

FTM

## QUARTERLY EXAMINATION - 2023-24

12 - Std

MATHEMATICS



Marks : 90

Time : 3.00 Hrs

## PART - I

20 X 1 = 20

I Choose the correct answer.

- If A is a non singular matrix such that  $A^{-1} \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ , then  $(A^T)^{-1} =$ 
  - $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$
  - $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$
  - $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$
  - $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$
- If  $\text{adj } A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$  and  $\text{adj } B = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$  then  $\text{adj } (AB)$  is
  - $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$
  - $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$
  - $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$
  - $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$
- The rank of the zero matrix of order  $2 \times 3$  is
  - 1
  - 2
  - 3
  - 0
- The solution of the equation  $|z| - z = 1 + 2i$  is
  - $\frac{3}{2} - 2i$
  - $-\frac{3}{2} + 2i$
  - $2 - \frac{3}{2}i$
  - $2 + \frac{3}{2}i$
- If  $\alpha$  and  $\beta$  are the roots of  $x^2 + x + 1 = 0$ , then  $\alpha^{2020} + \beta^{2020}$  is
  - 2
  - 1
  - 1
  - 2
- The centre of the circle  $|2z - 5 + 6i| = 6$  is
  - $\left(\frac{-5}{2}, 3\right)$
  - $\left(\frac{5}{2}, -3\right)$
  - (-5, 6)
  - (5, -6)
- The polynomial  $x^3 - kx^2 + 9x$  has three real zeros if and only if, k satisfies
  - $|k| \leq 6$
  - $k = 0$
  - $|k| > 6$
  - $|k| \geq 6$
- If  $x^3 + 12x^2 + 10ax + 1999$  definitely has a positive zero, if and only if
  - $a \geq 0$
  - $a > 0$
  - $a < 0$
  - $a \leq 0$
- If  $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$ ; then  $\cos^{-1}x + \cos^{-1}y$  is equal to
  - $\frac{2\pi}{3}$
  - $\frac{\pi}{3}$
  - $\frac{\pi}{6}$
  - $\pi$
- $\sin(\tan^{-1}x)$ ,  $|x| < 1$  is equal to
  - $\frac{x}{\sqrt{1-x^2}}$
  - $\frac{1}{\sqrt{1-x^2}}$
  - $\frac{1}{\sqrt{1+x^2}}$
  - $\frac{x}{\sqrt{1+x^2}}$

11. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is  
 a)  $\frac{4}{3}$                       b)  $\frac{4}{\sqrt{3}}$                       c)  $\frac{2}{\sqrt{3}}$                       d)  $\frac{3}{2}$
12. If  $p(x, y)$  be any point on  $16x^2 + 25y^2 = 400$  with foci  $F_1(3, 0)$  and  $F_2(-3, 0)$  then  $PF_1 + PF_2$  is    a) 8                      b) 6                      c) 10                      d) 12
13. Area of the greatest rectangle inscribed in the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  
 a)  $2ab$                       b)  $ab$                       c)  $\sqrt{ab}$                       d)  $\frac{a}{b}$
14. The latus rectum of the parabola  $y^2 = 8x$  is  
 a) 2                      b) 6                      c) 4                      d) 8
15. If  $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$  and  $A(\text{adj } A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$  then  $k = ?$   
 a) 0                      b)  $\sin \theta$                       c)  $\cos \theta$                       d) 1
16. If  $(1 + i)(1 + 2i)(1 + 3i) \dots \dots \dots (1 + ni) = x + iy$ , then  $2.5.10 \dots \dots (1 + n^2)$  is  
 a) 1                      b)  $i$                       c)  $x^2 + y^2$                       d)  $1 + n^2$
17. If  $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$ , where  $\vec{a}, \vec{b}, \vec{c}$  are any three vectors such that  $\vec{b} \cdot \vec{c} \neq 0$  and  $\vec{a} \cdot \vec{b} \neq 0$  then  $\vec{a}$  and  $\vec{c}$  are  
 a) perpendicular  
 b) parallel                      c) inclined at an angle  $\frac{\pi}{3}$                       d) inclined at an angle  $\frac{\pi}{6}$
18. Distance from the origin to the plane  $3x - 6y + 2z + 7 = 0$  is  
 a) 0                      b) 1                      c) 2                      d) 3
19. If the direction cosines of a line are  $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$  then  
 a)  $c = \pm 3$                       b)  $c = \pm \sqrt{3}$                       c)  $c > 0$                       d)  $0 < c < 1$
20. If  $\vec{a}, \vec{b}, \vec{c}$  are non-coplanar, non-zero vectors and  $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = 64$  then  $[\vec{a}, \vec{b}, \vec{c}]$  is  
 a) 4                      b) 6                      c) 8                      d) 16

## PART - II

II) i) Answer any seven questions.

ii) Question number 30 is compulsory.

