Tiruvannamalai District

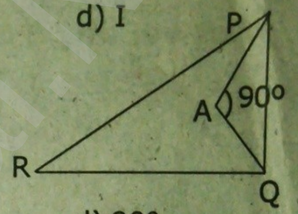
SECOND MID TERM TEST - 2023

10 - Std **MATHS**

Time: 1.30 Hrs. Marks: 50

Note: i) Answer all the questions. **7×1=7**

1) If A is a 2×3 matrix and B is a 3×4 matrix, how many columns does AB have
   a) 3 b) 4 c) 2 d) 5
2) Transpose of a column matrix is
   a) unit matrix b) diagonal matrix c) column matrix d) row matrix
3) The non-diagonal elements in any unit matrix are
   a) 0 b) 1 c) aij d) I
4) In the given figure PR = 26 cm, QR = 24 cm, $\angle PAQ = 90^\circ$, PA = 6 cm and QA = 8 cm. Find $\angle PQR$.



   a) 80° b) 85° c) 75° d) 90°

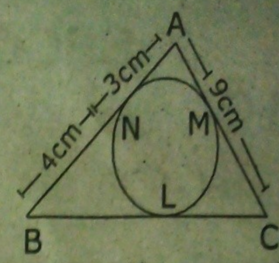
5. The angle of Elevation of cloud from a point h metres above a lake is β. The angle of depression of its reflection in the lake is 45° The height of location of the cloud from the lake is
   a) $\frac{h(1+\tan\beta)}{1-\tan\beta}$ b) $\frac{h(1-\tan\beta)}{1+\tan\beta}$ c) $h\tan(45^\circ-\beta)$ d) None of these
6. The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is a) 60π cm² b) 68π cm² c) 120π cm² d) 136π cm²
7. The totai surface area of cyclinder whose radius is $\frac{1}{3}$ of its height is
   a) $\frac{9\pi h^2}{8}$ sq.units. b) $24\pi h^2$ sq. units. c) $\frac{8\pi h^2}{9}$ sq. units. d) $\frac{56\pi h^2}{9}$ sq. units.

Part - II

Note: Answer any 5 questions. Question No. 14 is compulsory. **5×2=10**

8) Find the value of a, b, c, d from the equation $\begin{pmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$.
9) If $A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ then find the transpose of $-A$.
10) Find the length of the tangent dracon from a point whose distance from the centre of a circle is 5 cm and radius of the circle is 3 cm.
11) State Ceva's theorem.

12) In figure, ΔABC is circumscribing a circle. Find the length of BC.



13) From the top of a rock $50\sqrt{3}$ m high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock.

14) If $A = \begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}$, $B = \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$ find the value of 3A−9B.

Part - C

II. Answer any 5 of the following questions: Q.No.21 Compulsory . 5 x 5 = 25

15. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $A^2 - 5A + 7I_2 = 0$.

16. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{bmatrix}$ show that (AB).

17. State and prove Baudhayana theorem.

18. Two ships are sailing in the sea on either sides of a lighthouse. The angle of elevation of the top of the lighthouse as observed from the ships are 30° and 45° respectively. If the lighthouse is 200m high, find the distance between the two ships

19. The angle of elevation of the top of a cell phone tower from the foot of a high apartment is 60° and the angle of depression of the foot of the tower from the top of the apartment is 30° If the height of the apartment is 50 m, find the height of the cell phone tower. According to radiations control norms, the minimum height of a cell phone tower should be 120 m. state If the height of the above mentioned cell phone tower meets the radiation norms. $(\tan 40^{\circ} = 0.8391, \sqrt{3} = 1.732)$

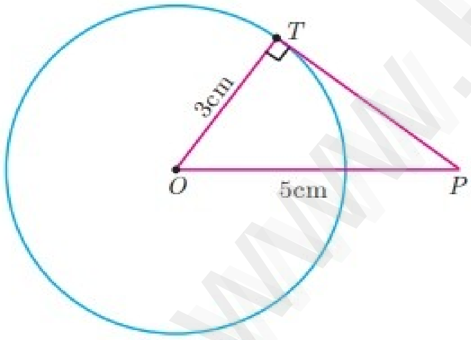
20. An Industrial metallic bucket is in the shape of the frustram of a right circular cone whose top and bottom diameters are 10m and 4m and whose height is ccc4m. Find the curved and total surface area of the bucket.

21. From the top of a lighthouse, the angle of depression of two ships on the opposite sides of it are observed to be 30° and 60°. If the height of the lighthouse is h meters and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ships is $\frac{4h}{\sqrt{3}}$ m.

Part - IV

Note : Answer the following question. **1×8=8**

22) a) Discuss the nature of solutions of the quadratic equation $x^2+x-12 = 0$. **(OR)**
b) Take a point which is 11 cm away from the centre of a circle of radius 4 cm and draw the two tangents to the circle from that point.

