Sri Raghavendra Tuition Center

ARTHI - HALF PORTION TEST

12th Standard

Maths

Date : 08-Oct-23

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Reg.No. :

Exam Time : 03:00:00 Hrs

Total Marks : 100

I . ANSWER ALL QUESTION

20 x 1 = 20

1) If A is a 3 × 3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then BB^T =

(a) A (b) B (c) I_3 (d) B^T

2) Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$. If B is the inverse of A, then the value of x is

(a) 2 (b) 4 (c) 3 (d) 1

3) The value of $\sum_{n=1}^{13}\left(i^n + i^{n-1}\right)$ is

(a) 1+ i (b) i (c) 1 (d) 0

4) The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

(a) $cis\frac{2\pi}{3}$ (b) $cis\frac{4\pi}{3}$ (c) $-cis\frac{2\pi}{3}$ (d) $-cis\frac{4\pi}{3}$

5) If f and g are polynomials of degrees m and n respectively, and if h(x) = (f ∘ g)(x), then the degree of h is

(a) mn (b) m+n (c) m^n (d) n^m

6) The polynomial $x^3 + 2x + 3$ has

(a) one negative and two imaginary zeros (b) one positive and two imaginary zeros (c) three real zeros (d) no zeros

7) If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$; then $\cos^{-1}x + \cos^{-1}y$ is equal to

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) π

8) If $\sin^{-1}\frac{x}{5} + \operatorname{cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$, then the value of x is

(a) 4 (b) 5 (c) 2 (d) 3

9) The differential equation representing the family of curves y = Acos(x + B), where A and B are parameters,is

(a) $\frac{d^2y}{dx^2} - y = 0$ (b) $\frac{d^2y}{dx^2} + y = 0$ (c) $\frac{d^2y}{dx^2} = 0$ (d) $\frac{d^2x}{dy^2} = 0$

10) If the solution of the differential equation $\frac{dy}{dx} = \frac{ax+3}{2y+f}$ represents a circle, then the value of a is

(a) 2 (b) -2 (c) 1 (d) -1

11) A rod of length $2l$ is broken into two pieces at random. The probability density function of the shorter of the two pieces is
\(f(x)= \begin{cases}\frac{1}{l} & 0
The mean and variance of the shorter of the two pieces are respectively.

(a) $\frac{l}{2}, \frac{l^2}{3}$ (b) $\frac{l}{2}, \frac{l^2}{6}$ (c) $l, \frac{l^2}{12}$ (d) $\frac{l}{2}, \frac{l^2}{12}$

12) If in 6 trials, X is a binomial variable which follows the relation 9P(X = 4) = P(X = 2), then the probability of success is

(a) 0.125 (b) 0.25 (c) 0.375 (d) 0.75

13) Subtraction is not a binary operation in

(a) R (b) Z (c) N (d) Q

14) Determine the truth value of each of the following statements:
(a) 4 + 2 = 5 and 6 + 3 = 9
(b) 3 + 2 = 5 and 6 + 1 = 7
(c) 4 + 5 = 9 and 1 + 2 = 4
(d) 3 + 2 = 5 and 4 + 7 = 11

(a)

(a)	(b)	(c)	(d)
F	T	F	T

(b)

(a)	(b)	(c)	(d)
T	F	T	F

(c)

(a)	(b)	(c)	(d)
T	T	F	F

(d)

(a)	(b)	(c)	(d)
F	F	T	T

15) If the system of equations x = cy + bz, y = az + cx and z = bx + ay has a non - trivial solution then ___________

(a) $a^2 + b^2 + c^2 = 1$ (b) abc ≠ 1 (c) a + b + c = 0 (d) $a^2 + b^2 + c^2 + 2abc = 1$

16) If $\sqrt{a+ib}$ = x + iy, then possible value of $\sqrt{a-ib}$ is ________

(a) x^2+y^2 (b) $\sqrt{x^2+y^2}$ (c) x+iy (d) x-iy

17) The quadratic equation whose roots are α and β is ________

(a) $(x-\alpha)(x-\beta)=0$ (b) $(x-\alpha)(x+\beta)=0$ (c) $\alpha+\beta=\frac{b}{a}$ (d) $\alpha\beta=\frac{-c}{a}$

18) If $sin^{-1}x - cos^{-1}x = \frac{\pi}{6}$ then ________

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{-1}{2}$ (d) none of these

19) If cosx is an integrating factor of the differential equation $\frac{dy}{dx}+Py=Q$, then P = ________

(a) -cot x (b) cot x (c) tan x (d) -tan x

20) IfF(x) is the probability distribution function, then $F(-\infty)$ is ________

(a) 1 (b) 2 (c) ∞ (d) 0

II. ANSWER ANY 8 QUESTION COMPULSORY ANSWER 30 QUS

10 x 2 = 20

21) Solve, by Cramer's rule, the system of equations
$x_1 - x_2 = 3,\ 2x_1 + 3x_2 + 4x_3 = 17,\ x_2 + 2x_3 = 7.$

22) Solve the following system of linear equations by matrix inversion method :
$2x - y = 8,\ 3x + 2y = -2.$

23) Prove the following properties
$Re(z) = \frac{z+\bar{z}}{2}$ and $Im(z) = \frac{z-\bar{z}}{2i}$

24) Find the square roots of
$-5-12i$.