7 X 2 = 14

21. Find the rank of the matrix  $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$ .

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22. Simplify :  $\sum_{r=1}^{12} r^r$ .
23. Find polynomial equation of minimum degree with rational coefficients, having  $2i + 3$  as a root.
24. Find the principal value of  $\sin^{-1}(2)$  if it exists.
25. Find the equations of tangent to the parabola  $x^2 + 6x + 4y + 5 = 0$  at  $(1, -3)$
26. Show that two straight lines  $\frac{x-4}{2} = \frac{y}{1} = \frac{z+1}{-2}$  and  $\frac{x-1}{4} = \frac{y+1}{-4} = \frac{z-2}{2}$  are perpendicular to each other.
27. Examine the position of the point  $(2, 3)$  with respect to the circle  $x^2 + y^2 - 6x - 8y + 12 = 0$ .
28. Solve the Cramer's rule :  $5x - 2y + 16 = 0, x + 3y - 7 = 0$ .
29. Show that the polynomial  $9x^9 + 2x^5 - x^4 - 7x^2 + 2$  has atleast six imaginary roots.
30. If  $\vec{a} = \vec{i} - \vec{k}, \vec{b} = \vec{j} - \vec{k}, \vec{c} = \vec{i} + \vec{j} + \vec{k}$  find  $\vec{a} \cdot (\vec{b} \times \vec{c})$ .

## PART - III

- III Note : i) Answer any seven questions. ii) Question number 40 is compulsory.

$$7 \times 3 = 21$$

31. If  $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$  verify that  $A(\text{adj } A) = (\text{adj } A)A = |A| I_2$ .
32. Find the square root of  $6 - 8i$ .
33. Construct a cubic equation with roots : 1, 2 and 3.
34. Show that  $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \sec^{-1}x, |x| > 1$ .
35. Find the centre, foci, and eccentricity of the hyperbola  $11x^2 - 25y^2 - 44x + 50y - 256 = 0$ .
36. Find the magnitude and the direction cosines of the torque about the point  $(2, 0, -1)$  of a force  $2\vec{i} + \vec{j} - \vec{k}$ , whose line of action passes through the origin.
37. Represent the complex number  $-1 - i$  polar form.
38. If the roots of  $x^3 + px^2 + qx + r = 0$  are  $\alpha, \beta, \gamma$  then find  $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ .
39. prove that  $\tan^{-1}\left(\frac{2}{11}\right) + \tan^{-1}\left(\frac{7}{24}\right) = \tan^{-1}\left(\frac{1}{2}\right)$ .
40. Find the inverse of the non-singular matrix  $A = \begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$  by Gaus - Jordan method.



## PART - IV

## IV Answer all the questions.

7 X 5 = 35

41. a) Using vector method, prove that  $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$ . (OR)  
 b) Investigate the values of  $\lambda$  and  $\mu$  the system of linear equations  $2x + 3y + 5z = 9$ ,  $7x + 3y - 5z = 8$ ,  $2x + 3y + \lambda z = \mu$  have  
 i) no solution ii) a unique solution iii) an infinite number of solutions
42. a) Find the equation of the circle passing through the points (1, 1), (2, -1) and (3, 2). (OR) b) If  $z = x + iy$  is a complex number such that  $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$ . Show that the locus of  $z$  is  $2x^2 + 2y^2 + x - 2y = 0$ .
43. a) Identify the types of conic and find centre, foci, vertices and directrices of the following.  $9x^2 - y^2 - 36x - 6y + 18 = 0$ . (OR)  
 b) If  $2 + i$  and  $3 - \sqrt{2}$  are roots of the equation  $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$ . Find all roots.
44. a) Solve  $(x - 4)(x - 7)(x - 2)(x + 1) = 16$ . (OR)  
 b) Find the parametric form of vector equation, and Cartesian equations of the plane passing through the points (2, 2, 1), (9, 3, 6) and perpendicular to the plane  $2x + 6y + 6z = 9$ .
45. a) If  $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$  and  $0 < x, y, z < 1$ , show that  $x^2 + y^2 + z^2 + 2xyz = 1$ . (OR)  
 b)  $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ , find the products AB and BA and hence solve the system of equations  $x + y + 2z = 1$ ,  $3x + 2y + z = 7$ ,  $2x + y + 3z = 2$ .
46. a) A bridge has a parabolic arch that is 10m high in the centre and 30m wide at the bottom. Find the height of the arch 6m from the centre, on either sides. (OR) b) Solve the equation  $z^3 + 8i = 0$ , where  $Z \in C$ .
47. a) If  $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$ ,  $\vec{b} = 3\vec{i} + 5\vec{j} + 2\vec{k}$ ,  $\vec{c} = -\vec{i} - 2\vec{j} + 3\vec{k}$  verify that  $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c}) \vec{b} - (\vec{b} \cdot \vec{c}) \vec{a}$ . (OR)  
 b) Find the value of  $\cos\left[\cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right)\right]$ .