V12M

Virudhunagar District Common Examinations

Common Quarterly Examination - 2023

Standard 12

MATHEMATICS

PART - I

Time: 3.00 Hours Marks: 90

Choose the correct Answer **20x1=20**

1) If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then B^{-1} =

a) $\begin{bmatrix} 2 & -3 \\ -3 & 8 \end{bmatrix}$ b) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ d) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

2) If $\rho(A) = \rho([A,B])$, then the system AX=B of linear equations is

a) Consistent and has a unique solution

b) Consistent

c) Consistent and has infinitely many solution

d) inconsistent

3) If A is a square matrix of order n, then |adj A| is

a) $|A|^2$ b) $|A|^n$ c) $|A|^{n-1}$ d) $|A|$

4) If $|z-2+i| \le 2$, then the greatest value of |z| is

a) $\sqrt{3}-2$ b) $\sqrt{3}+2$ c) $\sqrt{5}-2$ d) $\sqrt{5}+2$

5) The product of all four values of $\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{3/4}$ is

a) -2 b) -1 c) 1 d) 2

6) If W is a cube root of unity then the value of $(1 - W + W^2)^4 + (1 + W - W^2)^4$ is

a) 0 b) 32 c) -16 d) -32

7) The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satisfies

a) $|k| \le 6$ b) k = 0 c) $|k| > 6$ d) $|k| \ge 6$

8) The polynomial $x^3 + 2x + 3$ has

a) One negative and two imaginary zeros

b) One positive and two imaginary zeros

c) three real zeros

d) no zeros

9) If α, β and γ are the roots of $x^3 + px^2 + qx + r$, then $\sum\frac{1}{\alpha}$ is

a) $\frac{-q}{r}$ b) $\frac{-p}{r}$ c) $\frac{q}{r}$ d) $\frac{-q}{p}$

10) The value of $\sin^{-1}(\cos x)$, $0 \le x \le \pi$ is

a) $\pi - x$ b) $x - \frac{\pi}{2}$ c) $\frac{\pi}{2} - x$ d) $x - \pi$

11) The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

a) [1, 2] b) [-1, 1] c) [0, 1] d) [-1, 0]

12) The value of $\cos^{-1}(-1) + \tan^{-1}(\infty) + \sin^{-1}(1)$ is

a) $-\pi$ b) $\frac{3\pi}{2}$ c) $\frac{\pi}{6}$ d) 2π

13) The equation of the normal to the circle $x^2+y^2-2x-2y+1=0$ which is parallel to the line $2x + 4y = 3$ is

a) $x + 2y = 3$ b) $x + 2y + 3 = 0$

c) $2x + 4y + 3 = 0$ d) $x - 2y + 3 = 0$

14) The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is

a) 1 b) 3 c) $\sqrt{10}$ d) $\sqrt{11}$

15) Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

a) 2ab c) ab c) $\sqrt{ab}$ d) $\frac{a}{b}$

16) The eccentricity of the ellipse $9x^2 + 4y^2 = 36$ is

a) $\sqrt{\frac{5}{3}}$ b) $\sqrt{\frac{3}{5}}$ c) $\frac{\sqrt{3}}{5}$ d) $\frac{\sqrt{5}}{3}$

17) If $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that $\vec{a}$ is perpendicular to $\vec{b}$, and is parallel to $\vec{c}$, then $\vec{a}\times(\vec{b}\times\vec{c})$ is equal to

a) $\vec{a}$ b) $\vec{b}$ c) $\vec{c}$ d) $\vec{0}$

18) The distance between the planes $x + 2y + 3z + 7 = 0$ and $2x + 4y + 6z + 7 = 0$ is

a) $\frac{\sqrt{7}}{2\sqrt{2}}$ b) $\frac{7}{2}$ c) $\frac{\sqrt{7}}{2}$ d) $\frac{7}{2\sqrt{2}}$

19) The value of $\left[\vec{i}-\vec{j}, \vec{j}-\vec{k}, \vec{k}-\vec{i}\right]$ is

a) 0 b) 1 c) 2 d) 3

20) The area of the parallelogram having a diagonal $3\vec{i}+\vec{j}-\vec{k}$ and a side $\vec{i}-3\vec{j}+4\vec{k}$ is

a) $10\sqrt{3}$ b) $6\sqrt{30}$ c) $\frac{3}{2}\sqrt{30}$ d) $3\sqrt{30}$

PART - II

Answer any 7 questions. (30 is compulsory **7x2=14**

21) Verify the property

$$\left(A^T\right)^{-1} = \left(A^{-1}\right)^T \text{ with } A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$$

22) A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution? (Use Cramer's rule)

23) Find the square root of $-5 - 12i$

24) Construct a cubic equation with roots 1, 1 and -2.

