KS ACADEMY, SALEM

PG TRB, POLY TRB, UG TRB & AP TRB

COACHING CENTRE FOR PHYSICS

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UG TRB -Physics - Unit -9 Material

ALTERNATIONG CURRENT

An alternating voltage or current is one that periodically changes polarity. That is, it regularly changes from a positive value to a negative value, back to positive and so on

(1) **Equation :** Alternating current or voltage varying as sine function can be written as

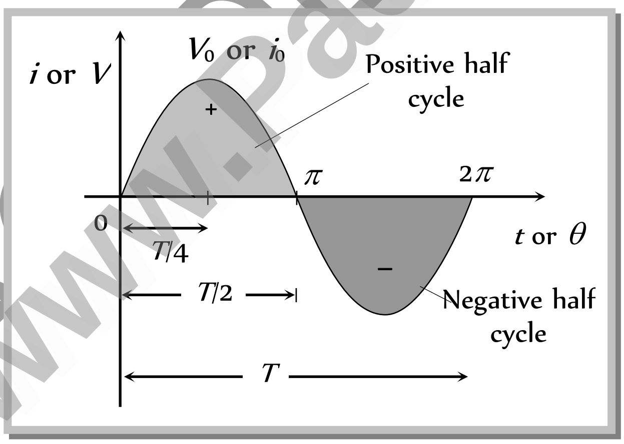
$i = i_0 \sin\omega t = i_0 \sin 2\pi\nu t = i_0 \sin \frac{2\pi}{T} t$

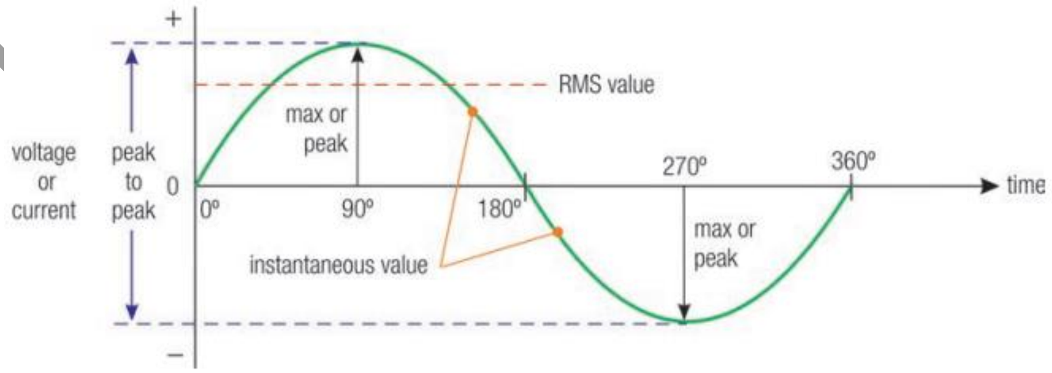
and $V = V_0 \sin \omega t = V_0 \sin 2\pi\nu t = V_0 \sin \frac{2\pi}{T} t$

where i and V = Instantaneous values of current and voltage,

i_0 and V_0 = Peak values of current and voltage

$$\text{since } \nu = \frac{1}{T}, \omega = 2\pi\nu$$





ω = Angular frequency in rad/sec, ν = Frequency in Hz and T = time period

(2) **cycle**

(i) The time taken to complete one cycle of variations is called the periodic time or time period.

(ii) Alternating quantity is positive for half the cycle and negative for the rest half. Hence average value of alternating quantity (i or V) over a complete **cycle is zero.**

(iii) Area under the positive half cycle is **equal to** area under negative cycle.

(iv) The value of alternating quantity is **zero** or **maximum 2ν times** every second. The direction also changes 2ν times every second.

(v) Generally sinusoidal waveform is used as alternating current/voltage.

(vi) At $t = \frac{T}{4}$ from the beginning, i or V reaches to their **maximum** value.

Important Values of Alternating Quantities.

(1) **Peak value (i_0 or V_0)**

The maximum value of alternating quantity (i or V) is defined as peak value or amplitude.

(2) **Mean square value ($\overline{V^2}$ or $\overline{i^2}$)**

The ***average of square of instantaneous values*** in one cycle is called mean square value.

It is always positive for one complete cycle. e.g. $\overline{V^2} = \frac{1}{T}\int_0^T V^2 dt = \frac{V_0^2}{2}$ or $\overline{i^2} = \frac{i_0^2}{2}$

(3) **Root mean square (r.m.s.) value**

Root of mean of square of voltage or current in an ac circuit for one complete cycle is called r.m.s. value. It is denoted by V_{rms} or i_{rms}

The RMS value for any waveform can be found by:

- calculating a number of instantaneous values for the waveform
- calculating the square of each of these values
- adding the squared values
- dividing their sum by the number of values to get an average (or mean)
- finding the square root of the mean

$$i_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + \ldots\ldots}{n}} = \sqrt{\overline{i^2}}$$

$$i_{rms} = \frac{i_{max}}{\sqrt{2}} = 0.707\, i_0 = 70.7\%\ of\ i_0$$

similarly $V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707\ V_0\ = 70.7\%$ of V_0

(i) The r.m.s. value of alternating current is also called virtual value or effective value.

(ii) In general when values of voltage or current for alternating circuits are given, these are r.m.s. value.

(iii) ac ammeter and voltmeter are always measure r.m.s. value. Values printed on ac circuits are r.m.s. values.

(iv) In our houses ac is supplied at 220 V, which is the r.m.s. value of voltage. It's peak value is $\sqrt{2} \times 200\ = 311\,V.$

(v) r.m.s. value of ac is equal to that value of dc, which when passed through a resistance for a given time will produce the same amount of heat as produced by the alternating current when passed through the same resistance for same time.

The RMS value gives us a way of calculating the heat (or power) because an RMS current of one ampere has the same heating effect as a DC current of one ampere.

Note : ❑ r.m.s. value of a complex current wave (e.g. i = a sinω t + b cosω t) is equal to the square root of the sum of the squares of the r.m.s. values of it's individual components i.e.

$$i_{rms} = \sqrt{\left(\frac{a}{\sqrt{2}}\right)^2 + \left(\frac{b}{\sqrt{2}}\right)^2} = \frac{1}{\sqrt{2}}\left(\sqrt{a^2 + b^2}\right).$$

(4) Mean or Average value (i_{av} or V_{av})

The average of instantaneous values of current or voltage in one cycle is called it's mean value. The average value of alternating quantity for one complete cycle is zero.

The average value of ac over half cycle (t = 0 to T/2)

$$i_{av} = \frac{\int_0^{T/2} i\,dt}{\int_0^{T/2} dt} = \frac{2i_0}{\pi} = 0.637\, i_0 = 63.7\%$$ of i_0,

Similarly $V_{av} = \frac{2V_0}{\pi} = 0.637\, V_0 = 63.7\%$ of V_0.

