QUARTERLY EXAMINATION - SEP 2023

12th Std. **MATHEMATICS** Register No

Time : 3.00 Hrs MARKS:90

PART – I

Answer all the questions . 20 x 1 = 20

1. If $A=\begin{bmatrix}2 & 0\\ 1 & 5\end{bmatrix}$ and $B=\begin{bmatrix}1 & 4\\ 2 & 0\end{bmatrix}$ then $|adj\,(AB)|$ = ----------
   1) – 40 2) – 80 3) – 60 4) – 20
2. If A^TA^{-1} is symmetric, then A^2 is ----------
   1) A^{-1} 2) $(A^T)^2$ 3) $(A)^T$ 4) $(A^{-1})^2$
3. The rank of the matrix $\begin{bmatrix}1 & 2 & 3 & 4\\ 2 & 4 & 6 & 8\\ -1 & -2 & -3 & -4\end{bmatrix}$ is ----------
   1) 1 2) 2 3) 4 4) 3
4. The product of all four values of $\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is
   (1)-2 (2)-1 (3)1 (4)2
5. The principal argument of $(sin40°+icos40°)^5$ is
   (1)-110^0 (2) -70^0 (3) 70^0 (4) 110^0
6. If α and β are the roots of $x^2+x+1=0$, then $\alpha^{2020}+\beta^{2020}$ is
   (1)-2 (2)-1 (3)1 (4)2
7. If $x^3+12x^2+10ax+1999$ definitely has a positive root, if and only if
   (1)$a\geq 0$ (2) $a>0$ (3) $a<0$ (4) $a\leq 0$
8. The polynomial x^3-kx^2+9x has three real zeros if and only if k satisfies
   (1)$|k|\leq 6$ (2) $k=0$ (3) $|k>6|$ (4)$|k|\geq 6$
9. If $\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3}$; *then* $\cos^{-1}x+\cos^{-1}y$ is equal to .....
   1) $\frac{2\pi}{3}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{6}$ 4) π
10. The domain of the function defined by $f(x)=\sin^{-1}\sqrt{x-1}$ is.....
    1) [1,2] 2) $[-1,1]$ 3) $[0,1]$ 4) $[-1,0]$
11. $\sin^{-1}(2cos^2x-1)+\cos^{-1}(1-2sin^2x)$ =.....
    1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{6}$
12. If $\sin^{-1}\frac{x}{5}+\text{cosec}^{-1}\frac{5}{4}=\frac{\pi}{2}$, then the value of x is .....
    1) 4 2) 5 3) 2 4) 3
13. The radius of the circle passing through the point (6,2) two of whose diameter are $x+y=6$ and $x+2y=4$ is
    (1) 10 (2) 25 (3) 6 (4) 4
14. If $x+y=K$ is a normal to the parabola $y^2=12x$, then the value of k is
    (1) 3 (2) -1 (3) 1 (4) 9
15. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is
    (1) 2ab (2) ab (3)$\sqrt{ab}$ (4) $\frac{a}{b}$

16. If the coordinates at one end of a diameter of the circle $x^2 + y^2 - 8x - 4y + c = 0$ are (11,2) the coordinates of the other end are
(1) (-5,2) (2) (2,-5) (3) (5, −2) (4) (−2,5)

17. The angle between the line $\vec{r} = (\hat{i} + 2\hat{j} - 3\vec{k}) + t(2\hat{i} + \hat{j} - 2\vec{k})$ and the plane $\vec{r}.(\hat{i} + \hat{j}) + 4 = 0$ is
(1) 0^0 (2) 30^0 (3) 45^0 (4) 60^0

18. . The distance between the planes $x + 2y + 3z + 7 = 0$ and $2x + 4y + 6z + 7 = 0$ is
(1) $\frac{\sqrt{7}}{2\sqrt{2}}$ (2) $\frac{7}{2}$ (3) $\frac{\sqrt{7}}{2}$ (4) $\frac{\sqrt{7}}{2\sqrt{2}}$

19. If $\sin^{-1}x + \cot^{-1}\left(\frac{4}{3}\right) = \frac{\pi}{2}$, then x is equal to .....
1) $\frac{4}{5}$ 2) $\frac{3}{5}$ 3) $\frac{3}{4}$ 4) $\frac{5}{4}$

20. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a},\vec{b},\vec{c}] = 2$,then $[\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}]^2$ is equal to
(1) 4 (2) 16 (3) 2 (4) 6

PART – II

Answer any seven questions. Question No.30 is compulsory. **7 x 2 = 14**

21. If $\text{adj}(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}.

22. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

23. For any vector $\vec{a}$, prove that $\hat{i}\times(\vec{a}\times\hat{i}) + \hat{j}\times(\vec{a}\times\hat{j}) + \hat{k}\times(\vec{a}\times\hat{k}) = 2\vec{a}$.

