ST. ANNE'S ACADEMY
(MATHS & PHYSICS TUITION CENTRE)
I FLOOR, JAFRO DENTAL CLINIC, HOLY CROSS COLLEGE ROAD, PUNNAI NAGAR, NAGERCOIL – 629004

Common Quarterly Exam (2023 – 24) – Model Question Paper
CLASS – XII - MATHEMATICS

Time Allowed : 3 Hrs Maximum Marks : 90

PART – I

I. Answer ALL questions. **20x1 = 20**

1) The area of the triangle formed by the complex numbers $z, iz,$ and $z+iz$ in the Argand's diagram is

(1) $\frac{1}{2}|z|^2$ (2) $|z|^2$ (3) $\frac{3}{2}|z|^2$ (4) $2|z|^2$

2) According to the rational root theorem, which number is not possible rational zero of $4x^7+2x^4-10x^3-5$?

(1) -1 (2) $\frac{5}{4}$ (3) $\frac{4}{5}$ (4) 5

3) If A^TA^{-1} is symmetric, then $A^2 =$

(1) A^{-1} (2) $(A^T)^2$ (3) A^T (4) $(A^{-1})^2$

4) If $|\text{adj}(\text{adj } A)| = |A|^{16}$, then the order of the square matrix A is

(1) 3 (2) 4 (3) 2 (4) 5

5) The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

(1) 1 (2) 2 (3) 4 (4) 3

6) If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then x is

(1) 15 (2) 12 (3) 14 (4) 11

7) The principal argument of $\frac{3}{-1+i}$ is

(1) $\frac{-5\pi}{6}$ (2) $\frac{-2\pi}{3}$ (3) $\frac{-3\pi}{4}$ (4) $\frac{-\pi}{2}$

8) If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

(1) z (2) $\bar{z}$ (3) $\frac{1}{z}$ (4) 1

9) The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

(1) $cis\frac{2\pi}{3}$ (2) $cis\frac{4\pi}{3}$ (3) $-cis\frac{2\pi}{3}$ (4) $-cis\frac{4\pi}{3}$

10) The polynomial x^3-kx^2+9x has three real zeros if and only if, k satisfies

(1) $|k|\leq 6$ (2) $k=0$ (3) $|k|>6$ (4) $|k|\geq 6$

11) The circle passing through $(1,-2)$ and touching the axis of x at $(3,0)$ passing through the point

(1) $(-5,2)$ (2) $(2,-5)$ (3) $(5,-2)$ (4) $(-2,5)$

12) If $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-5\hat{k}$, $\vec{c}=3\hat{i}+5\hat{j}-\hat{k}$, then a vector perpendicular to $\vec{a}$ and lies in the plane containing $\vec{b}$ and $\vec{c}$ is

(1) $-17\hat{i}+21\hat{j}-97\hat{k}$ (2) $17\hat{i}+21\hat{j}-123\hat{k}$

(3) $-17\hat{i}-21\hat{j}+97\hat{k}$ (4) $-17\hat{i}-21\hat{j}-97\hat{k}$

13) If the direction cosines of a line are $\frac{1}{c},\frac{1}{c},\frac{1}{c}$, then

(1) $c=\pm 3$ (2) $c=\pm\sqrt{3}$ (3) $c>0$ (4) $0<c<1$

14) If α and β are the roots of $x^2+x+1=0$, then $\alpha^{2020}+\beta^{2020}$ is

(1) -2 (2) -1 (3) 1 (4) 2

15) The principal argument of $(\sin 40^\circ+i\cos 40^\circ)^5$ is

(1) -110° (2) -70° (3) 70° (4) 110°

16) If z is a complex number such that $z\in\mathbb{C}\setminus\mathbb{R}$ and $z+\frac{1}{z}\in\mathbb{R}$, then $|z|$ is

(1) 0 (2) 1 (3) 2 (4) 3

17) Distance from the origin to the plane $3x-6y+2z+7=0$ is

(1) 0 (2) 1 (3) 2 (4) 3

18) If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+\hat{j}$, $\vec{c}=\hat{i}$ and $(\vec{a}\times\vec{b})\times\vec{c}=\lambda\vec{a}+\mu\vec{b}$, then the value of $\lambda+\mu$ is

(1) 0 (2) 1 (3) 6 (4) 3

19) The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2},z=2$ and $\frac{x-1}{1}=\frac{2y+5}{3}=\frac{z+5}{2}$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

20) If $x+y=k$ is a normal to the parabola $y^2=12x$, then the value of k is

(1) 3 (2) -1 (3) 1 (4) 9

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PART – II

II. Answer any SEVEN questions. [Question 30 is compulsory] **7x2 = 14**

21) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it to a row-echelon form.