The average of one cycle of a symmetrical sinewave is zero, the average of a half cycle is the Maximum voltage multiplied by 0.637

If you know the RMS value of a sinewave you can find its maximum and peak-to-peak values. The equation is simply a rearrangement of the previous equation. That is

$$V_{max} = \sqrt{2}\, V_{rms} = 1.41 V_{rms}$$

$$V_{P-P} = 2\, V_{rms} \;\; OR \;\; 2\sqrt{2}\, V_{rms} = 2.828\, V_{rms}$$

EX: 1 The period of a waveform is 20 milliseconds. What is its frequency?

Specific Examples

Currents	Average value (For complete cycle)	Peak value	r.m.s. value	Angular frequency
$i = i_0 \sin\omega t$	0	i_0	$\frac{i_0}{\sqrt{2}}$	ω
$i = i_0 \sin\omega t \cos\omega t$	0	$\frac{i_0}{2}$	$\frac{i_0}{2\sqrt{2}}$	2ω
$i = i_0 \sin\omega t + i_0 \cos\omega t$	0	$\sqrt{2}\, i_0$	i_0	ω

Phase.

Physical quantity which represents both the instantaneous value and direction of alternating quantity at any instant is called it's phase. It's a dimensionless quantity and it's unit is radian.

If an alternating quantity is expressed as $X = X_0 \sin(\omega t \pm \phi_0)$ then the argument of $\sin(\omega t + \phi)$ is called it's phase. Where ω t = instantaneous phase (changes with time) and

ϕ_0 = initial phase (constant w.r.t. time)

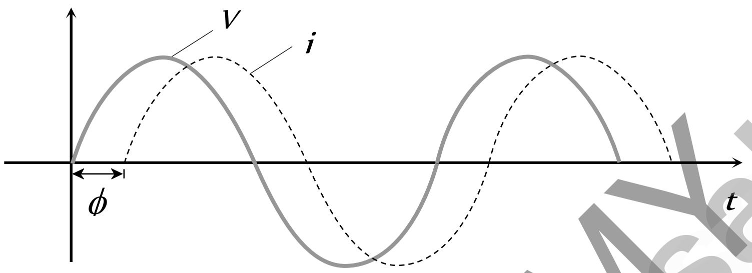
(1) **Phase difference** (Phase constant)

The difference between the phases of currents and voltage is called phase difference. If alternating voltage and current are given by $V = V_0 \sin(\omega t + \phi_1)$ and $i = i_0 \sin(\omega t + \phi_2)$ then phase difference $\phi = \phi_1 - \phi_2$ (relative to current) or $\phi = \phi_2 - \phi_1$ (relative to voltage)

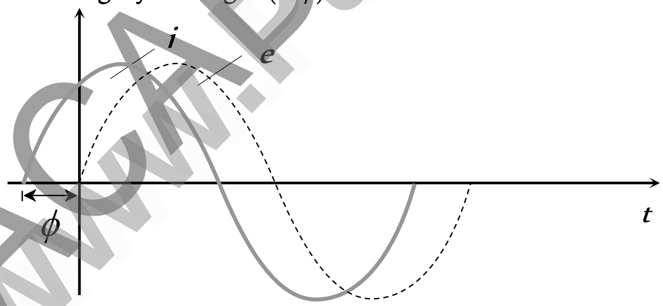
Note : ❑ Phase difference, generally is given relative to current.

❑ The quantity with higher phase is supposed to be leading and the other quantity is taken to be lagging.

(2) **Graphical representation**



Voltage $(V) = V_0 \sin \omega t$
Current $(i) = i_0 \sin(\omega t - \phi)$
Phase difference $= 0 - (-\phi) = +\phi$
i.e. voltage is leading by an angle $(+\phi)$ *w.r.t.* current



Voltage $(V) = V_0 \sin \omega t$
Current $(i) = i_0 \sin(\omega t + \phi)$
Phase difference $= 0 - (+\phi) = -\phi$
i.e. voltage is leading by an angle $(-\phi)$ *w.r.t.* current (or) voltage lags by an angle of ϕ *w.r.t current*

(3) **Time difference**

If phase difference between alternating current and voltage is ϕ then time difference between them is given as
$$\text{T.D.} = \frac{T}{2\pi} \times \phi$$

(4) **Phasor and phasor diagram**

AC circuits is much simplified by considering alternating current and alternating voltage as vectors with the angle between the vectors equals to the phase difference between the current and voltage.

The current and voltage are more appropriately called phasors. A diagram representing alternating current and alternating voltage (of same frequency) as vectors (phasors) with the phase angle between them is called a phasor diagram.

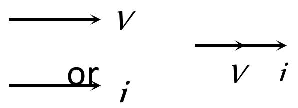
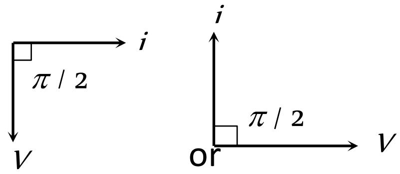
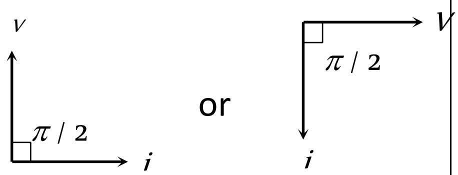
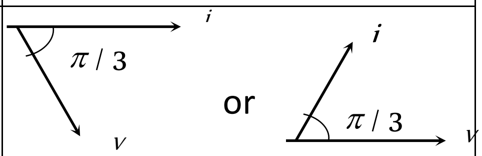
While drawing phasor diagram for a pure element (e.g. R, L or C) either of the current or voltage can be plotted along X-axis.

But when phasor diagram for a combination of elements is drawn then quantity which remains constant for the combination must be plotted along X-axis so we observe that

(a) In series circuits current has to be plotted along X-axis – because I=Constant

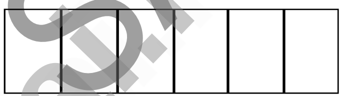
(b) In parallel circuits voltage has to be plotted along X-axis- because V=Constant

Specific Examples

Equation of V and i	**Phase difference ϕ**	**Time difference T.D.**	**Phasor diagram**
$V = V_0 \sin\omega t$ $i = i_0 \sin\omega t$	0	0	
$V = V_0 \sin\omega t$ $i = i_0 \sin(\omega t + \frac{\pi}{2})$	$-\frac{\pi}{2}$	$\frac{T}{4}$	
$V = V_0 \sin\omega t$ $i = i_0 \sin(\omega t - \frac{\pi}{2})$	$+\frac{\pi}{2}$	$\frac{T}{4}$	
$V = V_0 \sin\omega t$ $i = i_0 \sin(\omega t + \frac{\pi}{3})$	$-\frac{\pi}{3}$	$\frac{T}{6}$	

Measurement of Alternating Quantities.