25) Find the value of $\tan^{-1}\left(-\sqrt{3}\right)$

26) Examine the position of the point (2, 3) with respect to the circle $x^2 + y^2 - 6x - 8y + 12 = 0$.

27) The parabolic communication antenna has a focus at 2m distance from the vertex of the antenna. Find the width of the antenna 3m from the vertex.

28) For any vector $\vec{a}$, prove that $\vec{i} \times (\vec{a} \times \vec{i}) + \vec{j} \times (\vec{a} \times \vec{j}) + \vec{k} \times (\vec{a} \times \vec{k}) = 2\vec{a}$

29) Find the length of the perpendicular from the point (1, −2, 3) to the plane $x - y + z = 5$.

30) Find the least positive integer n such that $\left(\frac{1+i}{1-i}\right)^n = 1$

PART - III

Answer any 7 of the following. (40 is compulsory) **7x3=21**

31) Find the condition on a, b and c so that the following system of linear equations has one parameter family of solutions:
$x + y + z = a$, $x + 2y + 3z = b$, $3x + 5y + 7z = C$.

32) Find the rank of the matrix by row reduction method. $\begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$

33) Obtain the cartesian form of the locus of z for $|z| = |z - i|$

34) Solve the equation $x^4 - 9x^2 + 20 = 0$

35) Find the domain of $\cos^{-1}\left(\frac{2 + \sin x}{3}\right)$

36) Prove that $\tan^{-1}\frac{2}{\pi} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

37) If $y = 2\sqrt{2x} + C$ is a tangent to the circle $x^2 + y^2 = 16$, find the value of C.

38) Find the equations of tangent and normal to the parabola $x^2 + 6x + 4y + 5 = 0$ at (1, −3).

39) Find the magnitude and direction cosines of the torgue of a force represented by $3\vec{i} + 4\vec{j} - 5\vec{k}$ about the point with position vector $2\vec{i} - 3\vec{j} + 4\vec{k}$ acting through a point whose position vector is $4\vec{i} + 2\vec{j} - 3\vec{k}$.

40) If $\vec{a} = 2\vec{i} + 3\vec{j} - 5\vec{k}$, $\vec{b} = -\vec{i} + \vec{j} + 2\vec{k}$ and $\vec{c} = 4\vec{i} - 2\vec{j} + 3\vec{k}$, show that $(\vec{a} \times \vec{b}) \times \vec{c} \neq \vec{a} \times (\vec{b} \times \vec{c})$

PART - IV

Answer all the questions. **7x5=35**

41) a) Solve the following system of equations, using matrix inversion method.
$2x_1 + 3x_2 + 3x_3 = 5$, $x_1 - 2x_2 + x_3 = -4$, $3x_1 - x_2 - 2x_3 = 3$

(OR)

b) Find all the cube roots of $\sqrt{3} + i$.

42) a) Solve the equation $z^3 + 27 = 0$

(OR)

b) Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros.

43) a) Solve the folloiwng equations.

$x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

(OR)

b) Solve : $\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right) = \sin\left[\cot^{-1}\left(\frac{3}{4}\right)\right]$

44) a) Find the vertex, focus, equation of directrix and length of the latus rectum of the following equation. $y^2 - 4y - 8x + 12 = 0$

(OR)

b) Find the non - parametric form of vector equation and cartesian equation of the plane passing through the point (1, –2, 4) and perpendicular to the plane $x + 2y - 3z = 11$ and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$

45) a) Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact.

(OR)

b) If $a_1, a_2, a_3, \ldots, a_n$ is an arithmetic progression with common difference d, prove that

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \ldots + \tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = \frac{a_n - a_1}{1+a_1a_n}$$

46) a) Two coast guard stations are located 600 km apart at points A(0, 0) and B (0, 600). A distress signal from a ship at P is received at slightly different times by two stations. It is determined that the ship is 200 km forther from station A than it is from station B. Determine the equation of hyperbola that passes through the location of the ship.

(OR)

b) By vector method, Prove that $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

47) a) If $ax^2 + bx + c$ is divided by $x + 3$, $x - 5$ and $x - 1$, the remainders are 21, 61 and 9 respectively. Find a, b and c (Use Gaussian elimination method)

(OR)

b) Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar. Also find the equation of the plane containing these two lines.

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