Class : 11

Register Number

SECOND MID TERM TEST-NOVEMBER-2023

Time Allowed : 1.30 Hours] **MATHEMATICS** **[Max. Marks : 45**

Part - I

I. Choose the correct answer from the given four alternatives. **10x1=10**

1. If $A = \begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix}$ then for what value of λ, $A^2 = 0$?

   (a) 0 (b) ± 1 (c) -1 (d) 1

2. If A is a square matrix, then which of the following is not symmetric?

   (a) $A+A^T$ (b) AA^T (c) A^TA (d) $A-A^T$

3. If the points (x,-2) (5,2) (8,8) are collinear, then x is equal to

   (a) -3 (b) 1/3 (c) 1 (d) 3

4. The value of $\begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix}$ is

   (a) 0 (b) -Sin θ (c) Cos θ (d) 1

5. If $\begin{vmatrix} x-1 & x & x-2 \\ 0 & x-2 & x-3 \\ 0 & 0 & x-3 \end{vmatrix} = 0$ then the value of x is

   (a) x = 1, 2,3 (b) x=1, -1, -2 (c) x = 0, 2,-3 (d) x = -1, -2, -3

6. If $|\bar{a}| = 13$, $|\bar{b}| = 5$ and $\bar{a} \cdot \bar{b} = 60^\circ$ then the value of $|\bar{a} \times \bar{b}|$

   (a) 15 (b) 35 (c) 45 (d) 25

7. If $\bar{a}$ and $\bar{b}$ bowing same magnitude and angle between them is 60° and their scalar product is ½ then $|\bar{a}|$ is

   (a) 2 (b) 3 (c) 7 (d) 1

8. If $\bar{a} = \hat{i} + \hat{j} + \hat{k}$ $\bar{b} = 2\hat{i} + x\hat{j} + \hat{k}$, $\bar{c} = \hat{i} - \hat{j} + 4\hat{k}$ and $\bar{a} \cdot (\bar{b} \times \bar{c}) = 70$ then x is equal to

   (a) 5 (b) 7 (c) 26 (d) 10

9. $\lim_{x \to \infty} \frac{\sin x}{x} =$

   (a) 1 (b) 0 (c) ∞ (d) $-\infty$

10. $\lim_{x \to 3} [x] =$ ----------

    (a) 2 (b) 3 (c) No value (d) 0

PART – II

II. Answer any four questions. Q.No.17 is compulsory question. **4X2=8**

11. Simplify : $\sec\theta \begin{bmatrix} \sec\theta & \tan\theta \\ \tan\theta & \sec\theta \end{bmatrix} - \tan\theta \begin{bmatrix} \tan\theta & \sec\theta \\ \sec\theta & \tan\theta \end{bmatrix}$

12. Construct an m x n matrix A = [aij] where aij is given by $a_{ij} = \frac{(i-2j)^2}{2}$ with m = 2 and n = 3.

13. Compute |A| using Sarrus rule if $A = \begin{bmatrix} 3 & 4 & 1 \\ 0 & -1 & 2 \\ 5 & -2 & 6 \end{bmatrix}$

14. Find a unit vector along the direction of the vector $5\hat{i} - 3\hat{j} + 4\hat{k}$.

15. If $\bar{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\bar{b} = \hat{i} + \lambda\hat{j} + 3\hat{k}$ are parallel vectors then find the value of λ.

16. Let $f(x) = \begin{cases} x+1, & x > 0 \\ x-1, & x < 0 \end{cases}$ Verify the existence of limit as $x \to 0$.

17. Evaluate the limit $\lim\limits_{x \to 2} \dfrac{x^4 - 16}{x - 2}$

PART – III

III. Answer any four questions. Question No.24 is compulsory Question. **4X3=12**

18. Determine the value of x + y if $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$

19. Show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)\ (y-z)\ (z-x)$

20. For any vector $\bar{r}$ Prove that $\bar{r} = (\hat{r}.\hat{i})\hat{i} + (\hat{r}.\hat{j})\hat{j} + (\hat{r}.\hat{k})\hat{k}$.

21. Find the angle between the vectors $2\hat{i} + 3\hat{j} + 6\hat{k}$ and $6\hat{i} - 3\hat{j} + 2\hat{k}$

22. Find the area of the parallelogram whose adjacent sides are $\bar{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $b = \hat{i} - \hat{j} + \hat{k}$.