Alternating current shows heating effect only, hence meters used for measuring ac are based on heating effect and are called hot wire meters (Hot wire ammeter and hot wire voltmeter)

ac measurement	dc measurement
(1) All ac meters read r.m.s. value.	(1) All dc meters read average value
(2) All ac meters are based on heating effect of current.	(2) All dc meters are based on magnetic effect of current
(3) Deflection in hot wire meters : $\theta \propto i_{rms}^2$  (non-linear scale)	(3) Deflection in dc meters : $\theta \propto i$  (Linear scale)

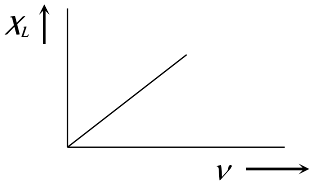
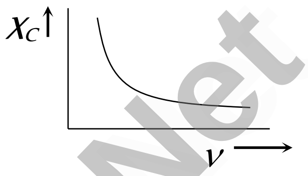
Note : ❑ ac meters can be used in measuring ac and dc both while dc meters cannot be used in measuring ac because the average value of alternating current and voltage over a full cycle is zero.

Terms Related to ac Circuits.

(1) **Resistance (R) :** The opposition offered by a conductor to the flow of current through it is defined as the resistance of that conductor. Reciprocal of resistance is known as conductance (G) i.e. $G = \frac{1}{R}$

(2) **Impedance (Z) :** The opposition offered by the capacitor, inductor and conductor to the flow of ac through it is defined as impedance. It's unit is ohm(Ω). $Z = \frac{V_0}{i_0} = \frac{V_{rms}}{i_{rms}}$

(3) **Reactance (X) :** The opposition offered by inductor or capacitor or both to the flow of ac through it is defined as reactance. It is of following two type –

Inductive reactance (X_L)	Capacitive reactance (X_C)
(i) Offered by inductive circuit(Inductor-Coil)	(i) Offered by capacitive circuit (capacitor)
(ii) $X_L = \omega L = 2\pi\nu L$	(ii) $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$
(iii) $\nu_{dc} = 0$ so for dc, $X_L = 0$	(iii) For dc $X_C = \infty$
(iv) X_L-ν Graph 	(iv) X_C - ν Graph 

Note : ❑ Resultant reactance of LC circuit is defined as $X = X_L \sim X_C$.

(4) **Admittance (Y) :** Reciprocal of impedance is known as admittance $\left(Y = \frac{1}{Z}\right)$. It's unit is mho.

(5) **Susceptance (S) :** the reciprocal of reactance is defined as susceptance $\left(S = \frac{1}{X}\right)$. It is of two type

(i) inductive susceptance $S_L = \frac{1}{X_L} = \frac{1}{2\pi\nu L}$ and (ii) Capacitive susceptance, $S_C = \frac{1}{X_C} = \omega C = 2\pi\nu C$.

Power and Power Factor.

The power is defined as the rate at which work is being done in the circuit.

In dc circuits power is given by P = Vi.

But in ac circuits, since there is some phase angle between voltage and current, therefore power is defined as the product of voltage and that component of ***the current which is in phase with the voltage.***

Thus $\boldsymbol{P = V\,i\cos\phi}$; where V and i are r.m.s. value of voltage and current.

(1) **Types of power**

There are three terms used for power in an ac circuit

(i) **Instantaneous power :** Suppose in a circuit $V = V_0 \sin\omega t$ and $i = i_0 \sin(\omega t + \phi)$ then

$$P_{\text{instantaneous}} = Vi = V_0 i_0 \sin\omega t \sin(\omega t + \phi)$$

(ii) **Average power (True power) :** The average of instantaneous power(average power) in an ac circuit over a full cycle is called true power.

It's unit is watt

i.e.

$$P_{av} = \overline{P}_{inst} \Rightarrow P_{av} = V_{rms} i_{rms} \cos\phi = \frac{V_0}{\sqrt{2}} \cdot \frac{i_0}{\sqrt{2}} \cos\phi = \frac{1}{2} V_0 i_0 \cos\phi$$

$$Power = i_{rms}{}^2 R = \frac{V_{rms}{}^2 R}{Z^2} \qquad Z = \frac{V_{rms}}{i_{rms}}$$

$$P_{av} = V_{rms} i_{rms} \cos\emptyset$$

P_{av} = Apparent power x Power factor

(iii) **Apparent or virtual power :** The product of apparent voltage and apparent current in an electric circuit is called apparent power. This is always positive

$$P_{app} = V_{rms} i_{rms} = \frac{V_0 i_0}{2}$$

(2) **Power factor :** It may be defined as

(i) Cosine of the angle of lag or lead

(ii) The ratio $\dfrac{\text{True power}}{\text{Apparent power}} = \dfrac{W}{VA} = \dfrac{kW}{kVA} = \cos\phi$

(iii) The ratio $\dfrac{R}{Z} = \dfrac{\text{Resistance}}{\text{Impedance}}$

Note : ❑ Power factor is a dimensionless quantity and it's value lies between 0 and 1.

❑ For a pure resistive circuit R = Z $\Rightarrow$ p.f. = cosϕ = 1

Wattless Current.

In an ac circuit R = 0 $\Rightarrow$ cosϕ = R/Z $\Rightarrow$ cosϕ = 0 so P_{av} = 0 i.e. in resistance less circuit the power consumed is zero. Such a circuit is called the wattless circuit and the current flowing is called the wattless current.

Note : ❑ The component of ac which remains in phase with the alternating voltage is defined as the effective current. The peak value of effective current is $i_0 \cos\phi$ and it's r.m.s. value is $i_{rms} \cos\phi = \dfrac{i_0}{\sqrt{2}} \cos\phi$.

Skin Effect

A direct current flows uniformly throughout the cross-section of the conductor. An alternating current, on the other hand, flows mainly along the surface of the conductor. This effect is known as skin effect. the reason is that when alternating current flows through a conductor, the flux changes in the inner part of the conductor are higher. Therefore the inductance of the inner part is higher than that of the outer part. Higher the frequency of alternating current, more is the skin effect.

The depth upto which ac current flows through a wire is called skin depth (δ).

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

Example: 1 The equation of an alternating current is $i = 50\sqrt{2} \times \sin 400\,\pi t$ ampere then the frequency and the root mean square of the current are respectively

(a) 200 Hz, 50 amp (b) 400 π Hz, $50\sqrt{2}$ amp

(c) 200 Hz, $50\sqrt{2}$ amp (d) 50 Hz, 200 amp

Solution: (a) Comparing the given equation with $i = i_0 \sin \omega t$

$\Rightarrow \omega = 400\,\pi \Rightarrow 2\pi\nu = 400\pi \Rightarrow \nu = 200$ Hz. Also $i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{50\sqrt{2}}{2} = 50$ A.

