SECOND MID TERM TEST - 2023

Standard X

Reg.No. | | | | | | |

MATHEMATICS

Time : 1.30 hrs ***Marks : 50***

I. Choose the correct answer: **7 x 1 = 7**

1. For the given matrix $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \\ 9 & 11 & 13 & 15 \end{bmatrix}$ the order of the matrix A^T is

   a) 2 x 3 b) 3 x 2 c) 3 x 4 d) 4 x 3

2. If number of columns and rows are not equal in a matrix then it is said to be a

   a) diagonal matrix b) rectangular matrix
   c) square matrix d) identity matrix

3. A tangent is perpendicular to the radius at the

   a) centre b) point of contact c) Infinity d) chord

4. The two tangents from an external points P to a circle with centre at O are PA and PB. If $\angle APB = 70^\circ$ then the value of $\angle AOB$ is

   a) 100° b) 110° c) 120° d) 130°

5. The electric pole subtends an angle of 30° at a point on the same level as its foot. At a second point 'b' meters above the first, the depression of the foot of the pole is 60°. The height of the pole (in metres) is equal to

   a) $\sqrt{3}\,b$ b) $b/3$ c) $b/2$ d) $b/\sqrt{3}$

6. If two solid hemispheres of same base radius 'r' units are joined together along their bases, then curved surface area of this new solid is

   a) $4\pi r^2$ sq.units b) $6\pi r^2$ sq.units c) $3\pi r^2$ sq.units d) $8\pi r^2$ sq.units

7. The total surface area of a cylinder whose radius is $\frac{1}{3}$ of its height is

   a) $\frac{9\pi h^2}{8}$ sq.units b) $24\pi h^2$ sq.units c) $\frac{8\pi h^2}{9}$ sq.units d) $\frac{56\pi h^2}{9}$ sq.units

II. Answer any 5 questions. (Q.No.14 is compulsory) **5 x 2 = 10**

8. Construct a 3 x 3 matrix whose elements are given by $A = [a_{ij}]$, $a_{ij} = |i - 2j|$

9. If $A = \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{pmatrix}$, then verify that $A + (-A) = (-A) + A = 0$

10. Find X and Y if $X + Y = \begin{pmatrix} 7 & 0 \\ 3 & 5 \end{pmatrix}$ and $X - Y = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$

11. A tower stands vertically on the ground. From a point on the ground which is 48 m away from the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower.

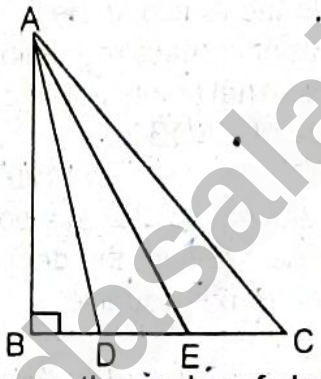
12. Find the angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of a tower of height $10\sqrt{3}$ m.
13. If the total surface area of a cone of radius 7 cm is 704 cm^2, then find its slant height.
14. State Ceva's theorem.

III. Answer any 5 questions. (Q.No.21 is compulsory) **5 x 5 = 25**

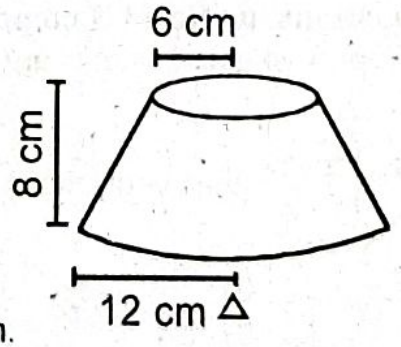
15. Find the value of x,y,z if $\begin{pmatrix} x-3 & 3x-z \\ x+y+7 & x+y+z \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 6 \end{pmatrix}$

16. If $A = \begin{pmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{pmatrix}$, $B = \begin{pmatrix} \sin\theta & 0 \\ 0 & \sin\theta \end{pmatrix}$, then show that $A^2 + B^2 = I$

17. In the adjacent figure, ABC is right angled triangle with right angle at B and points D, E trisect BC. Prove that $8AE^2 = 3AC^2 + 5AD^2$



18. To a man standing outside his house, the angles of elevation of the top and bottom of a window or 60^O and 45^O respectively. if the height of the man is 180 cm and if he is 5m away from the wall, what is the height of the window? $(\sqrt{3} = 1.732)$
19. A girl wishes to prepare birthday caps in the form of right circular cones for her birthday party, using a sheet of paper whose area is 5720 cm^2, how many caps can be made with radius 5 cm and height 12 cm.
20. The frustum shaped outer portion of the table lamp has to be painted including the top part. Find the total cost of painting the lamp if the cost of painting 1 sq.cm is ₹2.



