St.ANNE'S ACADEMY
Punnai Nagar, Nagercoil - 4.
(Maths - Chapters 8, 9 & 10)

Time : 3 Hrs Marks : 90

PART – I

I. Answer ALL questions **20x1 = 20**

1) If $u(x,y)=e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

(1) $e^{x^2+y^2}$ (2) $2xu$ (3) x^2u (4) y^2u

2) If $v(x,y)=\log(e^x+e^y)$, then $\frac{\partial v}{\partial x}+\frac{\partial v}{\partial y}$ is equal to

(1) e^x+e^y (2) $\frac{1}{e^x+e^y}$ (3) 2 (4) 1

3) Linear approximation for $g(x)=\cos x$ at $x=\frac{\pi}{2}$ is

(1) $x+\frac{\pi}{2}$ (2) $-x+\frac{\pi}{2}$ (3) $x-\frac{\pi}{2}$ (4) $-x-\frac{\pi}{2}$

4) If $f(x,y,z)=xy+yz+zx$, then f_x-f_z is equal

(1) $z-x$ (2) $y-z$ (3) $x-z$ (4) $y-x$

5) If $g(x,y)=3x^2-5y+2y^2, x(t)=e^t$ and $y(t)=\cos t$, then $\frac{dg}{dt}$ is equal to

(1) $6e^{2t}+5\sin t-4\cos t\sin t$
(2) $6e^{2t}-5\sin t+4\cos t\sin t$
(3) $3e^{2t}+5\sin t+4\cos t\sin t$
(4) $3e^{2t}-5\sin t+4\cos t\sin t$

6) If $f(x,y)=e^{xy}$, then $\frac{\partial^2 f}{\partial x\partial y}$ is equal to

(1) xye^{xy} (2) $(1+xy)e^{xy}$
(3) $(1+y)e^{xy}$ (4) $(1+x)e^{xy}$

7) The value of $\int_{-1}^{2}|x|\,dx$ is

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{7}{2}$

8) If $f(x)=\int_0^x t\cos t\,dt$, then $\frac{df}{dx}=$

(1) $\cos x-x\sin x$ (2) $\sin x+x\cos x$
(3) $x\cos x$ (4) $x\sin x$

9) The area between $y^2=4x$ and its latus rectum is

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{8}{3}$ (4) $\frac{5}{3}$

10) If $\frac{\Gamma(n+2)}{\Gamma(n)}=90$ then n is

(1) 10 (2) 5 (3) 8 (4) 9

11) The value of $\int_0^\infty e^{-3x}x^2dx$ is

(1) $\frac{7}{27}$ (2) $\frac{5}{27}$ (3) $\frac{4}{27}$ (4) $\frac{2}{27}$

12) The value of $\int_0^a\left(\sqrt{a^2-x^2}\right)^3dx$ is

(1) $\frac{\pi a^3}{16}$ (2) $\frac{3\pi a^4}{16}$ (3) $\frac{3\pi a^2}{8}$ (4) $\frac{3\pi a^4}{8}$

13) The general solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}$ is

(1) $xy=k$ (2) $y=k\log x$ (3) $y=kx$ (4) $\log y=k$

14) The solution of the differential equation $2x\frac{dy}{dx}-y=3$ represents

(1) straight lines (2) circles
(3) parabola (4) ellipse

15) The number of arbitrary constants in the general solutions of order n and $n+1$ are respectively

(1) $n-1,\ n$ (2) $n,\ n+1$
(3) $n+1,\ n+2$ (4) $n+1,\ n$

16) The degree of the differential equation

$$y(x)=1+\frac{dy}{dx}+\frac{1}{1\cdot2}\left(\frac{dy}{dx}\right)^2+\frac{1}{1\cdot2\cdot3}\left(\frac{dy}{dx}\right)^3+\ldots \text{ is}$$

(1) 2 (2) 3 (3) 1 (4) 4

17) The solution of the differential equation $\frac{dy}{dx}=2xy$ is

(1) $y=Ce^{x^2}$ (2) $y=2x^2+C$
(3) $y=Ce^{x^2}+C$ (4) $y=x^2+C$

18) If the solution of the differential equation $\frac{dy}{dx}=\frac{ax+3}{2y+f}$ represents a circle, then the value of a is

(1) 2 (2) –2 (3) 1 (4) –1

19) For the harmonic function $V(x,y)=e^x(x\cos y-y\sin y)$

$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=$

(1) -1 (2) -3 (3) 0 (4) 1

20) The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}x\cos x\,dx$ is

(1) 0 (2) 5 (3) 1 (4) 9

PART – II

II. Answer any seven questions. **7x2 = 14**

Question No.30 is Compulsory

21) If the radius of a sphere, with radius 10 cm, has to decrease by 0.1 cm, approximately how much will its volume decrease?