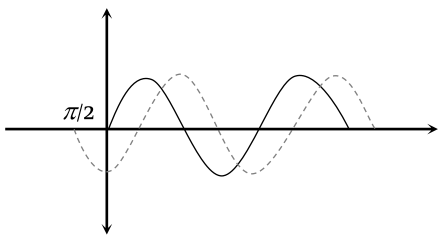
Example: 2 What will be the equation of ac of frequency 75 Hz if its r.m.s. value is 20 A

(a) $i = 20 \sin 150\,\pi t$ (b) $i = 20\sqrt{2} \sin(150\,\pi t)$

(c) $i = \frac{20}{\sqrt{2}} \sin(150\,\pi t)$ (d) $i = 20\sqrt{2} \sin(75\,\pi t)$

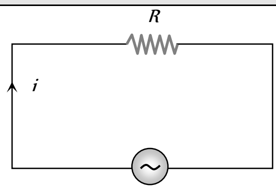
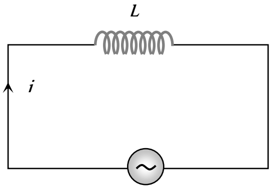
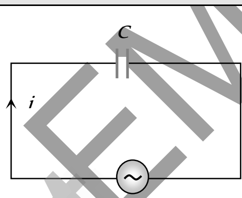
Solution: (b) By using $i = i_0 \sin \omega t = i_0 \sin 2\pi\nu\, t = i_{rms}\sqrt{2} \sin 2\pi\nu\, t$

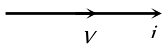
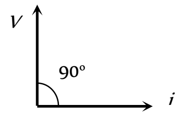
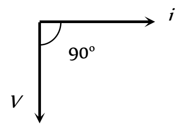
$\Rightarrow i = 20\sqrt{2} \sin(150\,\pi t)$.



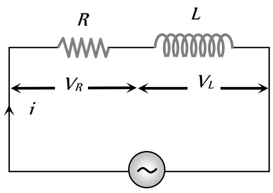
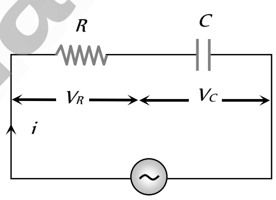
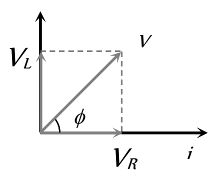
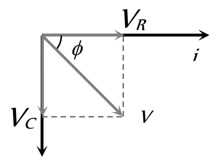
Different ac Circuit.

(1) **R, L and C circuits**

Circuit characteristics	**Purely resistive circuit (R-circuit)**	**Purely inductive circuit (L-circuit)**	**Purely capacitive circuit (C-circuit)**
(i) Circuit	 $V = V_0 \sin\omega t$	 $V = V_0 \sin\omega t$	 $V = V_0 \sin\omega t$
Definition	-	Inductance is a component that opposes a change in electric current.	Capacitor behaves very differently, as the periodic reversal of the voltage causes the capacitor to continually charge and discharge. It allow current.
(ii) Current	$i = i_0 \sin\omega t$	$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$	$i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$
(iii) Peak current	$i_0 = \frac{V_0}{R}$	$i_0 = \frac{V_0}{X_L} = \frac{V_0}{\omega_L} = \frac{V_0}{2\pi\nu L}$	$i_0 = \frac{V_0}{X_C} = V_0\omega C = V_0(2\pi\nu C)$
(iv) Phase difference	ϕ = 0°	$\phi = 90^o$ (or $+\frac{\pi}{2}$)	$\phi = 90^o$ (or $-\frac{\pi}{2}$)
(v) Power factor	$\cos\phi = 1$	$\cos\phi = 0$	$\cos\phi = 0$
(vi) Power	$P = V_{rms}i_{rms} = \frac{V_0 i_0}{2}$	P = 0	P = 0
(vii) Time difference	TD = 0	$\text{TD} = \frac{T}{4}$	$\text{TD} = \frac{T}{4}$
(viii) Leading quantity	Both are in same phase	Voltage (the Voltage leads the voltage by a phase angle of	Current(the current leads the voltage by a phase angle of

		π/2)	π/2)
(ix) lagging quantity	Both are in same phase	The current lags the voltage by a phase angle of π/2	the Voltage lags the voltage by a phase angle of π/2
(x) Phasor diagram			

(2) **RL, RC and LC circuits**

Circuit characterstics	**RL-circuit**	**RC-circuit**
(i) Circuit	 $V_R = iR$, $V_L = iX_L$ $V = V_0 \sin\omega t$	 $V_R = iR$, $V_C = iX_C$ $V = V_0 \sin\omega t$
(ii) Current	$i = i_0 \sin(\omega t - \phi)$	$i = i_0 \sin(\omega t + \phi)$
(iii) Peak current	$i_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + X_L^2}}$ $= \frac{V_0}{\sqrt{R^2 + 4\pi^2 \nu^2 L^2}}$	$i_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + X_C^2}}$ $= \frac{V_0}{\sqrt{R^2 + \frac{1}{4\pi^2 \nu^2 C^2}}}$
(iv) Phasor diagram		
(v) Applied voltage	$V = \sqrt{V_R^2 + V_L^2}$	$V = \sqrt{V_R^2 + V_C^2}$

(vi) Impedance	$Z=\sqrt{R^2+X_L^2}=\sqrt{R^2+\omega^2L^2}$ $=\sqrt{R^2+4\pi^2\nu^2L^2}$	$Z=\sqrt{R^2+X_C^2}=\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}$
(vii) Phase difference	$\phi=\tan^{-1}\frac{X_L}{R}=\tan^{-1}\frac{\omega L}{R}$	$\phi=\tan^{-1}\frac{X_C}{R}=\tan^{-1}\frac{1}{\omega CR}$
(viii) Power factor	$\cos\phi=\frac{R}{\sqrt{R^2+X_L^2}}$	$\cos\phi=\frac{R}{\sqrt{R^2+X_C^2}}$
(ix) Leading quantity	Voltage	Current

Example: 1 In a resistive circuit R = 10 Ω and applied alternating voltage V = 100 sin 100 π t. Find the following

(i) Peak current (ii) r.m.s. current (iii)Average current (iv) Frequency

(v) Time period (vi)Power factor (vii)Power dissipated in the circuit

(viii) Time difference

Solution: (i) Peak current $i_0=\frac{V_0}{R}=\frac{100}{10}=10\,A$

(ii) r.m.s. current $i_{rms}=\frac{i_0}{\sqrt{2}}=\frac{10}{\sqrt{2}}=5\sqrt{2}A$

