Tsl12M

Tenkasi District Common Examinations
Common Quarterly Examination - 2023

Standard 12
MATHEMATICS
PART - A

Time: 3.00 Hours | Marks: 90

Choose the correct Answer 20x1=20

1) If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, B = adj A and C = 3A, then $\frac{|adj\ B|}{|C|}$ =

a) $1/3$ b) $1/9$ c) $1/4$ d) 4

2) If A^TA^{-1} is symmetric, then A^2 =

a) A^{-1} b) $(A^T)2$ c) A^T d) $(A^{-1})^2$

3) If $\rho(A) = \rho\left(\left[\frac{A}{B}\right]\right) <$ n, number of unknowns, then the system AX = B of linear equations is

a) Consistent and has a unique solution b) Consistent

c) Consistent and has infinitely many solution d) inconsistent

4) If $|Z-2+i| \le 2$, then the greatest value of $|Z|$ is

a) $\sqrt{3}-2$ b) $\sqrt{3}+2$ c) $\sqrt{5}-2$ d) $\sqrt{5}+2$

5) The value of $\left[\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right]^{10}$ is

a) $\text{cis}\left(\frac{2\pi}{3}\right)$ b) $\text{cis}\left(\frac{4\pi}{3}\right)$ c) $-\text{cis}\left(\frac{2\pi}{3}\right)$ b) $-\text{cis}\left(\frac{4\pi}{3}\right)$

6) If Z is a non-zero complex number, then which one of the following is not true

a) $\bar{\bar{Z}} = Z$ b) $\text{Re}(Z) = \frac{Z-\bar{Z}}{2}$ c) $|Z| = |\bar{Z}|$ d) $\text{Im}(Z) = \frac{Z-\bar{Z}}{2i}$

7) A polynomial equation in x of degree 'n' always has

a) n distinct roots b) n real roots

c) n imaginary roots d) atmost one root

8) The number of positive zeros of the polynomial $\sum_{r=0}^{n} {}^nC_r (-1)^r x^r$ is

a) 0 b) n c) < n d) r

9) If $\sin^{-1}x = 2\sin^{-1}\alpha$ has a solution, then

a) $|\alpha| \le \frac{1}{\sqrt{2}}$ b) $|\alpha| \ge \frac{1}{\sqrt{2}}$ c) $-\pi/2 \le x \le \pi/2$ d) $-\pi/4 \le x \le 3\pi/4$

10) If $|x| \le 1$, then $2\tan^{-1}x - \sin^{-1}\frac{2x}{1+x^2}$ is equal to

a) $\tan^{-1}x$ b) $\sin^{-1}x$ c) 0 d) π

11) If $x \in [-1, 1]$, then the $\cos^{-1}x$ =

a) $\cos^{-1}x - \pi$ b) $\pi - \cos^{-1}x$ c) $\pi + \cos^{-1}x$ d) $\sin^{-1}x$

12) The area of quadrilateral formed with foci of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ is

a) $4(a^2+b^2)$ b) $2(a^2+b^2)$ c) a^2+b^2 d) $\frac{1}{2}(a^2+b^2)$

13) The eccentricity of the ellipse $(x-3)^2+(y-4)^2=\frac{y^2}{9}$ is

a) $\frac{\sqrt{3}}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{3\sqrt{2}}$ d) $\frac{1}{\sqrt{3}}$

14) The equation of the parabola with focus $(-\sqrt{2},0)$ and directrix $x=\sqrt{2}$ is

a) $x^2=2\sqrt{2}x$ b) $y^2=-4\sqrt{2}x$ c) $y^2=4\sqrt{2}x$ d) $x^2=-4\sqrt{2}x$

15) If $\vec{a},\vec{b},\vec{c}$ are three unit vectors such that $\vec{a}$ is perpendicular to $\vec{b}$ and is parallel to $\vec{c}$, then $\vec{a}\times(\vec{b}\times\vec{c})$ is equal to

a) $\vec{a}$ b) $\vec{b}$ c) $\vec{c}$ d) $\vec{0}$

16) If $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\times\vec{c}$, where $\vec{a},\vec{b},\vec{c}$ are any three vectors such that $\vec{b}.\vec{c}\neq 0$ and $\vec{a}.\vec{b}\neq 0$, then $\vec{a}$ and $\vec{c}$ are

a) perpendicular b) parallel

c) inclined at an angle $\frac{\pi}{3}$ d) inclined at an angle $\frac{\pi}{6}$

17) If the length of the perpendicular from the origin to the plane $2x+3y+\lambda z=1$, $\lambda>0$ is $\frac{1}{5}$, then the value of λ is

a) $2\sqrt{3}$ b) $3\sqrt{2}$ c) 0 d) 1

18) The slope of the line normal to the curve $f(x)=2.\cos 4x$ at $x=\frac{\pi}{12}$ is

a) $-4\sqrt{3}$ b) -4 c) $\frac{\sqrt{3}}{12}$ d) $4\sqrt{3}$

19) Angle between $y^2=x$ and $x^2=y$ at the origin is

a) $\tan^{-1}\frac{3}{4}$ b) $\tan^{-1}\frac{4}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

20) The maximum value of the function x^2e^{-2x}, $x>0$ is

a) $\frac{1}{e}$ b) $\frac{1}{2e}$ c) $\frac{1}{e^2}$ d) $\frac{4}{e^4}$

PART - B

7x2=14

Answer any 7 questions. Qn.no. 30 is compulsory.