1	(b) 4
2	(d) row matrix
3	(a) 0
4	(d) 90°
5	(a) $\frac{h(1+\tan\beta)}{1-\tan\beta}$
6	(d) 136π cm^2
7	(c) $\frac{8\pi h^2}{9}$ sq. units
8	***Solution*** First, we add the two matrices on both left, right hand sides to get $\begin{pmatrix} d+3 & 8+a \\ 3b-2 & a-4 \end{pmatrix} = \begin{pmatrix} 2 & 2a+1 \\ b-5 & 4c \end{pmatrix}$ Equating the corresponding elements of the two matrices, we have $d+3=2$ gives $d=-1$ $8+a=2a+1$ gives $a=7$ $3b-2=b-5$ gives $b=-3/4$ Substituting $a=7$ in $a-4=4c$ gives $c=3/4$ Therefore, $a=7$, $b=-3/2$, $c=3/4$, $d=-1$.
9	**Solution :** $-A = \begin{bmatrix} \sqrt{7} & 3 \\ \sqrt{5} & -2 \\ -\sqrt{3} & 5 \end{bmatrix}$ $\therefore -A^T = \begin{bmatrix} -\sqrt{7} & \sqrt{5} & -\sqrt{3} \\ 3 & -2 & 5 \end{bmatrix}$
10	***Solution*** Given OP = 5 cm, radius $r = 3$ cm To find the length of tangent *PT*.  Fig. 4.62 In right angled ΔOTP, $OP^2 = OT^2 + PT^2$ (by Pythagoras theorem) $5^2 = 3^2 + PT^2$ gives $PT^2 = 25 - 9 = 16$ Length of the tangent $PT = 4$ cm.
11	**Ceva's Theorem (without proof)** **Statement** Let ABC be a triangle and let *D,E,F* be points on lines *BC*, *AB* *CA*, respectively. Then the

cevians *AD*, *BE*, *CF* are concurrent if and only if $\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$ where the lengths are directed. This also works for the reciprocal of each of the ratios *B* as the reciprocal of 1 is 1.

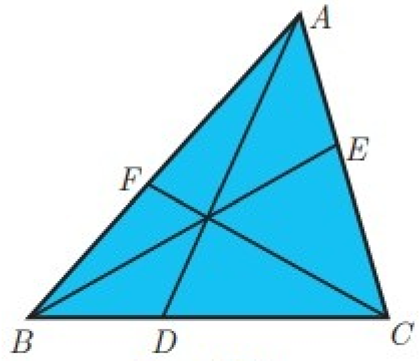


Fig. 4.70

12 ***Solution***

$AN = AM = 3$ cm (Tangents drawn from same external point are equal)

$BN = BL = 4$ cm

$CL = CM = AC - AM = 9 - 3 = 6$ cm

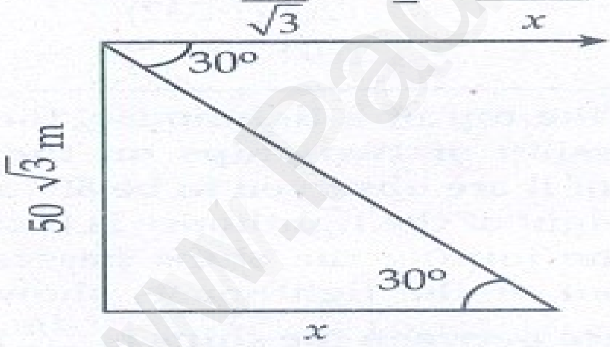
Gives *BC* ⁖ *BL*, *CL* ⁖ 4, 6 ⁖ 10 cm

13 **Solution:**

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 30^\circ = \frac{50\sqrt{3}}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{x}$$



$x = 50\sqrt{3} \times \sqrt{3} = 50\times 3$

$x = 150$ m

14

If $A = \begin{bmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{bmatrix}$ find the value of

i) $B - 5A$ ii) $3A - 9B$

Solution :

i) $B - 5A = \begin{bmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{bmatrix} - 5\begin{bmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{bmatrix}$

$= \begin{bmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{bmatrix} + \begin{bmatrix} 0 & -20 & -45 \\ -40 & -15 & -35 \end{bmatrix} = \begin{bmatrix} 7 & -17 & -37 \\ -39 & -11 & -26 \end{bmatrix}$

ii) $3A - 9B = 3\begin{bmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{bmatrix} - 9\begin{bmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{bmatrix}$

$= \begin{bmatrix} 0 & 12 & 27 \\ 24 & 9 & 21 \end{bmatrix} + \begin{bmatrix} -63 & -27 & -72 \\ -9 & -36 & -81 \end{bmatrix}$