(iii) Average current $i_{av}=\frac{2}{\pi}.i_0=\frac{2}{\pi}\times 10=6.37\;A$

(iv) Frequency $\nu=\frac{\omega}{2\pi}=\frac{100\pi}{2\pi}=50\,Hz$

(v) Time period $T=\frac{1}{\nu}=\frac{1}{50}=0.02\,sec$

(vi)Phase difference in resistive circuit $\phi=0$ so p.f. = cosϕ = 1

(vii)Power dissipated in the circuit $P=\frac{1}{2}V_0 i_0\cos\phi=\frac{1}{2}\times 100\times 10\times 1=500\,W$

(viii) Time difference T.D. $=\frac{T}{2\pi}\times\phi=\frac{T}{2\pi}\times 0=0$

Example: 2 In a purely inductive circuit if $L=\frac{100}{\pi}\times 10^{-3}H$ and applied alternating voltage is given by V = 100 sin 100 π t. Find the followings

(i) Inductive reactance (ii) Peak value, r.m.s. value and average value of current

(iii) Frequency and time period (iv) Power factor and power

(v) Time difference between voltage and current

Solution: (i) $X_L = \omega L = 100\pi \times \frac{100}{\pi} \times 10^{-3} = 10\ \Omega$

(ii) $i_0 = \frac{V_0}{X_L} = \frac{100}{10} = 10\,A$; $i_{rms} = \frac{10}{\sqrt{2}} = 5\sqrt{2}A$ and $i_{av} = \frac{2}{\pi} \times 10$ = 6.37 A

(iii)Frequency $\nu = \frac{100\pi}{2\pi} = 50\,Hz$ and $T = \frac{1}{50} = 0.02\,sec$

(iv)In purely L-circuit $\phi = 90^\circ$ so p.f. $\cos\phi = 0$

(v) Time difference T.D. $= \frac{T}{2\pi} \times \frac{\pi}{2} = \frac{T}{4}$.

Example: 3 An alternating voltage $E = 200\sqrt{2}\sin(100\ t)$ is connected to a 1 microfaracd capacitor through an ac ammeter. The reading of the ammeter shall be

(a) 10 mA (b) 20 mA (c) 40 mA (d) 80 mA

Solution: (b) Ammeter reads r.m.s. value so $i_{rms} = \frac{V_{rms}}{X_C} = V_{rms} \times \omega \times C$

$$\Rightarrow i_{rms} = \left(\frac{200\sqrt{2}}{\sqrt{2}}\right) \times 100 \times (1 \times 10^{-6}) = 2 \times 10^{-2} = 20\ mA.$$

Example: 4 An 120 volt ac source is connected across a pure inductor of inductance 0.70 henry. If the frequency of the source is 60 Hz, the current passing through the inductor is

(a) 4.55 amps (b) 0.355 amps (c) 0.455 amps (d) 3.55 amps

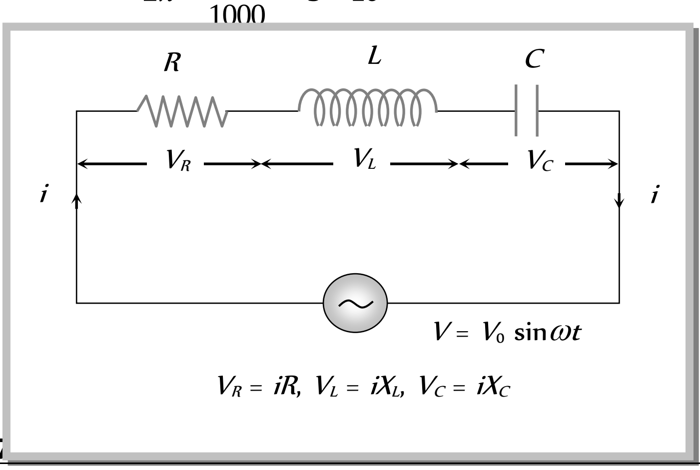
Solution: (c) $i_{rms} = \frac{V_{rms}}{X_L} = \frac{V_{rms}}{2\pi\nu L} = \frac{120}{2\pi \times 60 \times 0.7} = 0.455\ A$.

Example: 5 The frequency for which a $5\mu F$ capacitor has a reactance of $\frac{1}{1000}$ ohm is given by

(a) $\frac{100}{\pi}\,MHz$ (b) $\frac{1000}{\pi}\,Hz$ (c) $\frac{1}{1000}\,Hz$ (d) 1000 Hz

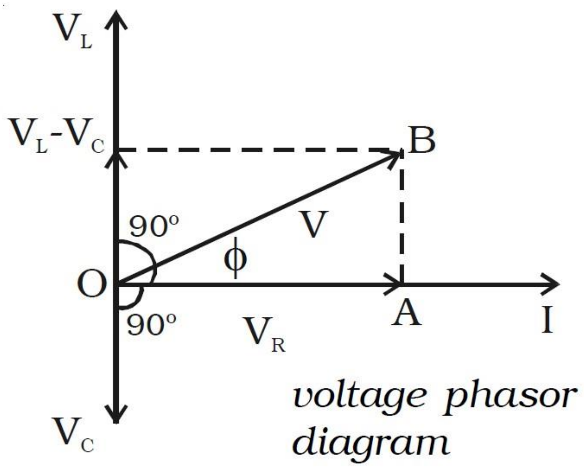
Solution: (a) $X_C = \frac{1}{2\pi\nu C} \Rightarrow \nu = \frac{1}{2\pi X_C(C)} = \frac{1}{2\pi \times \frac{1}{1000} \times 5 \times 10^{-6}} = \frac{100}{\pi}\,MHz$.

Series RLC Circuit.

