Common Quarterly Examination - September 2023

Standard 12

Time Allowed: 3.00 Hours **MATHEMATICS** Maximum Marks: 90

PART - I

Note: i) Answer all the questions. **20×1=20**

ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

1) If $|\text{adj}(\text{adj } A)| = |A|^9$, then the order of the square matrix A is

a) 3 b) 4 c) 2 d) 5

2) Which of the following is/are correct?

i) Adjoint of a symmetric matrix is also a symmetric matrix

ii) Adjoint of a diagonal matrix is also a diagonal matrix

iii) If A is a square matrix of order n and λ is a scalar, then $\text{adj}(\lambda A) = \lambda^n \text{adj}(A)$

iv) $A(\text{adj } A) = (\text{adj } A)A = |A|I$

a) only (i) b) (ii) and (iii) c) (iii) and (iv) d) (i) (ii) and (iv)

3) The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda-7 & \mu+5 \end{bmatrix}$

The system has infinitely many solutions if

a) $\lambda = 7, \mu \neq -5$ b) $\lambda = -7, \mu \neq 5$ c) $\lambda \neq 7, \mu \neq -5$ d) $\lambda = 7, \mu = -5$

4) If A is a 3×3 matrix such that $|3 \text{ adj } A| = 3$ then $|A|$ is equal to

a) $\pm\frac{1}{3}$ b) $\frac{1}{3}$ c) $-\frac{1}{3}$ d) ± 3

5) If Z is a non zero complex number, such that $2iZ^2 = \bar{Z}$, then $|Z|$ is

a) $\frac{1}{2}$ b) 1 c) 2 d) 3

6) If $\omega \neq 1$ is a cube root of unity and $(1+\omega)^7 = A+B\omega$, then (A, B) equals

a) (1, 0) b) (−1, 1) c) (0, 1) d) (1, 1)

7) Which of the following is incorrect?

a) $|Z_1+Z_2| \leq |Z_1|+|Z_2|$ b) $|Z_1-Z_2| \leq |Z_1|+|Z_2|$

c) $|Z_1-Z_2| \geq |Z_1|-|Z_2|$ d) $|Z_1+Z_2| \geq |Z_1|+|Z_2|$

8) The number of positive zeros of the polynomial $\sum_{r=0}^{n} nC_r (-1)^r x^r$ is

a) 0 b) n c) <n d) r

9) A polynomial equation of degree n always has

a) exactly n roots b) n distinct roots

c) n real roots d) n imaginary roots

10) If $x^3+12x^2+10ax+1999$ definitely has a positive zero, if and only if

a) $a \geq 0$ b) $a > 0$ c) $a < 0$ d) $a \leq 0$

11) If $\cot^{-1}x = \frac{2\pi}{5}$ for some $x \in R$, the value of $\tan^{-1}x$ is

a) $-\frac{\pi}{10}$ b) $\frac{\pi}{5}$ c) $\frac{\pi}{10}$ d) $-\frac{\pi}{5}$

12) If the function $f(x) = \sin^{-1}(x^2-3)$, then x belongs to

a) $[-1, 1]$ b) $[\sqrt{2}, 2]$ c) $[-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$ d) $[-2, -\sqrt{2}]$

13) $\sin^{-1}(2\cos^2 x-1) + \cos^{-1}(1-2\sin^2 x) =$

a) $\pi/2$ b) $\pi/3$ c) $\pi/4$ d) $\pi/6$

14) The radius of the circle $3x^2+by^2+4bx-6by+b^2 = 0$ is

a) 1 b) 3 c) $\sqrt{10}$ d) $\sqrt{11}$

15) Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

a) 2ab b) ab c) $\sqrt{ab}$ d) a/b

16) The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ having centre at (0, 3) is

a) $x^2+y^2-6y-7 = 0$ b) $x^2+y^2-6y+7 = 0$

c) $x^2+y^2-6y-5 = 0$ d) $x^2+y^2-6y-5 = 0$

17) If $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$, then the value of $[\vec{a}, \vec{b}, \vec{c}]$ is

a) $|\vec{a}||\vec{b}||\vec{c}|$ b) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ c) 1 d) −1

18) If $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\times\vec{c}$, where $\vec{a}, \vec{b}, \vec{c}$ are any three vectors such that $\vec{b}.\vec{c}\neq 0$ and $\vec{a}.\vec{b}\neq 0$, then $\vec{a}$ and $\vec{c}$ are

a) perpendicular b) parallel

c) inclined at an angle $\pi/3$ d) inclined at an angle $\pi/6$

19) If the direction consines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$ then

a) $c = \pm 3$ b) $c = \pm\sqrt{3}$ c) $c > 0$ d) $0 < c < 1$

20) If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a}, \vec{b}, \vec{c}] = 3$, then $\{[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}]\}^2$ is equal to

a) 81 b) 9 c) 27 d) 18

PART - II

Answer any seven questions. Question No. 30 is compulsory: **7×2=14**

21) Verify the property $(A^T)^{-1} = (A^{-1})^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$

22) Solve the following system of linear equations using matrix inversion method $2x-y = 8$; $3x+2y = -2$

23) Simplify the expression $i^{35}+\frac{1}{i^{35}}$ in the form of a+ib

24) Find the square root of $-6+8i$

25) Solve the equation $7x^3-43x^2 = 43x-7$

26) Find the equation of the parabola, whose vertex (1, −2) and focus (4, −2)

27) If $y = 2\sqrt{2}x+c$ is a tangent to the circle $x^2+y^2 = 16$, find the value of c.