25) Find the following $\left|\frac{i(2+i)^3}{(1+i)^2}\right|$

26) If α, β and γ are the roots of the cubic equation $x^3+2x^2+3x+4=0$, form a cubic equation whose roots are $-\alpha, -\beta, -\gamma$

27) Find the principal value of $sin^{-1}\left(sin\left(-\frac{\pi}{3}\right)\right)$

28) For each of the following differential equations, determine its order, degree (if exists)
$x^2\frac{d^2y}{dx^2}+\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}=0$

29) Write the statements in words corresponding to $\neg p$, $p \wedge q$, $p \vee q$ and $q \vee \neg p$, where p is 'It is cold' and q is 'It is raining'.

30) Compute P(X = k) for the binomial distribution, B(n, p) where
$n=10, p=\frac{1}{5}, k=4$

III. ANSWER ANY 8 QUESTION COMPULSORY ANSWER 40 QUS

10 x 3 = 30

31) If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$.

32) Find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method.

33) Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ into rectangular form

34) If $\omega \neq 1$ is a cube root of unity, then the show that $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}=-1$

35) Solve: $8x^{\frac{3}{2n}} - 8x^{\frac{-3}{2n}} = 63$

36) Find the domain of $\cos^{-1}\left(\frac{2+\sin x}{3}\right)$

37) Prove that
$tan^{-1}\left(\frac{2}{11}\right)+tan^{-1}\left(\frac{7}{24}\right)=tan^{-1}\left(\frac{1}{2}\right)$

38) Find the differential equation of the family of circles passing through the points (a, 0) and (-a, 0).

39) The probability density function of X is given by $f(x)=\begin{cases} kxe^{-2x} & for\ x>0 \\ 0 & for\quad x\le 0 \end{cases}$ Find the value of k.

40) In an algebraic structure the identity element (if exists) must be unique

IV. ANSWER 8 QUESTION

16 x 5 = 80

41) a) Four men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

(OR)

b) In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ? (Use Cramer's rule to solve the problem).

42) a) Let z_1, z_2 and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1+z_2+z_3 \neq 0$ prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right| = r$

(OR)

b) If z = x + iy is a complex number such that $Im\left(\frac{2z+1}{iz+1}\right)=0$ show that the locus of z is $2x^2+2y^2+x-2y=0$

43) a) If the equations $x^2+px+q=0$ and $x^2+p'x+q'=0$ have a common root, show that it must be equal to $\frac{pq'-p'q}{q-q'}$ or $\frac{q-q'}{p'-p}$.

(OR)

b) If $a_1, a_2, a_3, \ldots a_n$ is an arithmetic progression with common difference d, prove that tan
$\left[tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right]=\frac{a_n-a_1}{1+a_1a_n}$

44) a) Find the cube roots of unity.

(OR)

b) The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density function

$$f(x) = \begin{cases} 3e^{-3x} & x > 0 \\ 0 & elsewhere \end{cases}$$

Find the expected life of this electronic equipment.

45) a) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, show that $x + y + z = xyz$

(OR)

b) Find the value of $tan\left[\frac{1}{2}sin^{-1}\left(\frac{2a}{1+a^2}\right) + \frac{1}{2}cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$

46) a) If X is the random variable with probability density function f(x) given by,

$$f(x) = \begin{cases} x+1 & -1 \le x < 0 \\ -x+1 & 0 \le x < 1 \\ 0 & otherwise \end{cases}$$

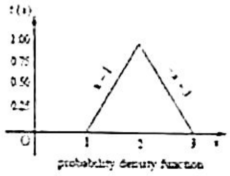
then find

(i) the distribution function F(x)

(ii) $P(-0.5 \le X \le 0.5)$

(OR)

b) If X is the random variable with probability density function f(x) given by,



$$f(x) = \begin{cases} x-1 & 1 \le x < 2 \\ -x+3 & 2 \le x < 3 \\ 0 & Otherwise \end{cases}$$

find

(i) the distribution function F(x)

(ii) $P(1.5 \le X \le 2.5)$

47) a) Verify

(i) closure property

(ii) commutative property

(iii) associative property

(iv) existence of identity and

(v) existence of inverse for the operation $\times_{11}$ on a subset A = {1, 3, 4, 5, 9} of the set of remainders {0,1, 2, 3, 4, 5, 6, 7, 8, 9,10}

(OR)

b) Let A be Q\{1}. Define * on A by x*y = x + y − xy. Is * binary on A? If so, examine the commutative and associative properties satisfied by * on A.

48) a) Solve the equation (x-2) (x-7) (x-3) (x+2)+19 = 0

(OR)

b) Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.

ALL THE BEST

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