21. State and prove Pythagores theorem.

IV. Answer any one. **1 x 8 = 8**

22. a) Draw the graph of $y = x^2 + 3x + 2$ and use it to solve $x^2 + 2x + 1 = 0$

(OR)

b) Draw the two tangents from a point which is 10 cm away from the centre of the circle of radius 5 cm. Also measure the lengths of the tangents.

/*

SECOND MID TERM TEST – 2023

STANDARD X

MATHEMATICS – KEY ANSWER

BY

A K RAJADHURAI, GRADUATE TEACHER, GGHSS, PODATURPET, TIRUVALLUR DISTRICT.

I. Choose the correct answer

1. For the given matrix $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \\ 9 & 11 & 13 & 15 \end{bmatrix}$ the order of the matrix $\mathbf{A^T}$ is

(A) 2 X3 (B) 3 X2 (C) 3X 4 **(D) 4X3**

2. If number of columns and rows are not equal in a matrix then it is said to be a

(A) diagonal matrix **(B) rectangular matrix** (C) square matrix (D) identity matrix

3. A tangent is perpendicular to the radius at the

(A) centre **(B) point of contact** (C) infinity (D) chord

4. The two tangents from an external points P to a circle with centre at O are PA and PB. If ∠APB =70° then the value of ∠AOB is

(A) 100° **(B) 110°** (C) 120° (D) 130°

5. The electric pole subtends an angle of 30° at a point on the same level as its foot. At a second point 'b' metres above the first, the depression of the foot of the pole is 60°. The height of the pole (in metres) is equal to

(A) √3 b **(B) b /3** (C) b/ 2 (D) b/√ 3

6. If two solid hemispheres of same base radius r units are joined together along their bases, then curved surface area of this new solid is

(A) $4\pi r^2$ sq. units (B) $6\pi r^2$ sq. units (C) $3\pi r^2$ sq. units (D) $8\pi r^2$ sq. units

7. The total surface area of a cylinder whose radius is 1 3 of its height is

(A) $9\pi h^2/8$ sq.units (B) $24\pi h^2$ sq.units **(C) $8\pi h^2/9$ sq.units** (D) $56\pi h^2/9$ sq.units

II. Answer any 5 questions (Q.No.14 is compulsory)

8. Construct a 3 × 3 matrix whose elements are given by A=$[a_{ij}]$
a_{ij} = |i – 2j|
Solution:

(i) $a_{ij} = |i - 2j|$

$a_{11} = |1 - 2 \times 1| = |1 - 2| = |-1| = 1$

$a_{12} = |1 - 2 \times 2| = |1 - 4| = |-3| = 3$

$a_{13} = |1 - 2 \times 3| = |1 - 6| = |-5| = 5$

$a_{21} = |2 - 2 \times 1| = |2 - 2| = 0$

$a_{22} = |2 - 2 \times 2| = |2 - 4| = |-2| = 2$

$a_{23} = |2 - 2 \times 3| = |2 - 6| = |-4| = 4$

$a_{31} = |3 - 2 \times 1| = |3 - 2| = |1| = 1$

$a_{32} = |3 - 2 \times 2| = |3 - 4| = |-1| = 1$

$a_{33} = |3 - 2 \times 3| = |3 - 6| = |-3| = 3$

$\therefore \begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ 1 & 1 & 3 \end{bmatrix}$ is the required 3 × 3 matrix.