$= \begin{bmatrix} -63 & -15 & -45 \\ 15 & -27 & -60 \end{bmatrix}$

15

Solution :

$A^2 = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$

$= \begin{pmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{pmatrix}$

$= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix}$

$-5A = -5\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$

$= \begin{pmatrix} -15 & -5 \\ 5 & -10 \end{pmatrix}$

$7I_2 = 7\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

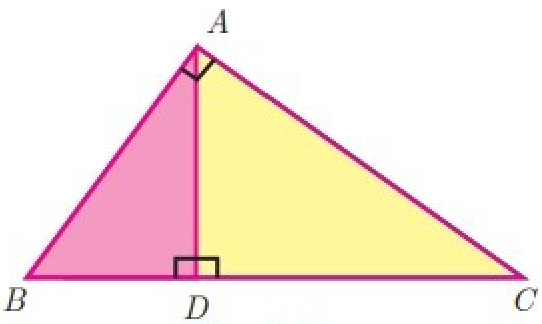
$= \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$

$\therefore A^2 - 5A + 7I_2 = \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} + \begin{pmatrix} -15 & -5 \\ 5 & -10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$= 0$

Hence proved

16	$AB = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}_{2\times3} \times \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}_{3\times2}$ $= \begin{pmatrix} 2-2+0 & -1+8+2 \\ 4+1+0 & -2-4+2 \end{pmatrix} = \begin{pmatrix} 0 & 9 \\ 5 & -4 \end{pmatrix}$
17	**Pythagoras Theorem** **Statement** In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.  Fig. 4.46 **Proof** **Given :** In ΔABC, $\angle A = 90°$ **To prove:** $AB^2 + AC^2 = BC^2$ **Construction :** Draw $AD \perp BC$

	No.	**Statement**	**Reason**
	1.	Compare ΔABC and ΔABD $\angle B$ is common $\angle BAC = \angle BDA = 90^\circ$ Therefore, $\Delta ABC \sim \Delta ABD$ $\frac{AB}{BD} = \frac{BC}{AB}$ $AB^2 = BC \times BD$ ... (1)	Given $\angle BAC = 90^\circ$ and by construction $\angle BDA = 90^\circ$ By AA similarity
	2.	Compare ΔABC and ΔADC $\angle C$ is common $\angle BAC = \angle ADC = 90^\circ$ Therefore, $\Delta ABC \sim \Delta ADC$ $\frac{BC}{AC} = \frac{AC}{DC}$ $AC^2 = BC \times DC$ ...(2)	Given $\angle BAC = 90^\circ$ and by construction $\angle CDA = 90^\circ$ By AA similarity

Adding (1) and (2) we get

$AB^2 + AC^2 = BC \times BD + BC \times DC$

$= BC(BD + DC) = BC \times BC$

$AB^2 + AC^2 = BC^2$

Hence the theorem is proved.

18 ***Solution***

Let AB be the lighthouse. Let C and D be the positions of the two ships.

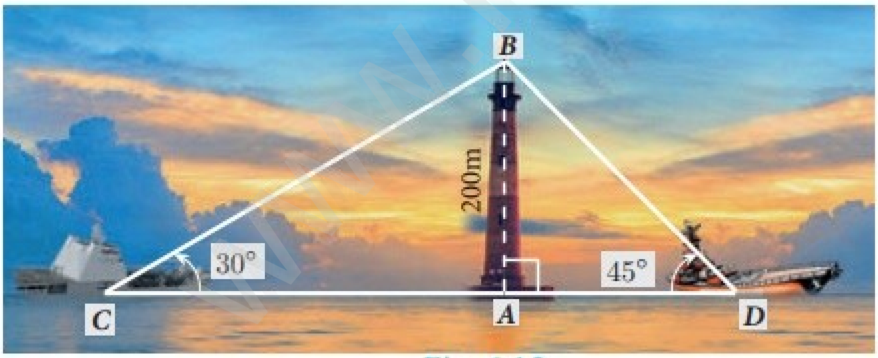


Fig. 6.15

Then, $AB = 200$ m.

$\angle ACB = 30^\circ$, $\angle ADB = 45^\circ$

In right triangle BAC, $\tan 30^\circ = AB/AC$

$1/\sqrt{3} = 200/AC$ gives $AC = 200\sqrt{3}$ ...(1)

In right triangle BAD, $\tan 45^\circ = AB/AD$

$1 = 200/AD$ gives $AD = 200$ ...(2)

Now, $CD = AC + AD = 200\sqrt{3} + 200$ [by (1) and (2)]

$CD = 200(\sqrt{3} + 1) = 200 \times 2.732 = 546.4$

Distance between two ships is 546.4 m.