22) Is $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ true? Justify your answer.

23) Represent the complex number $-1-i$ in polar form.

24) If $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$, find c.

25) If $2\hat{i} - \hat{j} + 3\hat{k},\ 3\hat{i} + 2\hat{j} + \hat{k},\ \hat{i} + m\hat{j} + 4\hat{k}$ are coplanar, find the value of m

26) Find the angle between the straight line $\vec{r} = \left(2\hat{i} + 3\hat{j} + \hat{k}\right) + t\left(\hat{i} - \hat{j} + \hat{k}\right)$ and the plane $2x - y + z = 5$.

27) If α, β, γ, and δ are the roots of the polynomial equation $2x^4 + 5x^3 - 7x^2 + 8 = 0$, find a quadratic equation with integer coefficients whose roots are $\alpha + \beta + \gamma + \delta$ and $\alpha\beta\gamma\delta$.

28) State Rouché-Capelli theorem.

29) Find the value of $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$.

30) Find the point of intersection of the tangents to the parabola $y^2 = 8x$ at $t_1=t$ and $t_2=3t$.

PART – III

III. Answer any SEVEN questions. [Question 40 is compulsory] **7x3 = 21**

31) Sketch the graphs $y = \sin x,\ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$ and $y = \sin^{-1} x,\ -1 \le x \le 1$

32) If $\frac{1+z}{1-z} = \cos 2\theta + i \sin 2\theta$, show that $z = i \tan\theta$.

33) Determine the values of λ for which the following system of equations $(3\lambda - 8)x + 3y + 3z = 0,\ 3x + (3\lambda - 8)y + 3z = 0,\ 3x + 3y + (3\lambda - 8)z = 0$ has a non-trivial solution.

34) With usual notations, in any triangle ABC, prove by vector method that $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$.

35) Find the equations of the two tangents that can be drawn from $(5,2)$ to the ellipse $2x^2+7y^2=14$.

36) Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$.

37) Solve : $2\sqrt{\frac{x}{a}}+3\sqrt{\frac{a}{x}}=\frac{b}{a}+\frac{6a}{b}$.

38) The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

39) If $\omega\neq1$ is a cube root of unity, show that the roots of the equation $(z-1)^3+8=0$ are $-1, 1-2\omega, 1-2\omega^2$.

40) Write the value of: $[\hat{i}+\hat{j}, \hat{j}+\hat{k}, \hat{k}+\hat{i}]$

PART – IV

IV. Answer ALL questions. **7x5 = 35**

41) a) Find the domain of $\sin^{-1}(2-3x^2)$

OR

b) If ax^2+bx+c is divided by $x+3, x-5$, and $x-1$, the remainders are $21, 61$ and 9 respectively. Find a, b and c. (Use Gaussian elimination method.)

42) a) Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point $(4,3,2)$ to the plane $x+2y+3z=2$.

OR

b) Solve the equation $z^3+27i=0$

43) a) If the roots of $x^3+px^2+qx+r=0$ are in H.P., prove that $9pqr=27r^2+2q^3$. Assume $p, q, r\neq0$

OR

b) Find the centre, foci, and eccentricity of the hyperbola $11x^2-25y^2-44x+50y-256=0$

44) a) By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8+O_2\rightarrow CO_2+H_2O$.

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b) Find the non-parametric form of vector equation and cartesian equation of the plane passing through the point $(1,-2,4)$ and perpendicular to the plane $x+2y-3z=11$ and parallel to the line $\frac{x+7}{3}=\frac{y+3}{-1}=\frac{z}{1}$.

45) a) If $a_1, a_2, a_3, \ldots a_n$ is an arithmetic progression with common difference d, prove that $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right]=\frac{a_n-a_1}{1+a_1a_n}$.

OR

b) If $z=x+iy$ and $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2}$, show that $x^2+y^2=1$.

46) a) Solve : $(x-5)(x-7)(x+6)(x+4)=504$

OR

b) Find the equation of the circle described on the chord $3x+y+5=0$ of the circle $x^2+y^2=16$ as diameter.

47) a) Two coast guard stations are located 600 km apart at points $A(0,0)$ and $B(0,600)$. A distress signal from a ship at P is received at slightly different times by two stations. It is determined that the ship is 200 km farther from station A than it is from station B. Determine the equation of hyperbola that passes through the location of the ship.

OR

b) If D is the midpoint of the side BC of a triangle ABC, show by vector method that $|\overrightarrow{AB}|^2+|\overrightarrow{AC}|^2=2(|\overrightarrow{AD}|^2+|\overrightarrow{BD}|^2)$.