A K RAJADHURAI, GRADUATE TEACHER, GGHSS, PODATURPET, TIRUVALLUR DISTRICT

9. If $A=\begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix}$ then verify that A+(-A)=(-A)+A=0

Solution:

$$A+(-A) = \begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix} + \begin{bmatrix} -1 & -9 \\ -3 & -4 \\ -8 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \quad \text{------(1)}$$

$$(-A)+A = \begin{bmatrix} -1 & -9 \\ -3 & -4 \\ -8 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \quad \text{------(2)}$$

From (1) and (2) A+(-A)=(-A)+A=0

Hence verified.

A K RAJADHURAI, GRADUATE TEACHER (MATHS), GGHSS, PODATURPET, TIRUVALLUR DISTRICT

10. Find X and Y if $X + Y = \begin{bmatrix} 7 & 0 \\ 3 & 5 \end{bmatrix}$ $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$

Solution:

$$X + Y = \begin{bmatrix} 7 & 0 \\ 3 & 5 \end{bmatrix} \text{--------(1)}$$

$$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} \text{--------(2)}$$

---

(1)+(2) ⇒

$$2X = \begin{bmatrix} 10 & 0 \\ 3 & 9 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} 5 & 0 \\ 3/2 & 9/2 \end{bmatrix}$$

$X + Y = \begin{bmatrix} 7 & 0 \\ 3 & 5 \end{bmatrix}$ --------(1)

$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$ --------(2)

---

(1)-(2) $\Rightarrow$

$2Y = \begin{bmatrix} 4 & 0 \\ 3 & 1 \end{bmatrix}$

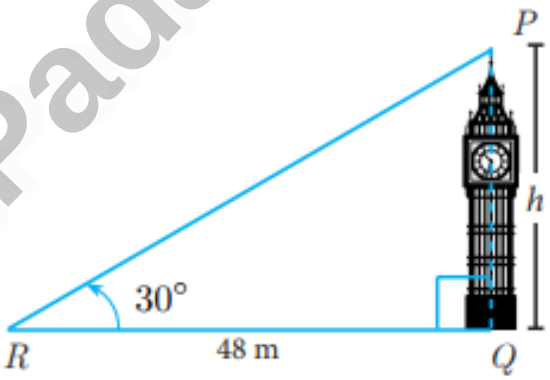
$$\mathbf{Y} = \begin{bmatrix} 2 & 0 \\ 3/2 & 1/2 \end{bmatrix}$$

11. A tower stands vertically on the ground. From a point on the ground, which is 48 m away from the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower.

Solution:

Let PQ be the height of the tower.

Take PQ = h and QR is the distance between the tower and the point R.



In the right angled ΔPQR, ∠ PRQ= 30°

$\tan \alpha = \frac{PQ}{QR}$

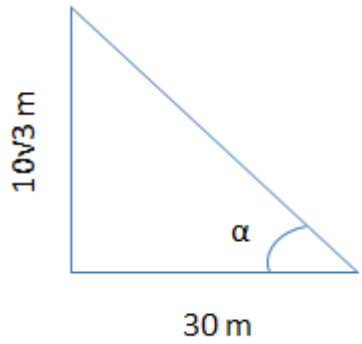
$\tan 30° = \frac{h}{48} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{48} \Rightarrow h = \frac{48}{\sqrt{3}} = 16\sqrt{3}$

Therefore, the height of the tower is 16√ 3 m

A K RAJADHURAI, GRADUATE TEACHER (MATHS), GGHSS, PODATURPET, TIRUVALLUR DISTRICT

12. Find the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of a tower of height 10√3 m.

Solution:



tan α = $\frac{opp.side}{adj.side} = \frac{10\sqrt{3}}{30} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

α = 30^0

13. If the total surface area of a cone of radius 7cm is 704 cm^2, then find its slant height.

Solution:

Given that, radius r = 7 cm

T.S.A of the cone = πr(l+r) sq. units

T.S.A. = 704 cm^2

$\frac{22}{7}$ x 7(l+7) = 704 ⇒ l+7 =32

l = 25 cm. Therefore, **slant height of the cone is 25 cm.**

14. State Ceva's Theorem

Statement: Let ABC be a triangle and let D,E,F be points on lines BC, CA, AB respectively. Then the cevians AD, BE, CF are concurrent if and only if $\frac{BD}{DC}$X $\frac{CE}{EA}$X $\frac{AF}{FB}$ = 1 where the lengths are directed. This also works for the reciprocal of each of the ratios as the reciprocal of 1 is 1.

