

SECOND MID TERM TEST - 2023

STD - XII

MATHS

TIME : 1.30 Hrs

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MARKS : 45

I. Choose the correct Answer.

10 x 1 = 10

- The tangent to the curve $y^2 - xy + 9 = 0$ is vertical
 - $y = 0$
 - $y = \pm \sqrt{3}$
 - $y = \frac{1}{2}$
 - $y = \pm 3$
- Angle between $y^2 = x$ and $x^2 = y$ at the origin is
 - $\tan^{-1} \frac{3}{4}$
 - $\tan^{-1} \frac{4}{3}$
 - $\frac{\pi}{2}$
 - $\frac{\pi}{4}$
- The position of a particle moving along a horizontal line of any time t is given $S(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is
 - $t = 0$
 - $t = 1/3$
 - $t = 1$
 - $t = 3$
- The value of $\lim_{x \rightarrow \infty} \frac{x}{\log x}$ is
 - 0
 - 1
 - 2
 - ∞
- The point of inflection of the curve $y = (x - 1)^3$ is
 - (0,0)
 - (0,1)
 - (1,0)
 - (1,1)
- If $w(x,y) = x^y$, $x > 0$ then $\frac{\partial w}{\partial x}$ is equal to
 - $x^y \log x$
 - $y \log x$
 - yx^{y-1}
 - $x \log y$
- The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is
 - $0.3 x dx \text{ m}^3$
 - $0.03 x \text{ m}^3$
 - $0.03 x^2 \text{ m}^3$
 - $0.03 x^3 \text{ m}^3$
- The percentage error of fifth root of 31 is approximately how many times the percentage error in 31.
 - $\frac{1}{31}$
 - $\frac{1}{5}$
 - 5
 - 31
- The maximum value of the function $x^2 e^{-2x}$, $x > 0$ is
 - $\frac{1}{e}$
 - $\frac{1}{2e}$
 - $\frac{1}{e^2}$
 - $\frac{4}{e^4}$
- If $f(x, y, z) = xy + yz + zx$, then $f_x - f_z$ is equal to
 - $z - x$
 - $y - z$
 - $x - z$
 - $y - x$

Part - II

Write any three questions. Q.No. 15 is compulsory

3 x 2 = 6

11. Evaluate : $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x^2 - 4x + 3}$

12. If $V(x, y, z) = x^3 + y^3 + z^3 + 3xyz$, then show that $\frac{\partial^2 V}{\partial y \partial z} = \frac{\partial^2 V}{\partial z \partial y}$

13. Find the absolute extrema of the function $f(x) = x^2 - 12x + 10$; [1, 2]

14. Write the Maclaurin series expansion of the function $\sin x$
 15. Find df for $f(x) = x^2 + 3x$ and evaluate it for $x = 2$ and $dx = 0.1$

Part - III

Write any three questions. Q.No.20 is compulsory

3 x 3 = 9

16. Use the linear approximation to find approximate value of $\sqrt[3]{26}$
 17. Find the intervals of monotonicity for the function $f(x) = x^{2/3}$
 18. Show that the value in the conclusion of the mean value theorem for $f(x) = 1/x$ on a closed interval of positive numbers $[a, b]$ is \sqrt{ab} .

19. if $g(x, y) = 2y + x^2$, $x = 2r - s$, $y = r^2 + 2s$, $r, s \in \mathbb{R}$ then find $\frac{\partial g}{\partial r}, \frac{\partial g}{\partial s}$

20. Find the equations of the tangent and normal to the curve $y = x^2 + 3x - 2$ at the point (1, 2)

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Part - IV

Answer all the questions.

4 x 5 = 20

21. a) Find the angle between the curves $y = x^2$ and $x = y^2$ at their points of intersections (0, 0) and (1, 1) (OR)
 b) Find the intervals of concavity and points of inflection for the function $f(x) = 4x^3 + 3x^2 - 6x + 1$
 22. a) The trunk of a tree has diameter 30cm. During the following year, the circumference grew 6cm.
 i) Approximately, how much did the tree's diameter grow?
 ii) What is the percentage increase in area of the tree's cross-section? (OR)
 b) A rectangular page is to contain 24cm² of print. The margins at the top and bottom of the page are 1.5cm and the margins at other sides of the page is 1cm. What should be the dimensions of the page. So that the area of the paper used is minimum.

23. a) Evaluate : $\lim_{x \rightarrow 1} \frac{1}{x^{1-x}}$ (OR)

b) if $w(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$, $(x, y, z) \neq (0, 0, 0)$ then show that $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = 0$.

24. a) Sketch the curve $y = \frac{x^3 - 3x}{x - 1}$ (OR)

- b) Prove that $f(x, y) = x^3 - 2x^2y + 3xy^2 + y^3$ is homogeneous ; what is the degree? Verify Euler's Theorem for f.