QUARTERLY EXAMINATION - 2023

12 - Std **MATHEMATICS**

Time : 3.00 Hrs Marks : 90

PART - I

Choose the correct answer. 20 X 1 = 20

1. If A is a non singular matrix such that $A^{-1}\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1} =$

   a) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ c) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

2. If $\text{adj } A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and $\text{adj } B = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ then adj (AB) is

   a) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ b) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ c) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ d) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

3. The rank of the zero matrix of order 2 x 3 is

   a) 1 b) 2 c) 3 d) 0

4. The solution of the equation $|z| - z = 1 + 2i$ is

   a) $\frac{3}{2} - 2i$ b) $-\frac{3}{2} + 2i$ c) $2 - \frac{3}{2}i$ d) $2 + \frac{3}{2}i$

5. If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{2020} + \beta^{2020}$ is

   a) -2 b) -1 c) 1 d) 2

6. The centre of the circle $|2z - 5 + 6i| = 6$ is

   a) $\left(\frac{-5}{2}, 3\right)$ b) $\left(\frac{5}{2}, -3\right)$ c) (-5, 6) d) (5, -6)

7. The polynomial $x^3 - kx^2 + 9x$ has three real zeros is and only if, K satisfies

   a) $|k| \leq 6$ b) $k = 0$ c) $|k| > 6$ d) $|k| \geq 6$

8. If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive zero, if and only if

   a) $a \geq 0$ b) $a > 0$ c) $a < 0$ d) $a \leq 0$

9. If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$; then $\cos^{-1}x + \cos^{-1}y$ is equal to

   a) $\frac{2\pi}{3}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) π

10. $\sin(\tan^{-1}x)$, $|x| < 1$ is equal to

    a) $\frac{x}{\sqrt{1-x^2}}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{1}{\sqrt{1+x^2}}$ d) $\frac{x}{\sqrt{1+x^2}}$

11. The eccentricity of the hyperbola whose latus rectum is 8 and conjugal axis is equal to half the distance between the foci is

a) $\frac{4}{3}$ b) $\frac{4}{\sqrt{3}}$ c) $\frac{2}{\sqrt{3}}$ d) $\frac{3}{2}$

12. If p (x, y) be any point on $16x^2 + 25y^2 = 400$ with foci F_1 (3, 0) and F_2 (−3, 0) then $PF_1 + PF_2$ is a) 8 b) 6 c) 10 d) 12

13. Area of the greatest rectangle inscribe in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

a) 2ab b) ab c) $\sqrt{ab}$ d) $\frac{a}{b}$

14. The latest rectum of the parabola $y^2 = 8x$

a) 2 b) 6 c) 4 d) 8

15. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin & \cos\theta\end{bmatrix}$ and $A(adj\,A)=\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}$ then k =

a) 0 b) sin θ c) cos θ d) 1

16. If (1 + i) (1 + 2i) (1 + 3i) .... (1 + ni) = x + iy , then 2.5.10 ..... $(1 + n^2)$ is

a) 1 b) i c) $x^2 + y^2$ d) $1 + n^2$

17. If $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\times\vec{c}$, where $\vec{a},\vec{b},\vec{c}$ are any three vectors such that $\vec{b}\cdot\vec{c}\neq 0$ and $\vec{a}\cdot\vec{b}\neq 0$ then $\vec{a}$ and $\vec{c}$ a) perpendicular

b) parallel c) inclined at an angle $\frac{\pi}{3}$ d) inclimed at an angle $\frac{\pi}{6}$

18. Distance from the origin to be plane 3x − 6y + 2z + 7 = 0 is

a) 0 b) 1 c) 2 d) 3

19. If the direction cosines of a line are $\frac{1}{c},\frac{1}{c},\frac{1}{c}$ then

a) $c=\pm 3$ b) $c=\pm\sqrt{3}$ c) c > 0 d) 0 < c < 1

20. If $\vec{a},\vec{b},\vec{c}$ are non - coplanar, non - zero vectors and $[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}]=64$ then $[\vec{a},\ \vec{b},\ \vec{c}]$ is

a) 4 b) 6 c) 8 d) 16

PART - II

i) Answer any seven questions.

ii) Question number 30 is compulsory. 7 X 2 = 14

21. Find the rank of the matrix $\begin{bmatrix}1 & -2 & 3\\ 2 & 4 & -6\\ 5 & 1 & -1\end{bmatrix}$.

22. Simplify : $\sum_{n=1}^{12} i^n$.

23. Find polynomial equation of minimum degree with rational coefficients, having $2i + 3$ as a root.

24. Find the principal value of $\sin^{-1}(2)$ if it exists.

25. Find the equation of tangent to the parabola $x^2 + 6x + 4y + 5 = 0$ at $(1, -3)$

26. Show that two straight lines $\frac{x-4}{2} = \frac{y}{1} = \frac{z+1}{-2}$ and $\frac{x-1}{4} = \frac{y+1}{-4} = \frac{z-2}{2}$ are perpendicular to each other.

