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Register No.

Time : 3.00 Hrs.

Quarterly Examination - 2023
MATHEMATICS

Marks : 90

PART - A

Answer all the following questions. **20 x 1 = 20**
Choose the most appropriate answer from the given four alternatives.

1. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ then $9I_2 - A =$ a) A^{-1} b) $\frac{A^{-1}}{2}$ c) $3A^{-1}$ d) $2A^{-1}$
2. If A and B are orthogonal matrix then $(AB)^T (AB)$ is a) A b) B c) I d) AB
3. If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ and $A(\text{adj } A) = \begin{pmatrix} K & O \\ O & K \end{pmatrix}$ then K = a) 0 b) $\sin\theta$ c) $\cos\theta$ d) 1
4. If A, B and C are invertible matrices of some order, then which one of the following is not true?
   a) $\text{adj}A = |A| A^{-1}$ b) $\text{adj}(AB) = (\text{adj}A)(\text{adj}B)$ c) $\det A^{-1} = (\det A)^{-1}$ d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$
5. If $z = x + iy$ then Re(1/z) is a) x b) y c) $\frac{x}{x^2+y^2}$ d) $\frac{y}{x^2+y^2}$
6. The value of $|i|$ is a) ± 1 b) $\pm i$ d) -1 d) 1
7. If $|z| = 1$ then the value of $\frac{1+z}{1+\bar{z}}$ is a) z b) $\bar{z}$ c) 1/z d) 1
8. The zero of $x^3 + 64$ is a) 0 b) 4 c) 4i d) -4
9. If the sum of the co-efficient of the polynomial is zero then one of its root is a) -1 b) 1 c) 0 d) none of the above
10. The number of positive zeros of the polynomial $\sum^{n} nC_r(-1)^r x^r$ is a) 0 b) n c) < n d) r
11. The value of $\tan^{-1}x + \cot^{-1}x$ is a) $\pi/4$ b) $\pi/2$ c) $\pi/6$ d) ∞
12. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is a) [1, 2] b) [–1, 1] c) [0, 1] d) [–1, 0]
13. If x = 1/5 the value of $\cos(\cos^{-1}x + 2\sin^{-1}x)$ is a) $-\sqrt{\frac{24}{25}}$ b) $\sqrt{\frac{24}{25}}$ c) $\frac{1}{5}$ d) $\frac{-1}{5}$
14. The radius of the circle passing through the point (6, 2) two of whose diameter are x + y = 6 and x + 2y = 4
    a) 10 b) $2\sqrt{5}$ c) 6 d) 4
15. The condition for the line y = mx + c to be a tangent to the parabola $y^2 = 4ax$ is
    a) $c^2 = a^2(1 + m^2)$ b) $c = \frac{a}{m}$ c) $c^2 = a^2$ d) $c = \frac{m}{a}$
16. The eccentricity of an ellipse $(x-3)^2 + (y-4)^2 = \frac{y^2}{9}$ is a) $\frac{\sqrt{3}}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{3\sqrt{2}}$ d) $\frac{1}{\sqrt{3}}$
17. The distance of the plane $3x - 6y + 2z + 7 = 0$ from the origin is a) 0 b) 1 c) 2 d) 3
18. If $\vec{a}$ and $\vec{b}$ are parallel vectors then the value of $[\vec{a}\ \vec{b}\ \vec{c}]$ is a) 2 b) -1 c) 1 d) 0
19. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ $\vec{b} = \hat{i} + \hat{j}$ $\vec{c} = \hat{i}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda\vec{a} + \mu\vec{b}$ then the value of $\lambda + \mu$ is a) 0 b) 1 c) 6 d) 3
20. The angle between the line $\vec{r} = (\hat{i} + 2\hat{j} - 3\hat{k}) + t(2\hat{i} + \hat{j} - 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} + \hat{j}) + 4 = 0$ is
    a) 0^0 b) 30^0 c) 45^0 d) 90^0

