ALPHA MATHS ACADAMY

JEE, CBSE AND BOARD EXAMINATION COACHING CENTER
TENKASI
MOBILE: 9489006077, 8778733955

UNIT TEST – CHAPTER 3

STANDARD 12

TIME: 3.00 HOURS **MATHEMATICS** **MARKS: 90**

PART 1

CHOOSE THE CORRECT ANSWER **$20 \times 1 = 20$**

1. A zero of $x^3 + 64$ is

   $(a)\ 0$ $(b)\ 4$ $(c)\ 4i$ $(d)\ -4$

2. If f and g are polynomials of degrees m and n respectively, and if $h(x) = (f \circ g)(x)$, then the degree of h is

   $(a)\ mn$ $(b)\ m+n$ $(c)\ m^n$ $(d)\ n^m$

3. A polynomial equation in x of degree n always has

   $(a)\ n$ distinct roots $(b)\ n$ real roots $(c)\ n$ complex roots (d) at most one root.

4. If α, β and γ are the zeros of $x^3 + px^2 + qx + r$, then $\sum \frac{1}{\alpha}$ is

   $(a)\ -\frac{q}{r}$ $(b)\ -\frac{p}{r}$ $(c)\ \frac{q}{r}$ $(d)\ -\frac{q}{p}$

5. According to the rational root theorem, which number is not possible rational zero of $4x^7 + 2x^4 - 10x^3 - 5$?

   $(a)\ -1$ $(b)\ \frac{5}{4}$ $(c)\ \frac{4}{5}$ $(d)\ 5$

6. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satisfies

   $(a)\ |k| \leq 6$ $(b)\ k = 0$ $(c)\ |k| > 6$ $(d)\ |k| \geq 6$

7. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

   $(a)\ 2$ $(b)\ 4$ $(c)\ 1$ $(d)\ \infty$

8. If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive zero, if and only if

   $(a)\ a \geq 0$ $(b)\ a > 0$ $(c)\ a < 0$ $(d)\ a \leq 0$

9. The polynomial $x^3 + 2x + 3$ has

(a) One negative and two imaginary zeros (b) one positive and two imaginary zeros

(c) three real zeros (d) no zeros

10. The number of positive zeros of the polynomial $\sum_{j=0}^{n} n_{C_r} (-1)^r x^r$ is

(a) 0 (b) n (c) $< n$ (d) r

11. If α, β and γ are the roots of the equation $x^3 + ax^2 + bx + c = 0$. the value of $(1+\alpha)(1+\beta)(1+\gamma)$?

(a) $(1+b)-(a+c)$ (b) $(1+b)+(a-c)$ (c) $(1+b)-(a-c)$ (d) $(1+b)+(a+c)$

12. $2x^3 - x^2 - 2x + 2 = Q(x)(2x-1) + R(x)$ for all values of x. The value of $R(x)$?

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

13. Roots of $x^3 + x^2 - 4x - 4 = 0$?

(a) 1 , -1 , 0 (b) 3 , -3 , 1 (c) 1 , 2 , 2 (d) 2 , -2 , 1

14. The value of x that satisfies $f(x) = 0$ is called the

(a) root of an equation $f(x) = 0$ (b) root of a equation $f(x)$

(c) zero of an equation $f(x)$ (d) none of the above

15. A monic polynomial which crosses the $x-$ axis at -4,0 and 2 ; lies below the $x-$axis between -4 and; lies above the $x-$axis between 0 and 2 is

(a) $x^3 + 2x^2 - 8x$ (b) $x^3 - 2x^2 - 8x$ (c) $-x^3 - 2x^2 + 8x$ (d) $-x^3 + 2x^2 + 8x$

16. A monic polynomial touches the $x-$ axis at 0 and crosses the $x-$axis at 3 ; lies above the $x-$axis between 0 and 3.

(a) $-x^3 - 3x^2$ (b) $x^3 + 3x^2$ (c) $x^3 - 3x^2$ (d) $-x^3 + 3x^2$

17. The list of all possible rational roots for $x^5 - 4x^2 + 6x + 5$

(a) $\pm 1, \pm 5$ (b) $\pm 5, \frac{1}{5}$ (c) $\pm 1, \pm \frac{1}{5}$ (d) $\pm \frac{1}{4}, \pm \frac{5}{4}, \pm 5$

18. The list of all possible rational roots $7x^3 - x^2 + 3$

(a) $\pm \frac{1}{7}, \pm \frac{3}{7}, \pm 1, \pm 3$ (b) $\pm \frac{1}{7}, \pm \frac{1}{3}, \pm 1, \pm 3, \pm 7$ (c) $\pm \frac{1}{7}, \pm \frac{3}{7}, \pm 1, \pm 3, \pm 7$ (d) $\pm \frac{1}{3}, \pm \frac{7}{3}, \pm 1, \pm 7$

19. Using Descartes Rule of Signs, the possible number of positive and negative real zeros of $p(x) = 6x^5 - 4x^2 + x + 4$

(a) 3 or 1 positive zeros, 3 or 1 negative zeros (b) 2 or 0 positive zeros, 1 or 0 negative zeros