23. Calculate $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{x^2(x^2 - 6x + 9)}$

24. Evaluate $\lim\limits_{x \to 0} |x|$

PART – IV

IV. Answer All the questions. 2. Each questions carries 5 marks. **3X5= 15**

25. (a) Prove that $|A| = \begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix} = 2pqr\ (p+q+r)^3$

(Or)

(b) Express the matrix $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ as the sum of a symmetric matrix and a skew symmetric matrix.

26. (a) Prove that the points whose position vectors $2\hat{i} + 4\hat{j} + 3\hat{k}$, $4\hat{i} + \hat{j} + 9\hat{k}$ and $10\hat{i} - \hat{j} + 6\hat{k}$ form a right angled triangle.

(Or)

(b) If $\bar{a}, \bar{b}, \bar{c}$ are position vectors of the vertices A, B, C of a triangle ABC, show that the area of the tirangle ABC is $\frac{1}{2}|\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}|$ Also deduce the condition for collinearity of the points A, B and C.

27. (a) Show that $\lim\limits_{x \to 0^+} x\left[\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots + \left[\frac{15}{x}\right]\right] = 120$

(Or)

(b) A tank contains 5000 litres of pure water. Brine (very salty water) that contains 30 gms of salt per litre of water is pumped into the tank at a rate of 25 litres per minute. The concentration of salt water t minutes (in grams per litre) is $C(t) = \dfrac{30t}{200+t}$. What happens to the concentration as $t \to \infty$?

9-11-2023

ST. JOSEPH'S HR. SEC. SCHOOL - CHENGALPATTU

SECOND MID TERM TEST - NOVEMBER - 2023

STD : 11 MATHEMATICS - KEY

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PART - I 10×1 = 10

I 1.	b	±1
2.	d	$A - A^T$
3	d	3
4.	d	1
5	a	x = 1, 2, 3
6	d	25
7	d	1
8	c	26
9	b	0
10	c	No value (OR) b. 3

PART - II 4×2 = 8

II 11. $A = \begin{bmatrix} \sec^2\theta & \sec\theta\tan\theta \\ \sec\theta\tan\theta & \sec^2\theta \end{bmatrix} - \begin{bmatrix} \tan^2\theta & \tan\theta\sec\theta \\ \sec\theta\tan\theta & \tan^2\theta \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

12. $A = \frac{1}{2}\begin{bmatrix} 1 & 9 & 25 \\ 0 & 4 & 16 \end{bmatrix}$ OR $A = \begin{bmatrix} 1/2 & 9/2 & 25/2 \\ 0 & 2 & 8 \end{bmatrix}$

13.
$$\begin{vmatrix} 3 & 4 & 1 & 3 & 4 \\ 0 & -1 & 2 & 0 & -1 \\ 5 & -2 & 6 & 5 & -2 \end{vmatrix}$$

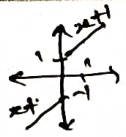
$|A| = 22 + 17 = 39$

14 $\frac{\vec{a}}{|\vec{a}|} = \frac{5\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{50}}$

15. $a = tb \Rightarrow 3\hat{i} + 2\hat{j} + q\hat{k} = t(\hat{i} + \lambda\hat{j} + 3\hat{k})$

$t = 3 \Rightarrow 2 = 3\lambda \Rightarrow \lambda = 2/3$

16. $\lim_{x\to 0^-} f(x) = -1$, $\lim_{x\to 0^+} f(x) = 1$



$\lim_{x\to 0} f(x)$ does not exist

17. $\lim_{x\to 2} \frac{x^4 - 2^4}{x - 2} = 4(2)^{4-1} = 32$

PART - III 4×3 = 12

III 18. $x = 2$, $y = 3 \Rightarrow x + y = 5$

19.
$$= \begin{vmatrix} 1 & 0 & 0 \\ x & y-x & z-x \\ x^2 & y^2-x^2 & z^2-x^2 \end{vmatrix} = (y-x)(z-x)\begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 1 \\ x^2 & y+x & z+x \end{vmatrix}$$

$= (x-y)(y-z)(z-x)$

20. $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{r}\cdot\hat{i} = (x\hat{i} + y\hat{j} + z\hat{k})\cdot\hat{i} = x$

If $\vec{r}\cdot\hat{j} = y$, $\vec{r}\cdot\hat{k} = z$

$(\vec{r}\cdot\hat{i})\hat{i} + (\vec{r}\cdot\hat{j})\hat{j} + (\vec{r}\cdot\hat{k})\hat{k} = x\hat{i} + y\hat{j} + z\hat{k} = \vec{r}$

21 $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}$, $\vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{a}\cdot\vec{b} = -9$, $|a| = 7$, $|b| = 7$

$\theta = \cos^{-1}(-9/49)$

22 $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix} = 5\hat{i} + \hat{j} - 4\hat{k}$