21) If adj $A=\begin{bmatrix}-1 & 2 & 2\\ 1 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$ find A^{-1}

22) Show that the system of equations, $2x+5y=7$, $6x+15y=13$ is inconsistent.

23) Find Z^{-1}, if $Z = (2+3i)(1-i)$

24) Find the polynomial equation of minimum degree with rational coefficients, having $2i+3$ as a root.

25) Find the period and amplitude of $y = 4\sin(-2x)$

26) Find $\tan^{-1}\left(\tan\frac{3x}{5}\right)$

27) If $y = 2\sqrt{2}x + c$ is a tangent to the circle $x^2+y^2 = 16$, find the value of C.

28) Find the angle between the line $\vec{r} = \left(2\hat{i} - \hat{j} + \hat{k}\right) + t\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$ and the plane

$\vec{r}.\left(6\hat{i} + 3\hat{j} + 2\hat{k}\right) = 8$

29) Verify Rolle's Theorem to the function $f(x) = \left|\frac{1}{x}\right| + 1$, $x \in [-1, 1]$

30) Find the square root of $7-24i$.

PART - C

7x3=21

Answer any 7 questions. Qn. no. 40 is compulsory.

31) Solve the system of linear equations by matrix inversion method:
$2x-y = 8$, $3x+2y = -2$

32) Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$ by row reduction method.

33) Simplify: $\left[\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right]^{18}$

34) For any two complex numbers Z_1 and Z_2 such that $|Z_1| = |Z_2| = 1$ and $Z_1.Z_2 \neq -1$, then show that $\frac{Z_1 + Z_2}{1 + Z_1 Z_2}$ is a real number.

35) Solve: $x^3-5x^2-4x+20 = 0$

36) Find the value of $\sin^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)$

37) Find the length of the Latus rectum of the parabola $y^2 = 4ax$

38) Show that the line $x-y+4 = 0$ is a tangent to the ellipse $x^2+3y^2 = 12$

39) Find the value of λ, if the value of the parallelepiped whose coterminus edges are $7\hat{i} + \lambda\hat{j} - 3\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$, $-3\hat{i} + 7\hat{j} + 5\hat{k}$ is 90 cubic units.

40) A ball is thrown vertically upwards, moves according to the law $S = 13.8t-4.9t^2$ where S is in metres and t is in seconds.
(i) Find the velocity at $t = 1$
(ii) Find the acceleration at $t = 1$
(iii) Find the maximum height reached by the ball?

PART - D

Answer all the questions:

41) a] The upward speed $v(t)$ of a rocket at time t is approximated by $v(t) = at^2+bt+c$, $0 \le t \le 100$ where a, b and c are constants. It has been found that the speed at times t = 3, t = 6 and t = 9 seconds are respectively, 64, 133 and 208 miles per second respectively. Find the speed at time t =15 seconds (use Gaussian Elimination Method)

(OR)

b] If $\frac{1+z}{1-z} = \cos 2\theta + i \sin 2\theta$, show that $z = i + \tan\theta$

42) a] If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $0 < x, y, z < 1$, show that $x^2+y^2+z^2+2xyz = 1$

(OR)

b] On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4 m. When it is 6 m away from the point of projection. Finally it reaches the ground 12 m away from the starting point. Find the angle of projection.

43) a] Find all cube roots of $\sqrt{3}+i$

(OR)

b] Identify the type of conic and find centre, foci and vertices of the following $y^2+8x-6y+1 = 0$

44) a] Investigate the values of λ and μ the system of linear equations $2x+3y+5z = 9$, $7x+3y-5z = 8$, $2x+3y+\lambda z = \mu$ have (i) No solution (ii) a unique solution (iii) an infinite number of solutions.

(OR)

b] Solve: $6x^4-35x^3+62x^2-35x+6 = 0$

45) a] Prove by vector method that $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

(OR)

b] Evaluate: $\lim_{x \to \pi/2} (\sin x)^{\tan x}$

46) a] Find the parametric form of vector equation and cartesian equations of the plane containing the line $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{3}$ and parallel to the line $\frac{x-1}{3} = \frac{-y+1}{2} = \frac{z+1}{1}$

(OR)

b] Find the angle between $y = x^2$ and $y = (x-3)^2$

47) a] Solve the equations: $(x-4)(x-7)(x-2)(x+1) = 16$

(OR)

b] If $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = 3\vec{i} + 5\vec{j} + 2\vec{k}$, $\vec{c} = -\vec{i} - 2\vec{j} + 3\vec{k}$ verify that

$$(\vec{a}\times\vec{b})\times\vec{c} = (\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a}$$

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TENKASI DISTRICT COMMON QUARTERLY EXAMINATION - 2023

XII - STD

PART - A

S.NO.	OPTION	ANSWER	S.NO.	OPTION	ANSWER
1	b	$1/9$	11	d	$\sin^{-1}x$
2	b	$(A^T)^2$	12	b	$2(a^2+b^2)$
3	c)	consistent and has infinitely many solution	13	b	$1/3$
4	d	$\sqrt{5}+2$	14	b	$y^2 = -4\sqrt{2}x$
5	a	$\text{cis}(2\pi/3)$	15	b	$\vec{b}$
6	b	$\text{Re}(z) = \frac{z-\bar{z}}{2}$	16	b	parallel
7	c	n imaginary roots	17	a	$2\sqrt{3}$
8	b	n	18	a	$-4\sqrt{3}$
9	a	$\lvert a\rvert \leq \frac{1}{\sqrt{2}}$	19	c	$\pi/2$
10	c	0	20	c	$1/e^2$