PART - B

Answer any 7 questions. Question number 30 is compulsory. **7 x 2 = 14**

21. Is it possible to find the inverse of the matrix $A = \begin{pmatrix} 2 & 3 \\ 6 & 9 \end{pmatrix}$? Give reason
22. Find the rank of $A = \begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$ by minor method.
23. Simplify : $i^{58} + \frac{1}{i^{59}}$
24. Find the centre and radius of the circle $|3z - 5 + i| = 4$

25. If α, β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$. Find the value of $\sum \frac{1}{\beta\gamma}$

26. Find the principal value of $\sin^{-1}(2)$ if it exists

27. If $\cot^{-1}\left(\frac{1}{7}\right) = \theta$ find the value of $\cos\theta$

28. Find the vaue of c if $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$

29. If the vectors $2\hat{i} - \hat{j} + 3\hat{k}$, $3\hat{i} + 2\hat{j} + \hat{k}$ and $\hat{i} + m\hat{j} + 4\hat{k}$ are coplanar find the value of m.

30. Find the acute angle between the planes $\vec{r} \cdot (2\hat{i} + 2\hat{j} + 2\hat{k}) = 11$ and $4x - 2y + 2z = 15$

PART - C

Answer any 7 questions. Q.No.40 is compulsory. **7 x 3 = 21**

31. Test for consistency of the given system of equation $2x + 2y + z = 5$ $x - y + z = 1$ $3x + y + 2z = 4$

32. If $adj(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$ find A^{-1}.

33. If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$ find the complex number z in the rectangular form.

34. Show that $(2 + i\sqrt{3})^{10} - (2 - i\sqrt{3})^{10}$ is purely imaginary.

35. Find the sum of the squares of the roots of $ax^5 + bx^4 + cx^3 + dx + e = 0$, $a \neq 0$

36. Discuss the maximum possible number of positive and negative roots of the polynomial equation.
$9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$

37. Obtain the equation of the circle for which (3, 4) and (2, –7) are the end points of a diameter.

38. Find the equations of tangent and normal to the ellipse $x^2 + 4y^2 = 32$ when $\theta = \pi/4$

39. Find the vector equation in parametric form and cartesian equations of the line passing through (–4, 2, –3) and is parallel to the line $\frac{-x-2}{4} = \frac{y+3}{-2} = \frac{2z-6}{3}$

40. Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$

PART - IV

Answer all the following questions. **7 x 5 = 35**

41. a) Solve the system of equation $x + y + z = 2$ $6x - 4y + 5z - 31 = 0$, $5x + 2y + 2z = 13$ by matrix inversion method. **(OR)** b) Solve : $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

42. a) Solve the equation $z^3 + 8i = 0$ where $z \in C$ **(OR)** b) Solve the equation $(x - 2)(x - 7)(x - 3)(x + 2) + 19 = 0$

43. a) Solve the following system of equation $x + 2y + 3z = 0$ $3x + 4y + 4z = 0$ $7x + 10y + 12z = 0$ **(OR)**
b) Solve : $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

44. a) If $z = x + iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$ then show that $x^2 + y^2 = 1$ **(OR)**
b) Prove by vector method $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

45. a) Show that the lines $\frac{x-3}{3} = \frac{y-3}{-1}$, $z - 1 = 0$ and $\frac{x-6}{2} = \frac{z-1}{3}$, $y - 2 = 0$ are intersecting lines and find the point of intersection. **(OR)**
b) Find the parametric vector and cartesian equation of the plane through the points (2, 2, 1) (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$

46. a) Find the centre, foci, vertices and directrices of the conic $18x^2 + 12y^2 - 144x + 48y + 120 = 0$ **(OR)**
b) If the normal at the point t_1 on the parabola $y^2 = 4ax$ meets the parabola again at the point t_2 then prove that $t_2 = -(t_1 + 2/t_1)$

47. a) On lighting the rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection. **(OR)**
b) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $0 < x, y, z < 1$ then show that $x^2 + y^2 + z^2 + 2xyz = 1$