(c) 2 or 0 positive zeros, 2 or 0 negative zeros (d) 2 or 0 positive zeros, 1 negative zeros

20. If $x = -1$ is a zero with multiplicity 2 of the polynomial $p(x) = x^4 + x^3 + x^2 + kx + k - 1$, then the value of k is

(a) 3 (b) 2 (c) 1 (d) 0

PART 2

ANSWER ANY 7 OF THE FOLLOWING QUESTIONS $(30^{TH}$ *QUESTION IS COMPULSARY*$)$ $7 \times 2 = 14$

21. Construct a cubic equation with roots 1 , 2 and 3.

22. Show that the equation $2x^2 - 6x + 7 = 0$ cannot be satisfied by any real values of x.

23. Find a polynomial equation of minimum degree with rational coefficients, having $2 + \sqrt{3}i$ as a root.

24. Solve the equation $x^4 - 9x^2 + 20 = 0$

25. Prove that a straight line and parabola cannot intersect at more than two points.

26. Determine the number of positive and negative roots of the equation $x^9 - 5x^8 - 14x^7 = 0$.

27. If α, β and γ are the root of the equation $x^3 + px^2 + qx + r = 0$,find the value of $\sum \frac{1}{\beta\gamma}$ in terms of the coefficients.

28. If $x^2 + 2(k + 2)x + 9k = 0$ has equal roots, find k.

29. If p and q are the roots of the equation $lx^2 + nx + n = 0$. Show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$.

30. Solve the equations $\sin^2 x - 5 \sin x + 4 = 0$.

PART 3

ANSWER ANY 7 OF THE FOLLOWING QUESTIONS $(40^{TH}$ *QUESTION IS COMPULSARY*$)$ $7 \times 3 = 21$

31. If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, show that it must be equal to $\frac{pq' - p'q}{q - q'}$ *or* $\frac{q - q'}{p' - p}$.

32. If α , β and γ are the roots of the cubic equation $x^3 + 2x^2 + 3x + 4 = 0$, form a cubic equation whose roots are $\frac{1}{\alpha}$, $\frac{1}{\beta}$, $\frac{1}{\gamma}$.

33. Solve the equation $x^3 - 3x^2 - 33x + 35 = 0$

34. Find all real numbers satisfying $4^x - 3(2^{x+2}) + 2^5 = 0$

35. If α, β, γ and δ are the roots of the polynomial equation $2x^4 + 5x^3 - 7x^2 + 8 = 0$, find a quadratic equation with integer coefficients whose roots are $\alpha + \beta + \gamma + \delta$ and $\alpha\beta\gamma\delta$.

36. Find the monic polynomial equation of minimum degree with real coefficients having $2 - \sqrt{3}i$ as a root.

37. If the sides of a cubic box are increased by 1, 2, 3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Find the volume of the cuboid.

38. Obtain the condition that the roots of $x^3 + p\,x^2 + qx + r = 0$ are in A.P.

39. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

40. If a complex number z_0 is a root of a polynomial equation with real coefficients, then its complex conjugate $\bar{z}_0$ is also a root.

PART 4

ANSWER ALL THE FOLLOWING QUESTIONS **$7 \times 5 = 35$**

41. (a) If the roots of $x^3 + px^2 + qx + r = 0$ are in H.P, Prove that $p\,q\,r = 2\,q^3 + 27\,r^2$ .Assume $p, q, r \neq 0$ **(or)**

(b) Determine k and solve the equation $2x^3 - 6x^2 - 3x + k = 0$ if one of its roots is twice the sum of the other two roots.

42. (a) Solve the equation $2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}} = \frac{b}{a} + \frac{6a}{b}$ **(or)**

(b) Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5} - \sqrt{3}$ as a root.

43. (a) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution. **(or)**

(b) If $2 + i$ and $3 - \sqrt{2}$ are roots of the equation

$x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140$

44. (a) Solve $(2x - 1)(x + 3)(x - 2)(2x + 3) + 20 = 0$. **(or)**

(b) Discuss the maximum possible number of positive and negative roots of the polynomial equation

$9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0.$

45. (a) A polynomial equation $a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0 =$; $a_n \neq 0$ is a reciprocal equation if, and only if, one of the following two statements is true :

$(i)\ a_n = a_0\ ,\ a_{n-1} = a_1\ , a_{n-2} = a_2\ , \ldots$

$(ii)\ a_n = -a_0\ ,\ a_{n-1} = -a_1\ , a_{n-2} = -a_2\ , \ldots$ $(\boldsymbol{or})$

(b) Find the roots of $2x^3 + 3x^2 + 2x + 3 = 0$

46. (a) Solve the cubic equation $2x^3 - x^2 - 18x + 9 = 0$ if sum of two of its roots vanishes. $(\boldsymbol{or})$

(b) Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if it is given that two of its roots are in the ratio 3:2

47. (a) Solve the equation $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$ $(\boldsymbol{or})$

(b) Solve the equation $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

********** ***ALL THE BEST*** **********

PREPARED BY
M.KARTHIGAI GANAPATHY M.Sc., M.Ed.,
PG ASST. MATHEMATICS
TENKASI-627802
CONTACT NUMBER: 9489006077
E.MAIL: kr.aashni@gmail.com