpha}\right)+\tan^{-1}\left(\sqrt{\sin\alpha}\right)=u$, then $\cos 2u$ is equal to

(1) $\tan^2\alpha$ (2) 0 (3) $-1$ (4) $\tan 2\alpha$

13) The eccentricity of the ellipse $(x-3)^2+(y-4)^2=\frac{y^2}{9}$ is

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{3\sqrt{2}}$ (4) $\frac{1}{\sqrt{3}}$

14) The number of positive zeros of the polynomial $\sum_{j=0}^{n}{}^{n}C_r(-1)^r x^r$ is

(1) 0 (2) $n$ (3) $< n$ (4) $r$

15) If $z=x+iy$ is a complex number such that $|z+2|=|z-2|$, then the locus of $z$ is

(1) real axis (2) imaginary axis (3) ellipse (4) circle

16) If $z=\frac{\left(\sqrt{3}+i\right)^3(3i+4)^2}{(8+6i)^2}$, then $|z|$ is equal to

(1) 0 (2) 1 (3) 2 (4) 3

17) If $A=\begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$, then $9I_2-A=$

(1) $A^{-1}$ (2) $\frac{A^{-1}}{2}$ (3) $3A^{-1}$ (4) $2A^{-1}$

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18) The distance between the planes $x+2y+3z+7=0$ and $2x+4y+6z+7=0$ is

(1) $\frac{\sqrt{7}}{2\sqrt{2}}$ (2) $\frac{7}{2}$ (3) $\frac{\sqrt{7}}{2}$ (4) $\frac{7}{2\sqrt{2}}$

19) If $(AB)^{-1}=\begin{bmatrix}12 & -17\\ -19 & 27\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}1 & -1\\ -2 & 3\end{bmatrix}$, then $B^{-1}=$

(1) $\begin{bmatrix}2 & -5\\ -3 & 8\end{bmatrix}$ (2) $\begin{bmatrix}8 & 5\\ 3 & 2\end{bmatrix}$ (3) $\begin{bmatrix}3 & 1\\ 2 & 1\end{bmatrix}$ (4) $\begin{bmatrix}8 & -5\\ -3 & 2\end{bmatrix}$

20) The solution of the equation $|z|-z=1+2i$ is

(1) $\frac{3}{2}-2i$ (2) $-\frac{3}{2}+2i$ (3) $2-\frac{3}{2}i$ (4) $2+\frac{3}{2}i$

**PART – II**

**II. Answer any SEVEN questions. Question 30 is compulsory** **7x2 = 14**

21) If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{c}=3\hat{i}+2\hat{j}+\hat{k}$ and $\vec{a}\times(\vec{b}\times\vec{c})=l\vec{a}+m\vec{b}+n\vec{c}$, find the values of $l,m,n$ .

22) Find the matrix $A$ for which $A\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}=\begin{bmatrix}14 & 7\\ 7 & 7\end{bmatrix}$.

23) Simplify: $ii^2i^3\cdots i^{2000}$

24) Find centre and radius of the following circle.

$$2x^2+2y^2-6x+4y+2=0$$

25) Find the intercepts cut off by the plane $\vec{r}\cdot(6\hat{i}+4\hat{j}-3\hat{k})=12$ on the coordinate axes.

26) Simplify: $\sin^{-1}[\sin 10]$

27) Find the angle between the line $\vec{r}=\left(2\hat{i}-\hat{j}+\hat{k}\right)+t\left(\hat{i}+2\hat{j}-2\hat{k}\right)$ and the plane $\vec{r}\cdot\left(6\hat{i}+3\hat{j}+2\hat{k}\right)=8$

28) Find $z^{-1}$, if $z=(2+3i)(1-i)$.

29) Find the equation of the ellipse with foci $(0,\pm 4)$ and end points of major axis are $(0,\pm 5)$.

30) Prove that the point of intersection of the tangents at '$t_1$' and '$t_2$' on the parabola $y^2 = 4ax$ is $\left[at_1t_2,\ a(t_1+t_2)\right]$.

## PART – III

**III. Answer any SEVEN questions. Question 40 is compulsory** **7x3 = 21**

31) Solve the following system of linear equations, using matrix inversion method: $5x+2y=3,\ 3x+2y=5$.

32) Show that $\left(\frac{19+9i}{5-3i}\right)^{15}-\left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

33) If $k$ is real, discuss the nature of the roots of the polynomial equation $2x^2+kx+k=0$ in terms of $k$.

34) Find the equation of the circle described on the chord $3x+y+5=0$ of the circle $x^2+y^2=16$ as diameter.

35) Find the value of $\tan^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$.

36) Determine the values of $\lambda$ for which the following system of equations $(3\lambda-8)x+3y+3z=0,\ 3x+(3\lambda-8)y+3z=0,\ 3x+3y+(3\lambda-8)z=0$ has a non-trivial solution.

37) A rod of length $1.2m$ moves with its ends always touching the coordinate axes. The locus of a point $P$ on the rod, which is $0.3m$ from the end in contact with $x$-axis is an ellipse. Find the eccentricity.

38) With usual notations, in any triangle $ABC$, prove by vector method that $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$.

39) Find the sum of the squares of the roots of $ax^4+bx^3+cx^2+dx+e = 0,\ a\neq 0$

40) Find the image of the point whose position vector is $\hat{i}+2\hat{j}+3\hat{k}$ in the plane $\vec{r}\cdot\left(\hat{i}+2\hat{j}+4\hat{k}\right)=38$ Find also the foot of the perpendicular drawn from the point $\hat{i}+2\hat{j}+3\hat{k}$

## PART – IV

**IV. Answer ALL questions.** **7x5 = 35**

41) a) Solve the following systems of linear equations by Cramer's rule:

$$\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0,\ \frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0,\ \frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$$

**OR**

b) Find the number of solutions of the equation $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}(3x)$.

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42) a) If the roots of $x^3 + px^2 + qx + r = 0$ are in H.P. , prove that $9pqr = 27r^2 + 2q^3$ . Assume $p, q, r \neq 0$

OR

b) Find all cube roots of $\sqrt{3} + i$.

43) a) Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent

OR

b) Prove that, the sum of the focal distances of any point "P"on the ellipse is equal to length of the major axis.

44) a) Solve: (i) $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1} x$ for $x > 0$.

(ii) $\sin^{-1} x > \cos^{-1} x$

OR

b) Find the parametric form of vector equation of a straight line passing through the point of intersection of the straight lines $\vec{r} = (\hat{i} + 3\hat{j} - \hat{k}) + t(2\hat{i} + 3\hat{j} + 2\hat{k})$ and $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+3}{4}$, and perpendicular to both straight lines.

45) a) Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the three non-collinear points $(3, 6, -2), (-1, -2, 6)$, and $(6, 4, -2)$.

OR

b) If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$, show that

(i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$ and

(ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$.

46) a) Solve the following equation: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

OR

b) Find the equation of the circle passing through the points $(1,1)$, $(2,-1)$, and $(3,2)$.

47) a) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of $4m$ when it is $6m$ away from the point of projection. Finally it reaches the ground $12m$ away from the starting point. Find the angle of projection.

OR

b) By using Gaussian elimination method, balance the chemical reaction equation: $C_2H_6 + O_2 \rightarrow H_2O + CO_2$

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