28) Show that the vectors $\hat{i}+2\hat{j}-3\hat{k}$, $2\hat{i}-\hat{j}+2\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ are coplanar.

29) Find the angle between the straight line $\vec{r}=\left(2\hat{i}+3\hat{j}+\hat{k}\right)+t\left(\hat{i}-\hat{j}+\hat{k}\right)$ and the plane $2x-y+z = 5$

30) Find the value of $\tan^{-1}\left(\sqrt{3}\right)-\sec^{-1}(-2)$

PART - III

Answer any seven questions. Question No. 40 is compulsory: **7×3=21**

31) Find the inverse by Gauss.Jordan method $\begin{bmatrix}5 & 2\\ 7 & 3\end{bmatrix}$

32) If $\omega \neq 1$ is a cube root of unity, show that $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = -1$.

33) Show that $\left(2+i\sqrt{3}\right)^{10}+\left(2-i\sqrt{3}\right)^{10}$ is real.

34) Find the condition that the roots of $ax^3+bx^2+cx+d = 0$ are in geometric progression. Assume $a, b, c, d \neq 0$

35) Find the domain of $\cos^{-1}\left(\frac{2+\sin x}{3}\right)$

36) Find the value of $\cos\left(\sin^{-1}\left(\frac{4}{5}\right)-\tan^{-1}\left(\frac{3}{4}\right)\right)$

37) The orbit of Halley's comet is an ellipse 36.18 astronomical units long and by 9.12 astronomiocal units wide. Find its eccentricity.

38) Let $\vec{a},\vec{b},\vec{c}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$. Show that $\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2 = \frac{1}{4}|\vec{a}|^2|\vec{b}|^2$

39) If the straight lines $\frac{x-5}{5m+2}=\frac{2-y}{5}=\frac{1-z}{-1}$ and $x=\frac{2y+1}{4m}=\frac{1-z}{-3}$ are perpendicular to each other, find the value of m.

40) Show that the equation $x^9-5x^5+4x^4+2x^2+1 = 0$ has atleast 6 imaginary solutions.

PART - IV

Note: Answer all the questions. 7×5=35

41) a) Solve the system of linear equations by Cramer's rule:

$$\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0,\quad \frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0,\quad \frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0.$$

(OR)

b) If $A=\frac{1}{7}\begin{bmatrix}6 & -3 & a\\ b & -2 & 6\\ 2 & c & 3\end{bmatrix}$ is orthogonal, find a, b and c and hence A^{-1}.

42) a) If Z_1, Z_2 and Z_3 are three Complex Numbers such that $|Z_1| = 1$, $|Z_2| = 2$, $|Z_3| = 3$ and $|Z_1+Z_2+Z_3| = 1$, show that $|9Z_1Z_2+4Z_1Z_3+Z_2Z_3| = 6$

(OR)

b) Suppose Z_1, Z_2 and Z_3 are the vertices of an equilateral triangle incribed in a circle $|Z| = 2$. If $Z_1 = 1+i\sqrt{3}$, then find Z_2 and Z_3.

43) a) If the eqations $x^2+px+q = 0$ and $x^2+p'x+q' = 0$ have a common root, show that it must be equal to $\frac{pq'-p'q}{q-q'}$ or $\frac{q-q'}{p'-p}$

(OR)

b) Solve: $x^4-10x^3+26x^2-10x+1 = 0$

44) a) Show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)=\sec^{-1}x, |x|>1$

(OR)

b) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, show that $x+y+z = xyz$.

45) a) Find the equation of the circle passing through the points (1, 0) (0, −1) and (0, 1)

(OR)

b) If the normal at the point 't_1' on the parabola $y^2 = 4ax$ meet the parabola again at the point 't_2' then prove that $t_2=-\left(t_1+\frac{2}{t_1}\right)$

46) a) A rod of length 1.2 m moves with its ends always touching the co-ordinate axes. The locus of a point P on the rod, which is 0.3 m from the end in contact with x-axis is an ellipse. Find the eccentricity.

(OR)

b) Prove by Vector Method that $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

47) a) Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points (−1, 2, 0) (2, 2, −1) and parallel to the straight line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$

(OR)

b) Investigate for what values of λ and μ the system of linear equations $x+2y+z = 7$, $x+y+\lambda z = \mu$, $x+3y-5z = 5$ has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.