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Register No.

Second Mid-Term Test - 2023

MATHEMATICS

Time : 1.30 Hrs. Marks : 50

PART - I

I. Choose the correct answer **10 x 1 = 10**

1. The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when a) $y = 0$ b) $y = \pm \sqrt{3}$ b) $y = \frac{1}{2}$ d) $y = \pm 3$
2. A stone is thrown up vertically. The height iit reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by a) 2 b) 2.5 c) 3 d) 3.5
3. The number given by the Rolle's theorem for the function $x^3 - 3x^2$, $x + [0, 3]$ is

   a) 1 b) $\sqrt{2}$ c) $\frac{3}{2}$ d) 2

4. The point of Inflection of the curve $y = \frac{3x}{x^2 - 1}$ is a) (0, 0) b) (0, 1) b) (1, 0) d) (1, 1)
5. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

   a) $\frac{1}{31}$ b) $\frac{1}{5}$ c) 5 d) 31

6. If $g(x, y) = 3x^2 - 5y + 2y^2$, $x(t) = e^t$ and $y(t) = \cos t$, then $\frac{dg}{dt}$ is equal to

   a) $6e^{2t} + 5 \sin t - 4 \cos t \sin t$ b) $6e^{2t} - 5\sin t + 4\cos t \sin t$ c) $3e^{2t} + 5\sin t + 4\cos t \sin t$
   d) $3e^{2t} - 5\sin t + 4\cos t \sin t$
7. If $f(x, y, z) = xy + yz + zx$, then $f_x - f_z$ is equal to a) $z - x$ b) $y - z$ c) $x - z$ d) $y - x$
8. The value of $\int_{-1}^{2} |x| \, dx$ is a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) $\frac{7}{2}$
9. If $\frac{\Gamma(n+2)}{\Gamma(n)} = 90$ then n is a) 10 b) 5 c) 8 d) 9
10. If $\int_{0}^{a} \frac{1}{4 + x^2} \, dx = \frac{\pi}{8}$ then a is a) 4 b) 1 c) 3 d) 2

II. Answer any 5 of the following. Question No.17 is compulsory **5 x 2 = 10**

11. If the volume of a cube of side length x is $v = x^3$. Find the rate of change of the volume with respect to x when x = 5 units.
12. Find the angle of intersection of the cure $y = \sin x$ with the positive x - axis.
13. Find the equations of tangent and normal to the curve $y = x^2 - x^4$ at the point (1, 0).
14. Let $g(x) = x^2 + \sin x$ calculate the differential dg.
15. Evaluate $\int_{0}^{1} x^3 (1-x)^4 \, dx$
16. Show that the percentage error in the nth root of a number is approximately $\frac{1}{n}$ times the percetage error in the number.

17. Explain why Rolle's theorem is not applicable to the following functions in the respective interval.

$f(x) = \sin x, \quad x \in [0, \frac{\pi}{2}]$

III. Answer any 5 of the following. Question number 24 is compulsory. **5 x 3 = 15**

18. If $w(x, y, z) = x^2y + y^2z + z^2x$, $x, y, z \in R$, find the differential dw.
19. Assuming $\log_{10} e = 0.4343$, find an approximate value of $\log_{10} 1003$.
20. Evaluate $\lim_{x \to \infty} \frac{2x^2 - 3}{x^2 - 5x + 3}$
21. Find the local extremum of the function $f(x) = x^4 + 32x$
22. Evaluate $\int_2^3 \frac{\sqrt{x}}{\sqrt{5 - x + \sqrt{x}}} dx$
23. If $v(x, y) = \log\left(\frac{x^2 + y^2}{x + y}\right)$ prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$
24. If $U(x, y, z) = \frac{x^2 + y^2}{xy} + 3z^2y$ find $\frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial z}$

IV. Answer all the questions. **3 x 5 = 15**

25. a) A particle moves along a horizontal line such that its position at any time $t > 0$ is given by $s(t) = t^3 - 6t^2 + 9t + 1$, where s is measured in metres and t in seconds?
i) At what time the particle is at rest?
ii) At what time the particle changes its direction?
iii) Find the total distance travelled by the particle in the first 2 seconds. **(OR)**
b) A rectangular page is to contain $24cm^2$ of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1cm. What should be the dimensions of the page so that the area of the paper used in minimum.

26. a) Evaluate $\lim_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$
b) Find the interals of monotonicites and hence find the local extremum of the function $f(x)\ 2x^3 + 3x^2 - 12x$

27. a) $W(x, y, z) = xy + yz + zx$, $x = u - v$, $y = uv$, $z = u + v$, $u, v \in R$. Find $\frac{\partial w}{\partial u}, \frac{\partial w}{\partial v}$ and evaluate them at $\left(\frac{1}{2}, 1\right)$

(OR)

b) Evaluate $\int_1^4 (2x^2 + 3)\, dx$ as the limit of a sum.