A K RAJADHURAI, GRADUATE TEACHER (MATHS), GGHSS, PODATURPET, TIRUVALLUR DISTRICT

III. Answer any 5 questions (Q.No.21 is compulsory)

15. Find the value of x,y,z if $\begin{bmatrix} x-3 & 3x-z \\ x+y+7 & x+y+z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 6 \end{bmatrix}$

Solution:

The given matrices are equal. Thus all corresponding elements are equal.

x-3=1 x+y+7=1 x+y+z=6

x=1+3 4+y+7=1 4+(-10)+z=6

x=4 y=1-11 z=6+6

y=-10 **z=12**

16. If A $= \begin{bmatrix} cos\theta & 0 \\ 0 & cos\theta \end{bmatrix}$ **, B =** $\begin{bmatrix} sin\theta & 0 \\ 0 & sin\theta \end{bmatrix}$ **then show that $A^2 + B^2 = I$**

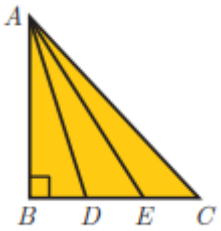
Solution:

$A^2 = A.A = \begin{bmatrix} cos\theta & 0 \\ 0 & cos\theta \end{bmatrix}\begin{bmatrix} cos\theta & 0 \\ 0 & cos\theta \end{bmatrix} = \begin{bmatrix} cos2\theta & 0 \\ 0 & cos2\theta \end{bmatrix}$

$B^2 = B.B = \begin{bmatrix} sin\theta & 0 \\ 0 & sin\theta \end{bmatrix}\begin{bmatrix} sin\theta & 0 \\ 0 & sin\theta \end{bmatrix} = \begin{bmatrix} sin2\theta & 0 \\ 0 & sin2\theta \end{bmatrix}$

$A^2 + B^2 = \begin{bmatrix} cos2\theta & 0 \\ 0 & cos2\theta \end{bmatrix} + \begin{bmatrix} sin2\theta & 0 \\ 0 & sin2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

17. In the adjacent figure, ABC is a right angled triangle with right angle at B and points D, E trisect BC.



Prove that $8AE^2 = 3AC^2 + 5AD^2$.

Solution:

Since the Points D, E trisect BC

BD = DE = CE

Let BD = DE = CE = x

BE = 2x and BC = 3x

A K RAJADHURAI, GRADUATE TEACHER (MATHS), GGHSS, PODATURPET, TIRUVALLUR DISTRICT

In the right ΔABD,

$AD^2 = AB^2 + BD^2$

$AD^2 = AB^2 + x^2$ .....(1)

In the right ΔABE,

$AE^2 = AB^2 + 2BE^2$

$AE^2 = AB^2 + 4X^2$ .....(2) (BE = 2x)

In the right ΔABC

$AC^2 = AB^2 + BC^2$

$AC^2 = AB^2 + 9x^2$ ..... (3) (BC = 3x)

R.H.S = $3AC^2 + 5AD^2$

$= 3[AB^2 + 9x^2] + 5[AB^2 + x^2]$ [From (1) and (3)]

$= 3AB^2 + 27x^2 + 5AB^2 + 5x^2$

$= 8AB^2 + 32x^2$

$= 8(AB^2 + 4x^2)$

$= 8AE^2$ [From (2)]

= **L.H.S.**

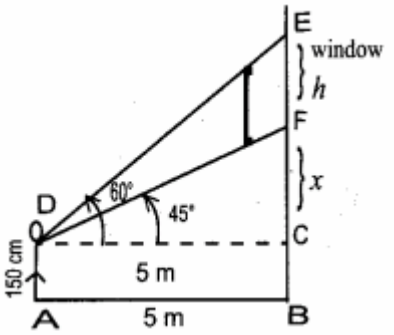
∴ $8AE^2 = 3AC^2 + 5AD^2$

A K RAJADHURAI, GRADUATE TEACHER (MATHS), GGHSS, PODATURPET, TIRUVALLUR DISTRICT

18. To a man standing outside his house, the angles of elevation of the top and bottom of a window are 60° and 45° respectively. If the height of the man is 180 cm and if he is 5 m away from the wall, what is the height of the window? (√3 = 1.732)