$|\vec{a}\times\vec{b}| = \sqrt{42}$ sq. units

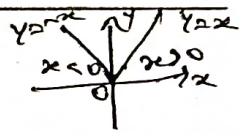
23 $\lim_{x\to 3^-} \frac{(x-3)(x+3)}{x^2(x-3)^2} = \lim_{x\to 3^-} \frac{x+3}{x^2(x-3)} = -\infty$

$f(3) \to -\infty$ as $x \to 3^-$

$\lim_{x\to 3^+} \frac{(x-3)(x+3)}{x^2(x-3)^2} = \lim_{x\to 3^+} \frac{(x+3)}{x^2(x-3)} = +\infty$

$f(3) \to \infty$ as $x \to 3^+$

24. $|x| = \begin{cases} -x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ x & \text{if } x > 0 \end{cases}$



$\lim_{x\to 0^-} |x| = 0 = \lim_{x\to 0^+} |x| \Rightarrow \lim_{x\to 0} |x| = 0$

PART - IV 3×5 = 15

IV 25 a. $p = 0 \Rightarrow |A| = 0 \Rightarrow (p-0)$ is a factor

$|A|$ is cyclic symmetric, p, q, r, also factor

$n = 6 - 5 = 1$

$$\begin{vmatrix} (p+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix} = k(p+q+r)(p+q+r)^2 \times pqr$$

$p = 1, q = 1, r = 1 \Rightarrow \begin{vmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{vmatrix} = k(3)^2(1)(1)(1)$

$\Rightarrow k = 2$

b. $A^T = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

$P = \frac{1}{2}(A + A^T) = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$, $P^T = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} = P$

P is a symmetric matrix

$Q = \frac{1}{2}(A - A^T) = \frac{1}{2}\begin{bmatrix} 0 & -5 & 3 \\ 5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$, $Q^T = \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix} = -Q$

Q is skew symmetric matrix

$A = P + Q$

26. a. $\vec{OA} = 2i+4j+3k$

$\vec{OB} = 4i+4j+9k$

$\vec{OC} = 10i-j+6k$

$\vec{AB} = 2i-3j+6k \Rightarrow |AB| = 7$

$\vec{BC} = 6i-2j-3k \Rightarrow |BC| = 7$

$\vec{CA} = -8i+5j-3k \Rightarrow |CA| = \sqrt{98}$

$CA^2 = BC^2 + AB^2$

$98 = 49+49$

$98 = 98$

b. $\vec{AB} = \vec{b}-\vec{a}$

$\vec{AC} = \vec{c}-\vec{a}$

Area of $\Delta ABC = \frac{1}{2}\left|\vec{AB} \times \vec{AC}\right|$

$= \frac{1}{2}\left|(\vec{b}-\vec{a}) \times (\vec{c}-\vec{a})\right|$

$= \frac{1}{2}\left|b\times c + a\times b + c\times a\right|$

$0 = \frac{1}{2}\left|a\times b + b\times c + c\times a\right|$

$a\times b + b\times c + c\times a = \vec{0}$

27 a. $\frac{1}{x} - 1 \le \left\lfloor \frac{1}{x} \right\rfloor \le \frac{1}{x} + 1$

$\vdots$

$\frac{15}{x} - 1 \le \left\lfloor \frac{15}{x} \right\rfloor \le \frac{15}{x} + 1$

$120 - 15x \le x\left[\left\lfloor \frac{1}{x} \right\rfloor + \left\lfloor \frac{2}{x} \right\rfloor + \cdots + \left\lfloor \frac{15}{x} \right\rfloor\right] \le 120 + 15x$

$\lim_{x\to 0^+}(120-15x) \le \lim_{x\to 0^+} x\left[\left\lfloor \frac{1}{x} \right\rfloor + \left\lfloor \frac{2}{x} \right\rfloor + \cdots + \left\lfloor \frac{15}{x} \right\rfloor\right] \le \lim_{x\to 0^+}(120-15x)$

$\lim_{x\to 0^+} x\left[\left\lfloor \frac{1}{x} \right\rfloor + \left\lfloor \frac{2}{x} \right\rfloor + \cdots + \left\lfloor \frac{15}{x} \right\rfloor\right] = 120$

b. $c(t) = \frac{30t}{200+t}$

$t \to \infty$, $c(t) = \lim_{t\to\infty} \frac{30t}{200+t}$

$= \lim_{t\to\infty} \frac{30t}{t\left(1+\frac{200}{t}\right)} = \frac{30}{1}$

$= 30$

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