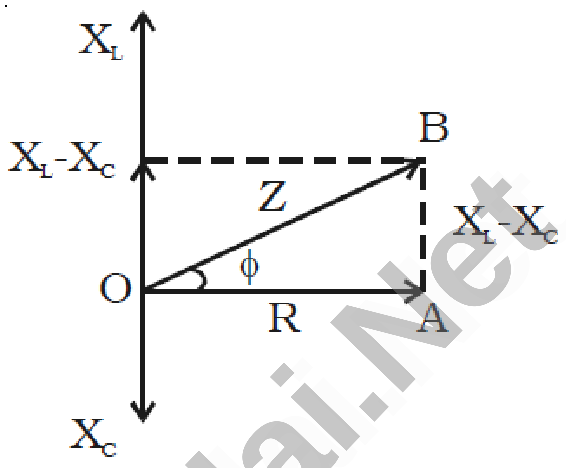


Let an alternating source of emf V (e) be connected to a series combination of a resistor of resistance R, inductor of inductance L and a capacitor of capacitance C

$\boldsymbol{V_R = IR}$ ($\boldsymbol{V_R}$ in phase with I)
$\boldsymbol{V_L = IX_L}$ ($\boldsymbol{V_L}$ leads I by π/2)
$\boldsymbol{V_C = IX_C}$ ($\boldsymbol{V_C}$ lags behind I by π/2)



voltage phasor diagram



Impedance diagram

V_L and V_C are 180^0 out of phase with each other and the resultant of V_L and V_C is $(\boldsymbol{V_L - V_C})$,
Assuming the circuit is ***predominantly inductive,***
The applied voltage 'V' equals to

$$OB^2 = OA^2 + AB^2$$
$$\boldsymbol{V^2 = V_R^2 + (V_L - V_C)^2}$$
$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$
$$V = \sqrt{IR^2 + (IX_L - IX_C)^2}$$
$$V = \sqrt{I^2R^2 + I^2(X_L - X_C)^2}$$
$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

Impedance of the circuit $\boldsymbol{Z = \frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}}$

The unit of impedance is ***ohm***

Phase angle ∅

$$\boldsymbol{\tan\emptyset = \frac{X_L - X_C}{R}}$$
$$\emptyset = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

(Or) $\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R} = \frac{2\pi\nu L - \frac{1}{2\pi\nu C}}{R}$

(1) **Equation of** Instantaneous current : $i = i_0 \sin(\omega t \pm \phi)$; where $i_0 = \frac{V_0}{Z}$

(2) **If net reactance is capacitive :** Circuit behave as CR circuit

$I_o \sin(\omega t + \emptyset)$ – **for capacitive dominant circuit current leads**

(3) **If net reactance is inductive :** Circuit behaves as LR circuit

$I_o \sin(\omega t - \emptyset)$ – **for inductive dominant circuit current lags**

Series resonance in RLC circuit

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$I = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

At a particular value of the angular frequency, the inductive reactance and the capacitive reactance will be equal to each other (OR)

net reactance is zero ⇒Means $X = X_L - X_C = 0$ ⇒ $X_L = X_C$. This is the condition of resonance

$$\omega L = \frac{1}{\omega C}$$

so that the impedance of the **circui** $= Z = \sqrt{R^2 + (X_L - X_C)^2}$ becomes minimum and it is given by Z = R.

Hence I is in phase with V

The particular frequency νo at which the impedance of the circuit becomes minimum and therefore the current becomes maximum is called Resonant frequency of the circuit. Such a circuit which admits maximum current is called series resonant circuit or acceptor circuit.

And Current in the circuit is maximum and it is $i_0 = \frac{V_0}{R}$

(i) At frequencies lower than resonance the capacitive reactance of the circuit is large as compared to the inductive reactance $\left(\frac{1}{\omega C} > \omega L\right)$ and the total reactance is capacitive.

(ii) At frequencies higher than resonance the inductive reactance is large as compared to the capacitive reactance $\left(\omega L > \frac{1}{\omega C}\right)$ and the total reactance is inductive.

(iii) At resonant frequency inductive and capacitive reactance are equal $\left(\omega L = \frac{1}{\omega C}\right)$

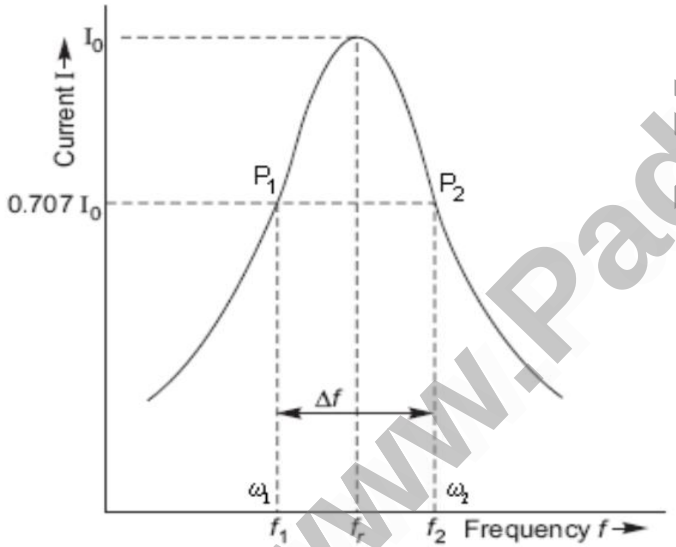
Band Width

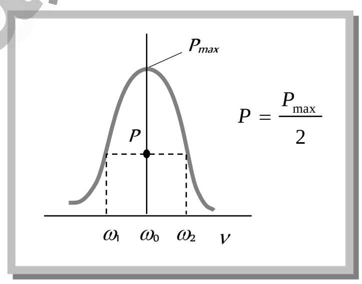
The band width is defined as range of frequencies specified between two points on either side of resonant frequency where current falls to $\frac{1}{\sqrt{2}}$ times or power falls to half of its value at resonance. P_1 and P_2 are known as half power points shown in fig on either side of the resonant frequency.

Half power frequencies:

The frequencies at which the power in the circuit is half of the maximum power (The power at resonance), are called half power frequencies.

(i) The current in the circuit at half power frequencies (HPF) is $\frac{1}{\sqrt{2}}$ or 0.707 or 70.7% of maximum current (current at resonance).





(ii) There are two half power frequencies.

(a) $\omega_1 \rightarrow$ called lower half power frequency. At this frequency the circuit is capacitive.

(b) $\omega_2 \rightarrow$ called upper half power frequency. It is greater than ω_0. At this frequency the circuit is inductive.

(iii) Band width ($\Delta\omega$) : The difference of half power frequencies ω_1 and ω_2 is called band width ($\Delta\omega$) and $\Delta\omega = \omega_2 - \omega_1$.

band width of series resonant circuit can be derived as $\Delta\omega = \left(\frac{R}{L}\right)$

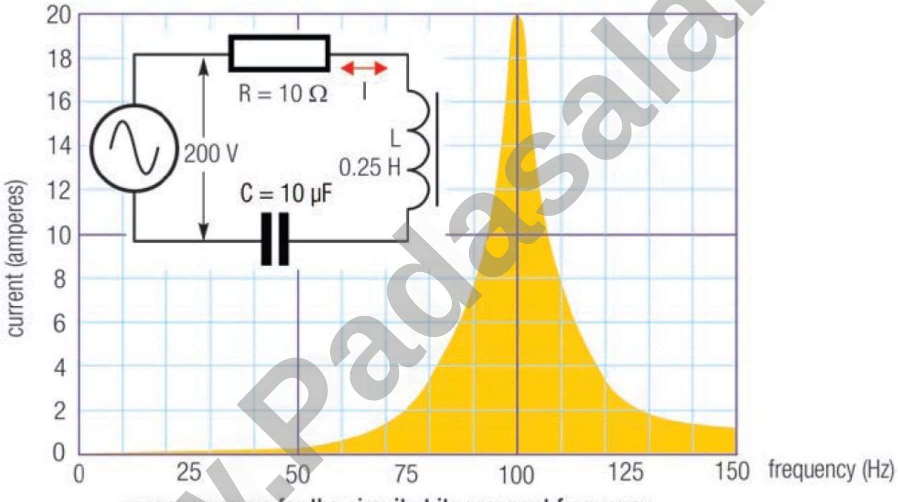
or $\text{Band width} = (f_2 - f_1) = \frac{R}{2\pi L}$

(12) **Quality factor (Q - factor) of series resonant circuit**

The characteristic of a series resonant circuit is determined by the quality factor (Q - factor) of the circuit.