22) Let $V(x, y, z) = xy + yz + zx, x, y, z \in \mathbb{R}$. Find the differential dV.

23) Evaluate $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x \cos x \, dx$

24) Evaluate $\int_{0}^{\frac{\pi}{2}} \left(\sin^2 x + \cos^4 x\right) dx$

25) Evaluate $\int_{0}^{1} x dx$, as the limit of a sum.

26) Determine the order and degree

$$3\left(\frac{d^2y}{dx^2}\right) = \left[4 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$$

27) Solve $(1+x^2)\frac{dy}{dx} = 1 + y^2$

28) Form the differential equation by eliminating the arbitrary constants A and B from

$$y = A\cos x + B\sin x$$

29) Prove that $\frac{dy}{dx} = \frac{x^3 + y^2}{2x^3 - xy^2}$ is not homogeneous.

30) Find Δf for the function f for the indicated values of $x, \Delta x$

$f(x) = x^2 + 2x + 3$; $x = -0.5, \Delta x = dx = 0.1$

PART – III

III. Answer any seven questions. **7x3 = 21**

Question No.40 is Compulsory

31) Solve $\frac{dy}{dx} - x\sqrt{25 - x^2} = 0$

32) Solve $\tan y \frac{dy}{dx} = \cos(x + y) + \cos(x - y)$

33) Solve $\frac{dy}{dx} + \frac{y}{x \log x} = \frac{\sin 2x}{\log x}$

34) Evaluate: $\int_{0}^{\frac{\pi}{2}} \frac{e^{-\tan x}}{\cos^6 x} dx$

35) Show that $\int_{0}^{\frac{\pi}{2}} \frac{dx}{4 + 5\sin x} = \frac{1}{3}\log_e 2$

36) Find the area of the region bounded by $2x - y + 1 = 0$, $y = -1$, $y = 3$ and y-axis

37) Show that the percentage error in the n^{th} root of a number is approximately $1/n$ times the percentage error in the number

38) Assuming $\log_{10} e = 0.4343$, find an approximate value of $\log_{10} 1003$.

39) If $U(x, y, z) = \log(x^3 + y^3 + z^3)$, find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial v} + \frac{\partial U}{\partial z}$

40) Solve $\frac{dy}{dx} + 2y = e^{-x}$.

PART – IV

IV. Answer all questions. **7x5 = 35**

41) a) Let $f(x, y) = \sin(xy^2) + e^{x^3 + 5y}$ for all $(x, y) \in \mathbb{R}^2$. Check whether $f_{xy} = f_{yx}$

Or

b) Find the area of the region bounded by $y = \tan x$, $y = \cot x$ and the lines $x = 0$, $x = \frac{\pi}{2}$, $y = 0$.

42) a) Find the volume of the spherical cap of height 'h' cut of from a sphere of radius r.

Or

b) $\left(x^2 + y^2\right)dy = xy\, dx$. It is given that $y(1) = 1$ and $y(x_0) = e$. Find the value of x_0.

43) a) Water at temperature $100^\circ C$ cools in 10 minutes to $80^\circ C$ in a room temperature of $25^\circ C$. Find

(i) The temperature of water after 20 minutes

(ii) The time when the temperature is $40^\circ C$

$$\left[\log_e \frac{11}{15} = -0.3101;\ \log_e 5 = 1.6094\right]$$

Or

b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$.

44) a) If $w(x, y, z) = x^2 + y^2 + z^2, x = e^t, y = e^t \sin t$ and $z = e^t \cos t$, find $\frac{dw}{dt}$.

Or

b) Evaluate by using properties of integration :

$$\int_{0}^{\sin^2 x} \sin^{-1}\sqrt{t}\, dt + \int_{0}^{\cos^2 x} \cos^{-1}\sqrt{t}\, dt$$

45) a) Solve $\left(1+2e^{x/y}\right)dx+2e^{x/y}\left(1-\frac{x}{y}\right)dy=0$.

Or

b) Find the area of the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

46) a) Evaluate $\int_0^{\infty}\frac{x^n}{n^x}dx$, where n is a positive integer ≥ 2.

Or

b) A tank contains 1000 litres of water in which 100 grams of salt is dissolved. Brine runs in a rate of 10 litres per minute, and each litre contains 5grams of dissolved salt. The mixture of the tank is kept uniform by stirring. Brine runs out at 10 litres per minute. Find the amount of salt at any time t.

47) a) Consider $f(x,y)=\frac{xy}{x^2+y^2}$ if $(x,y)\neq(0,0)$ and $f(0,0)=0$. Show that f is not continuous at $(0,0)$ and continuous at all other points of $\mathbb{R}^2$.

Or

b) Find the area of the region bounded by $y=\cos x, y=\sin x$, the lines $x=\frac{\pi}{4}$ and $x=\frac{5\pi}{4}$

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