24. If $\omega \neq 1$ is a cube root of unity, show that $(1 - \omega + \omega^2)^6 + (1 + \omega - \omega^2)^6 = 128$

25. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$.

26. Solve the equation: $x^4 - 14x^2 + 45 = 0$.

27. For what values of x, the inequality $\frac{\pi}{2} < \cos^{-1}(3x - 1) < \pi$ holds?

28. If $\cot^{-1}\left(\frac{1}{7}\right) = \theta$, find the value of $\cos\theta$

29. Obtain the equation of the circle for which (3,4) and (2, −7) are the ends of a diameter.

30. obtain the polar form of 1+itanα . where α is an acute angle

PART – III

Answer any Seven questions. Question No.40 is compulsory. 7 x 3 = 21

31. If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

32. Solve the following system of linear equations by Cramer's rule: $\frac{3}{x} + 2y = 12, \frac{2}{x} + 3y = 13$

33. Simplify $\left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)^{18}$.

34. If $|z| = 2$ show that $3 \leq |z + 3 + 4i| \leq 7$

35. Find the length of Latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

36. With usual notations, in any triangle ABC, prove by vector method that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin c}$.

37. Find the angle between the planes $\vec{r}.(\hat{i} + \hat{j} - 2\hat{k}) = 3$ and $2x - 2y + z = 2$.

38. The parabolic communications antenna has a focus at $2m$ distance from the vertex of the antenna. Find the width of the antenna $3m$ from the vertex.

39. Find the value of $\cos^{-1}\left(\cos\frac{\pi}{7}\cos\frac{\pi}{17} - \sin\frac{\pi}{7}\sin\frac{\pi}{17}\right)$.

40. Solve the equation $x^3 - 9x^2 + 26x - 24 = 0$.

PART – IV

Answer all the questions. 7 x 5 = 35

41. **a)** If $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x + y + 2z = 1, 3x + 2y + z = 7, 2x + y + 3z = 2$.

(OR)

b) Find the value of k for which the equations $kx - 2y + z = 1$, $x - 2ky + z = -2, x - 2y + kz = 1$ have

(i)no solution (ii)unique solution (iii)infinitely many solution

42. **a)** If $z = x + iy$ is a complex number such that $Im\left(\frac{2z+1}{iz+1}\right) = 0$, show that

$$2x^2 + 2y^2 + x - 2y = 0.$$

(OR)

b) Find the number of solutions of the equation

$$\tan^{-1}(x - 1) + \tan^{-1}x + \tan^{-1}(x + 1) = \tan^{-1}3x.$$

43.a) Solve the following equation: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

(OR)

b) Solve the equation $z^3 + 8i = 0$, where $z \in \mathbb{C}$.

44.a). If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ and $0 < x, y, z < 1$, then show that $x^2 + y^2 + z^2 + 2xyz = 1$.

(OR)

b) Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact.

45.a) Assume that water issuing from the end of a horizontal pipe, 7.5m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

(OR)

b) Find the condition that the roots of $ax^3 + bx^2 + cx + d = 0$ are in geometric progression. Assume $a, b, c, d \neq 0$.

46. a) Prove by vector method that $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$.

(OR)

b) Show that the lines $\vec{r} = (6\hat{\imath} + \hat{\jmath} + 2\hat{k}) + s(\hat{\imath} + 2\hat{\jmath} - 3\hat{k})$ and $\vec{r} = (3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}) + t(2\hat{\imath} + 4\hat{\jmath} - 5\hat{k})$ are skew lines and hence find the shortest distance between them.

47. a) Find the non-parametric form of vector equation and Cartesian equation of the plane passing through the point $(0,1,-5)$ and parallel to the straight lines $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + s(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ and $\vec{r} = (\hat{\imath} - 3\hat{\jmath} + 5\hat{k}) + t(\hat{\imath} + \hat{\jmath} - \hat{k})$.

(OR)

b) For the ellipse $4x^2 + y^2 + 24x - 2y + 21 = 0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2.

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