SECOND MID TERM TEST - 2023

MATHS

STD - XI **MARKS : 45**

TIME : 1.30 Hrs

I. Choose the correct Answer. **10 x 1 = 10**

1. If $A = \begin{pmatrix} a & x \\ y & a \end{pmatrix}$ and if xy = 1 then det(AA^T) is equal to

   a) $(a^2 + 1)^2$ b) $(a - 1)^2$ c) $a^2 - 1$ d) $(a^2 - 1)^2$

2. If $A = \begin{pmatrix} I & 1 \\ -1 & -I \end{pmatrix}$ then for what value of λ, $A^2 = 0$?

   a) 0 b) ± 1 c) -1 d) 1

3. Find the value of $\begin{vmatrix} Cosq & Sinq \\ -Sinq & Cosq \end{vmatrix}$ = a) 0 b) 1 c) -1 d) 2

4. If $\vec{a} + 2\vec{b}$ and $3\vec{a} + m\vec{b}$ are parallel then the value of m is

   a) 3 b) 1 / 3 c) 6 d) 1 /6

5. If $\vec{a} = \hat{i} + \hat{j} + 8\hat{k}$, $\vec{b} = 3\hat{j} - \hat{k}$ then the value of $\vec{a} - b$ is a) 11 b) -11 c) -5 d) 5

6. $|\vec{a}| = 13$ $|\vec{b}| = 5$ and $\vec{a}.\vec{b} = 60°$ then $|\vec{a} \times \vec{b}|$ is a) 15 b) 35 c) 45 d) 25

7. The value of $\lim_{x \to 0} \frac{Sinx}{\sqrt{x^2}}$ is a) 1 b) -1 c) 0 d) limit does not exist

8. $\lim_{x \to \infty} \left(\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + ... + \frac{n}{n^2} \right)$ is a) 1 / 2 b) 0 c) 1 d) ∞

9. $\lim_{x \to \infty} \frac{a^x - b^x}{x}$ = a) log ab b) $\log \frac{a}{b}$ c) $\log \frac{b}{a}$ d) $\frac{a}{b}$

10. $\lim_{x \to 0} \frac{e^{Sinx} - 1}{x}$ a) 1 b) e c) 1 / e d) 0

II. Answer any four questions. Q.No. 16 is compulsory **4 x 2 = 8**

11. $\begin{pmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{pmatrix} = \begin{pmatrix} 7 & 7y-13 \\ y & x+6 \end{pmatrix}$ find x + y.

12. Find $|\vec{a} \times \vec{b}|$ where $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$

13. Find the direction cosines of $3\hat{i} - 4\hat{k}$

14. Evaluate $\lim_{x \to 0} (1 + \sin x)^{2Cosec\, x}$

15. Evaluate $\lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2}$

16. Construct the matrix $A = [a_{ij}]_{3\times 3}$ where $a_{ij} = i - j$. State whether A is symmetric or skew - symmetric.

III. Answer any four questions. Q.No.22 is compulsory **4 x 3 = 12**

17. Determine if f defined by $f(x)=\begin{cases} x^2 \sin\frac{1}{x} & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$ is continuous in $\mathbb{R}$

18. If $\vec{a}, \vec{b}$ are unit vectors and θ is the angle between them, show that $\cot\frac{q}{2}=\frac{|\vec{a}+\vec{b}|}{|\vec{a}-\vec{b}|}$

19. Fine the area of the triangle whose Vertices are (-2, -3) (3, 2) (-1, -8)

20. If G is the centroid of a triangle ABC, prove that $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=\vec{0}$

21. If Cos 2θ = 0 determine $\begin{vmatrix} 0 & Cosq & Sinq \\ Cosq & Sinq & 0 \\ Sinq & 0 & Cosq \end{vmatrix}^2$

22. If f and g are continuous functions with f(3) = 5 and $\lim_{x\to 3}[2f(x)-g(x)]=4$ find g(3).

IV. Answer all the questions. **3 x 5 = 15**

23. a) Prove that the points whose position vectors $2\hat{i}+4\hat{j}+3\hat{k}$, $4\hat{i}+\hat{j}+9\hat{k}$ and $w\hat{i}-\hat{j}+6\hat{k}$ from a right angled triangle

(OR)

b) Show that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

24. a) Using Factor Theorem then show that $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$

(OR)

b) Show that $\lim_{n\to\infty} \frac{1^2+2^2+3^2+.....(3n)^2}{(1+2+3+....+(5n)(2n+3)} = \frac{9}{25}$

25. a) Prove that the medians of a triangle are concurrent.

(OR)

b) Prove that $\lim_{q\to 0} \frac{Sinq}{q} = 1$