SOLUTION:

Let the height of the window FE be "h" m



Let FC be "x" m

∴ In the right Δ CDF, **tan 45° =** $\frac{CF}{CD}$

$1 = \frac{x}{5} \Rightarrow x = 5$

In the right Δ CDE, **tan 60° =** $\frac{CE}{CD}$

$\sqrt{3} = \frac{x+h}{5} \Rightarrow x + h = 5\sqrt{3}$

$5 + h = 5\sqrt{3}$ (substitute the value of x)

$h = 5\sqrt{3} - 5 = 5 \times 1.732 - 5 = 8.66 - 5 = 3.66$

∴ Height of the window = 3.66 m.

19. A girl wishes to prepare birthday caps in the form of right circular cones for her birthday party, using a sheet of paper whose area is 5720 cm^2, how many caps can be made with radius 5 cm and height 12 cm.

Solution:

r = 5 cm h = 12 cm

$l^2 = h^2 + r^2$

$l^2 = 25 + 144$

$l^2 = 169$

l = 13 cm

Required No. of caps = $\frac{\text{Area of the paper}}{Area\ of\ 1\ cap}$

$= \frac{5720}{\pi rl}$

$= \frac{5720}{\frac{22}{7}X5X13}$

= 28 caps

A K RAJADHURAI, GRADUATE TEACHER (MATHS), GGHSS, PODATURPET, TIRUVALLUR DISTRICT

20. The frustum shaped outer portion of the table lamp has to be painted including the top part. Find the total cost of painting the lamp if the cost of painting 1 sq.cm is ₹ 2.

Solution: Here given that R = 12 m

r = 6 m h = 8 m

$l^2=h^2+(R-r)^2$

$l^2=8^2+(12-6)^2$

$l^2=8^2+6^2$ =64+36 =100

l = 10 cm

Surface area of the table lamp = $\pi(R+r)l + \pi r^2$

$= \pi [(12+6)10+36]$

$= \frac{22}{7}$ x 216 = $\frac{4752}{7}$

= 678.86 sq. cm

The cost of painting 1 sq.cm is ₹ 2.

The cost of painting 678.86 sq.cm = ₹ 678.86 X 2 =₹ 1357.72

A K RAJADHURAI, GRADUATE TEACHER (MATHS), GGHSS, PODATURPET, TIRUVALLUR DISTRICT

21. State and prove Pythagoras theorem

Theorem 5 : Pythagoras Theorem

Statement

In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

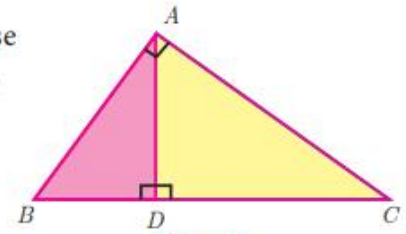


Fig. 4.46

Proof

Given : In ΔABC, $\angle A = 90^\circ$

To prove : $AB^2 + AC^2 = BC^2$

Construction : Draw $AD \perp BC$

No.	Statement	Reason
1.	Compare ΔABC and ΔDBA $\angle B$ is common $\angle BAC = \angle BDA = 90^\circ$ Therefore, $\Delta ABC \sim \Delta DBA$ $\frac{AB}{BD} = \frac{BC}{AB}$ $AB^2 = BC \times BD$ ... (1)	Given $\angle BAC = 90^\circ$ and by construction $\angle BDA = 90^\circ$ By AA similarity
2.	Compare ΔABC and ΔDAC $\angle C$ is common $\angle BAC = \angle ADC = 90^\circ$ Therefore, $\Delta ABC \sim \Delta DAC$ $\frac{BC}{AC} = \frac{AC}{DC}$ $AC^2 = BC \times DC$ ...(2)	Given $\angle BAC = 90^\circ$ and by construction $\angle ADC = 90^\circ$ By AA similarity

Adding (1) and (2) we get

$$AB^2 + AC^2 = BC \times BD + BC \times DC$$
$$= BC(BD + DC) = BC \times BC$$
$$AB^2 + AC^2 = BC^2.$$

Hence the theorem is proved.