27. Examine the position of the point $(2, 3)$ with respect to the circle $x^2 + y^2 - 6x - 8y + 12 = 0$.

28. Solve the Crammer's rule : $5x - 2y + 16 = 0$, $x + 3y - 7 = 0$.

29. Show that the polynomial $9x^9 + 2x^5 - x^4 \;\; 7x^2 + 2$ has atleast six imagining roots.

30. If $\vec{a} = \vec{i} - \vec{k}$, $\vec{b} = \vec{j} - \vec{k}$, $\vec{c} = \vec{i} + \vec{j} + \vec{k}$ find $\vec{a} \cdot (\vec{b} \times \vec{c})$.

PART - III

Note : i) Answer any seven questions. ii) Questions number 40 is compulsory. 7 x 3 = 21

31. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$ verify that $A(\text{adj} A) = (\text{adj } A)A = |A| I_2$.

32. Find the square root of $6 - 8i$.

33. Construct a cubic equation with roots : 1, 2 and 3.

34. Show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \sec^{-1} x, |x| > 1$.

35. Find the centre, foci, and eccentricity of the hyperbola $11x^2 - 25y^2 - 4x + 50y - 25b = 0$.

36. Find the magnitude and the direction cosines of the torque about the point (2, 0, −1) of the force $2\hat{i} + \hat{j} - \hat{k}$, whose line of action passes through the origin.

37. Represent the complex number −1 −1 polar form.

38. If the roots of $x^3 + px^2 + qx + r = 0$ are α, β, γ then find $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$.

39. prove that $\tan^{-1}\left(\frac{2}{11}\right) + \tan^{-1}\left(\frac{7}{14}\right) = \tan^{-1}\left(\frac{1}{2}\right)$.

40. Find the inverse of the non - singular matrix $A = \begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$ by Gaws - Jordan method.

PART - IV

Answer all the questions. 7 X 5 = 35

41. a) Using vector method, prove that $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$. ***(OR)*** b) Investigate the values of l and m the system of lineal equations $2x + 3y + 5z = 9$, $7x + 3y - 5z = 8$, $2x+3y+\lambda z = \mu$ have.

i) no solution ii) a unique solution iii) an infinite number of solutions

42. a) Find the equation of the circle passing through the points (1, 1), (2, -1) and (3, 2). ***(OR)*** b) If $z = x + iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$. Show that the locus of 2 is $2x^2 + 2y^2 + x - 2y = 0$.

43. a) Identify the types of conic and find centure foci, vertices and directical of the following. $9x^2 - y^2 - 36x - 6y + 18 = 0$. ***(OR)***

b) If $2 + i$ and $3 - \sqrt{2}$ are roots of the equation

$x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$. Find all roots.

44. a) Solve $(x - 4)(x - 7)(x - 2)(x + 1) = 16$. ***(OR)***

b) Find the parametric form of rector equation, and costesian equations of the plane passing through the point (2, 2, 1), (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$.

45. a) $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $0 < x, y, z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$. ***(OR)***

b) $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equation $x + y + 2z = 1$. $3x + 2y + z = 7$, $2x + y + 3z = 2$.

46. a) A bridge has C parabolic arch that is 10m high in the centre and 30m with at the bottom. Find the height of the arch 6m from the centre, on either sides. ***(OR)*** b) Solve the equation $z^3 + 8i = 0$, where $Z \in C$.

47. a) If $\vec{a} = 2\vec{i} + 2\vec{j} - \vec{k}$, $\vec{b} = 3\vec{i} + 5\vec{j} + 2\vec{k}$, $\vec{c} = -\vec{i} - 2\vec{j} + 3\vec{k}$ verify that $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}$. ***(OR)***

b) Find the value of $\cos\left(\cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right)\right)$