The selectivity or sharpness of tuning of a resonant circuit is measured by the quality factor or Q factor. It defines sharpness of i - ν curve at resonance when Q - factor is large, the sharpness of resonance curve is more and vice-versa.

As the graph shows, the current starts to increase rapidly over a certain range of frequencies centered around the resonant frequency. Current is a maximum at the resonant frequency, and the impedance of the circuit is a minimum, and simply the resistance of the circuit.



response curve for the circuit at its resonant frequency

One way of looking the Q Factor is to compare the voltage drops at resonance across the resistance and **either of the reactive components**. This gives a measure of the amplification effect caused at resonance, or the Q factor (quality factor). That is:

Q factor is defined as the ratio of voltage across L (or) C to the applied voltage.

$$Q = \frac{\text{Voltage across } L\,(or)\,C}{\text{Applied voltage}}$$

Applied Voltage = IR

Q - factor $= \frac{V_L}{V_R}$ or $\frac{V_C}{V_R}$

$$Q = \frac{\omega_0 L}{R} \text{ or } \frac{1}{\omega_0 CR} \Rightarrow Q\text{ - factor} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad \because \omega_o = \frac{1}{\sqrt{LC}}$$

Q - factor also defined as follows

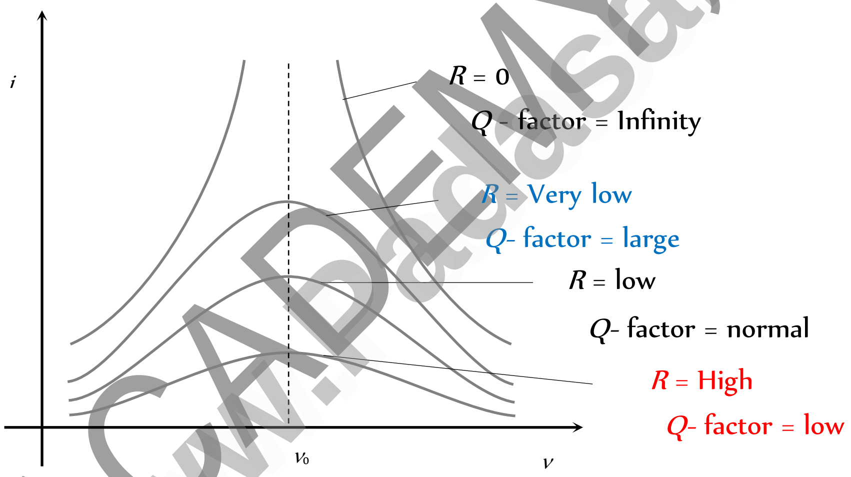
$$\text{Q factor} = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipation}} = \frac{2\pi}{T} \times \frac{\text{Maximum energy stored}}{\text{Mean power dissipated}}$$

$$= \frac{\text{Resonant frequency}}{\text{Band width}} = \frac{\omega_0}{\Delta\omega}$$

Thus the band width is inversely proportional to quality factor or Q-factor. As the value of Q increases, band width decreases and sharpness of resonance or selectivity of the circuit increases.

- Q is just a number having values between 10 to 100 for normal frequencies.

Circuit with high Q values would respond to a very narrow frequency range and vice versa. Thus a circuit with a high Q value is sharply tuned while one with a low Q has a flat resonance. Note: Q-factor can be increased by having a coil of large inductance but of small ohmic resistance.



Resonance curve

The variation of peak value of current with the frequency of applied emf is shown in fig (4). In figure two curves are plotted. One when the circuit resistance is low i.e., R and the other for the circuit resistance is high i.e., 2R. If the circuit resistance is low the curve of current has a sharp peak and the circuit is said to be sharply resonant or highly selective. If the circuit resistance is high the peak is broadened and the selectivity is poor. Selectivity of a circuit is given as

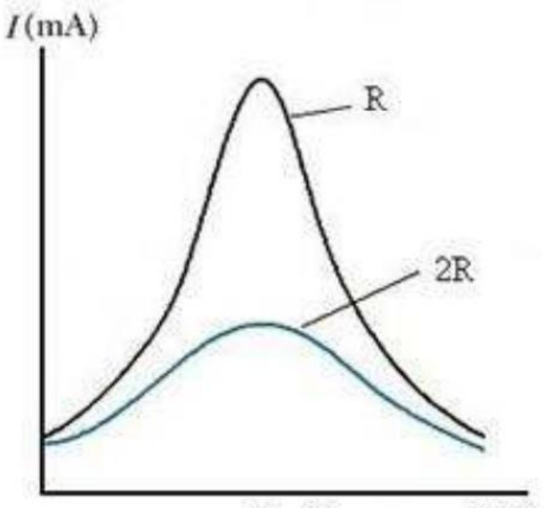


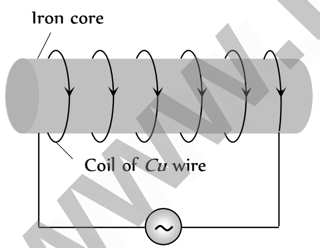
Fig (4)

Choke Coil.

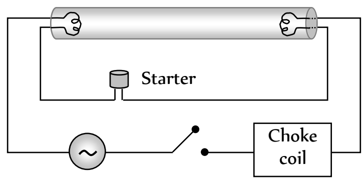
A choke coil (or ballast) is an inductance coil of very small resistance used for controlling current in an a.c. circuit.

Choke coil is a device having high inductance and negligible resistance. It is used to control current in ac circuits and is used in fluorescent tubes. The power loss in a circuit containing choke coil is least.

If a resistance is used to control current, there is wastage of power due to Joule heating effect in the resistance. On the other hand there is no dissipation of power when a current flows through a **pure** inductor.