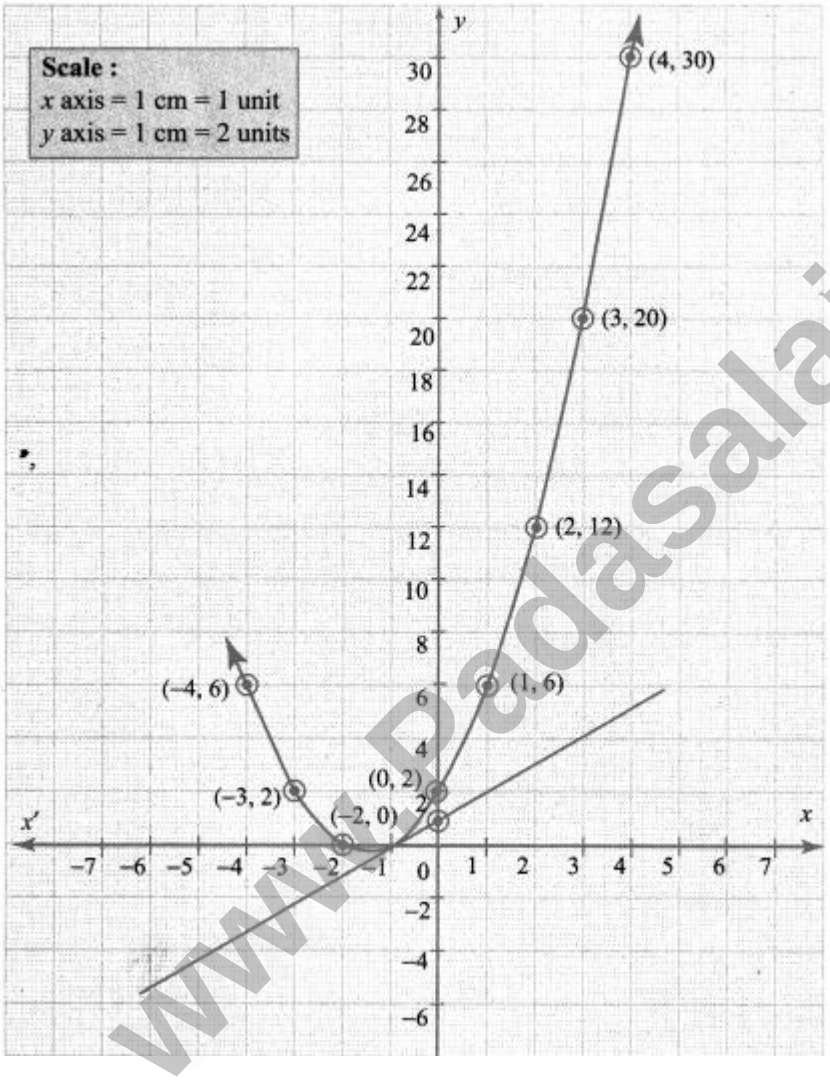
IV. Answer any one

22. a) Draw the graph of $x^2 + 3x + 2$ and use it to solve $x^2 + 2x + 1 = 0$

Table:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$3x$	–12	–9	–6	–3	0	3	6	9	12
2	2	2	2	2	2	2	2	2	2
$y = x^2 + 3x + 2$	6	2	0	0	2	6	12	20	30

Draw the parabola by plotting the point (-4, 6), (-3, 2), (-2, 0), (-1, 0), (0, 2), (1, 6), (2, 12), (3, 20), (4, 30).



To solve $x^2 + 2x + 1 = 0$, subtract $x^2 + 2x + 1 = 0$ from $y = x^2 + 3x + 2$

$$y = x^2 + 3x + 2$$

$$0 = x^2 + 2x + 1$$

(–) (–) (–) (–)

$y = x+1$

x	-2	0	2
1	1	1	1
$y = x + 1$	-1	1	3

Draw the straight line by plotting the points (-2, -1), (0, 1), (2, 3)

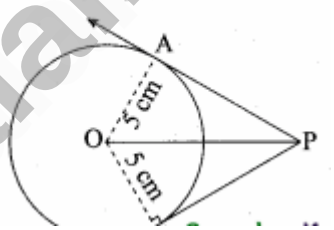
The straight line touches the parabola at the point (-1,0)

Therefore the x coordinate -1 is the only solution of the given equation

Solution = {-1,-1}

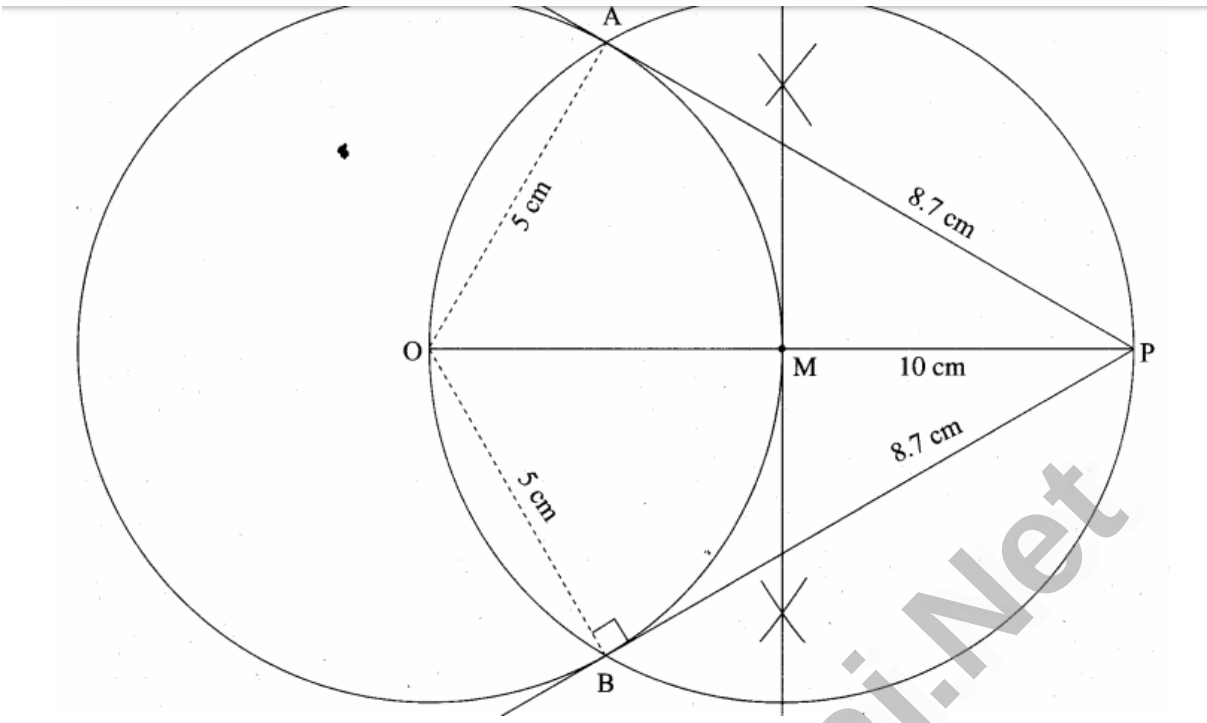
b) Draw the two tangents from a point which is 10cm away from the centre of the circle of radius 5 cm. Also measure the lenths of the tangents.

Rough Diagram



A K RAJADHURAI, GRADUATE TEACHER (MATHS), GGHSS, PODATURPET, TIRUVALLUR DISTRICT

Diagram



Length of tangent

$PA^2 = OP^2 + OA^2$

$PA^2 = 10^2 + 5^2 = 100+25 = 125$

PA = 8.7 cm

BY

A K RAJADHURAI,

GRADUATE TEACHER (MATHS),

GGHSS,

PODATURPET,

TIRUVALLUR DISTRICT,

MOBILE:9843938366