Choke coil



Application of choke coil

(1) It consist of a Cu coil wound over a soft iron laminated core.

(2) Thick Cu wire is used to reduce the resistance (R) of the circuit.

(3) Soft iron is used to improve inductance (L) of the circuit.

(4) The inductive reactance or effective opposition of the choke coil is given by $X_L = \omega L = 2\pi\nu L$

(5) For an ideal choke coil r = 0, no electric energy is wasted

For an ideal inductor the current lags behind the emf by a phase angle $\frac{\pi}{2}$

The average power consumed by the choke coil over a complete cycle is

$P_{av} = E_{rms} I_{rms} \cos \pi/2 = 0$

i.e. average power P = 0.

(6) In actual practice choke coil is equivalent to a R – L circuit.

$$P_{av} = E_{rms} I_{rms} \cos \phi$$

$$P_{av} = E_{rms} I_{rms} \frac{r}{\sqrt{r^2 + \omega^2 L^2}}$$

Where $\frac{r}{\sqrt{r^2 + \omega^2 L^2}}$ is the power factor.

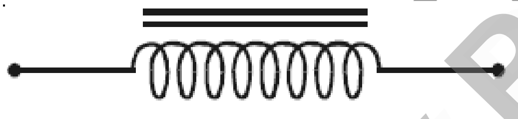
The value of average power dissipated in the choke coil is much smaller than the power loss $I^2 R$ in a resistance R.

i.e In practice, a choke coil of inductance L possesses a small resistance r. Hence it may be treated as a series combination of an inductor and small resistance r. In that case the average power consumed by the choke coil over a complete cycle is

(7) Choke coil for different frequencies are made by using different core material.

For low frequency a.c. circuit the value L should be large thus iron core choke coil is used.

For high frequency(radio frequencies) ac circuit, L should be small, so air cored choke coil is used.



A.F Choke

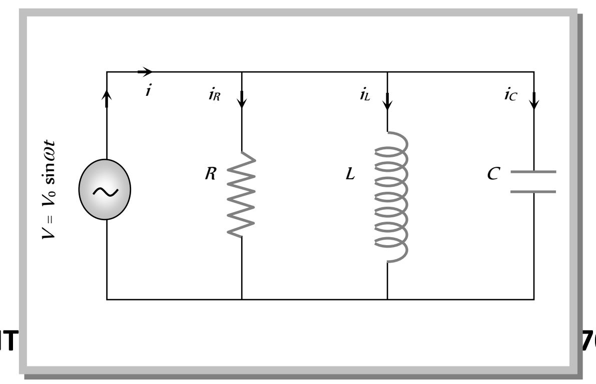
R.F. Choke

Concepts

- Series RLC circuit also known as acceptor circuit (or tuned circuits or filter circuit) as at resonance it most readily accepts that current out of many currents whose frequency is equal to it's natural frequency.
- The choke coil can be used only in ac circuits not in dc circuits, because for dc frequency $\nu = 0$ hence $X_L = 2\pi\nu L = 0$, only the resistance of the coil remains effective.
- Choke coil is based on the principle of wattless current.

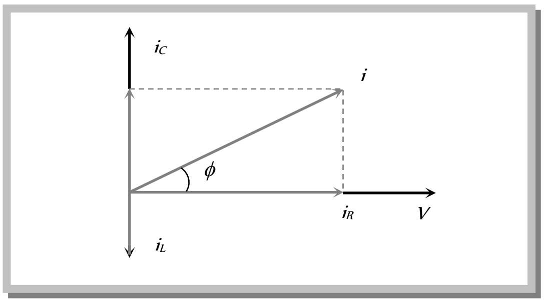
Parallel RLC Circuits.

$$i_R = \frac{V_0}{R} = V_0 G$$



$$i_L = \frac{V_0}{X_L} = V_0 S_L$$

$$i_C = \frac{V_0}{X_C} = V_0 S_C$$



(1) **Current and phase difference**

From phasor diagram current $i = \sqrt{i_R^2 + (i_C - i_L)^2}$ and phase difference

$$\phi = \tan^{-1}\frac{(i_C - i_L)}{i_R} = \tan^{-1}\frac{(S_C - S_L)}{G}$$

(2) **Admittance (Y) of the circuit**

From equation of current $\frac{V_0}{Z} = \sqrt{\left(\frac{V_0}{R}\right)^2 + \left(\frac{V_0}{X_L} - \frac{V_0}{X_C}\right)^2}$

$$\Rightarrow \frac{1}{Z} = Y = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2} = \sqrt{G^2 + (S_L - S_C)^2}$$

(3) **Resonance**

At resonance (i) $i_C = i_L \quad \Rightarrow i_{\min} = i_R$

(ii) $\frac{V}{X_C} = \frac{V}{X_L} \quad \Rightarrow S_C = S_L \Rightarrow \Sigma S = 0$

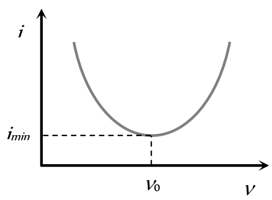
(iii) Impedance is maximum $Z_{\max} = \frac{V}{i_R} = R$

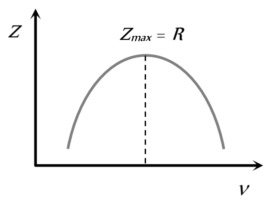
(iv) $\phi = 0 \quad \Rightarrow$ power factor = cosϕ = 1 = maximum

(v) Resonant frequency

$$\Rightarrow \nu = \frac{1}{2\pi\sqrt{LC}}$$

(4) **Current resonance curve**





Note :
- At resonant frequency due to the property of rejecting the current, parallel resonant circuit is also known as anti-resonant circuit or rejecter circuit.
- Due to large impedance, parallel resonant circuits are used in radio.

Characteristic	Series Resonance	Parallel Resonance
Impedance	Minimum at resonance	Maximum at resonance
Admittance	Maximum at resonance (Acceptor circuit)	Minimum at resonance (Rejector circuit)
Current	Maximum at resonance	Minimum at resonance
Phase angle	0° at resonance	0° at resonance
Behavior of circuit	Acceptor circuit	Rejector circuit
Amplification	This circuit amplifies the voltage in the circuit	This circuit amplifies the current in the circuit
Equation of effective impedance	Z=R	$Z = \frac{L}{CR}$
Quality factor	$Q = \frac{\omega_0 L}{R} \quad or \quad = \frac{1}{\omega_0 RC}$	$Q = \frac{R}{\omega_0 L} \quad or \quad = \omega_0 RC$
Applications	Band-pass filters, tuned circuits, oscillators circuits and Voltage amplifiers	Band-stop filters, power factor correction, voltage regulators, Current amplifiers, frequency amplifiers, and induction heating.