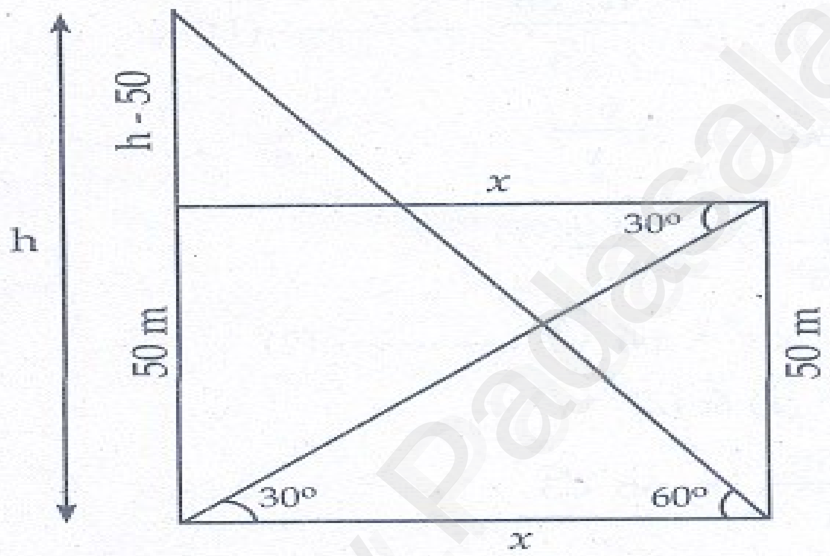
19	**Solution:** $\tan\theta = \frac{opp}{adj}$ $\tan 30° = \frac{50}{x}$ $\frac{1}{\sqrt{3}} = \frac{50}{x}$ $x = 50\sqrt{3}$ ---------------- (1) $\tan 60° = \frac{h}{x}$ $\sqrt{3} = \frac{h}{x}$ $x = \frac{h}{\sqrt{3}}$  from (1) & (2) $\frac{h}{\sqrt{3}} = 50\sqrt{3}$ $h = 50\sqrt{3} \times \sqrt{3} = 50 \times 3 = 150$ **h = 150 m** Yes, meets the radiation norm.
20	***Solution*** Let *h, l, R* and *r* be the height, slant height, outer radius and inner radius of the frustum.

Fig. 7.24

Given that, diameter of the top =10 m; radius of the top $R = 5$ m.
diameter of the bottom = 4 m; radius of the bottom $r = 2$ m, height $h = 4$ m

Now, $l = \sqrt{h^2 + (R - r)^2}$

$= \sqrt{4^2 + (5 - 2)^2}$

$l = \sqrt{16 + 9} = \sqrt{25} = 5\text{m}$

Here, C.S.A. $= \pi(R + r)l$ sq. units

$= \frac{22}{7}(5 + 2) \times 5 = 110\, m^2$

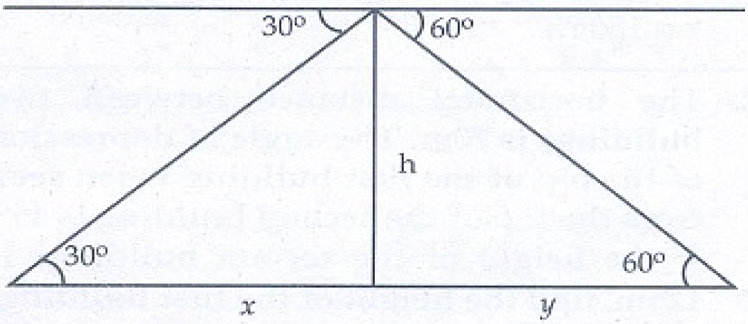
T.S.A. $= \pi(R + r)l + \pi R^2 + \pi r^2$ sq. units

$= \frac{22}{7}\left[(5 + 2)5 + 25 + 4\right] = \frac{1408}{7} = 201.14$

Therefore, C.S.A. = 110 m^2 and T.S.A. = 201.14 m^2

21

Solution:



$$\tan\theta = \frac{opp}{adj}$$

$$\tan 30^\circ = \frac{h}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$x = \sqrt{3}\,h \quad \text{------(1)}$$

$$\tan 60^\circ = \frac{h}{y}$$

$$\sqrt{3} = \frac{h}{y}$$

$$y = \frac{h}{\sqrt{3}}$$

$$\text{Distance} = x+y$$

$$= \frac{h}{\sqrt{3}} + h\sqrt{3}$$

$$= \frac{h+3h}{\sqrt{3}}$$

$$x+y = \frac{4h}{\sqrt{3}}\ \text{m}$$

22 a)

Solution

(i) $x^2 + x - 12 = 0$

Step 1

Prepare the table of values for the equation $y = x^2 + x - 12$.

x	−5	−4	−3	−2	−1	0	1	2	3	4
y	8	0	−6	−10	−12	−12	−10	−6	0	8

Step 2

Plot the points for the above ordered pairs (x, y) on the graph using suitable scale.

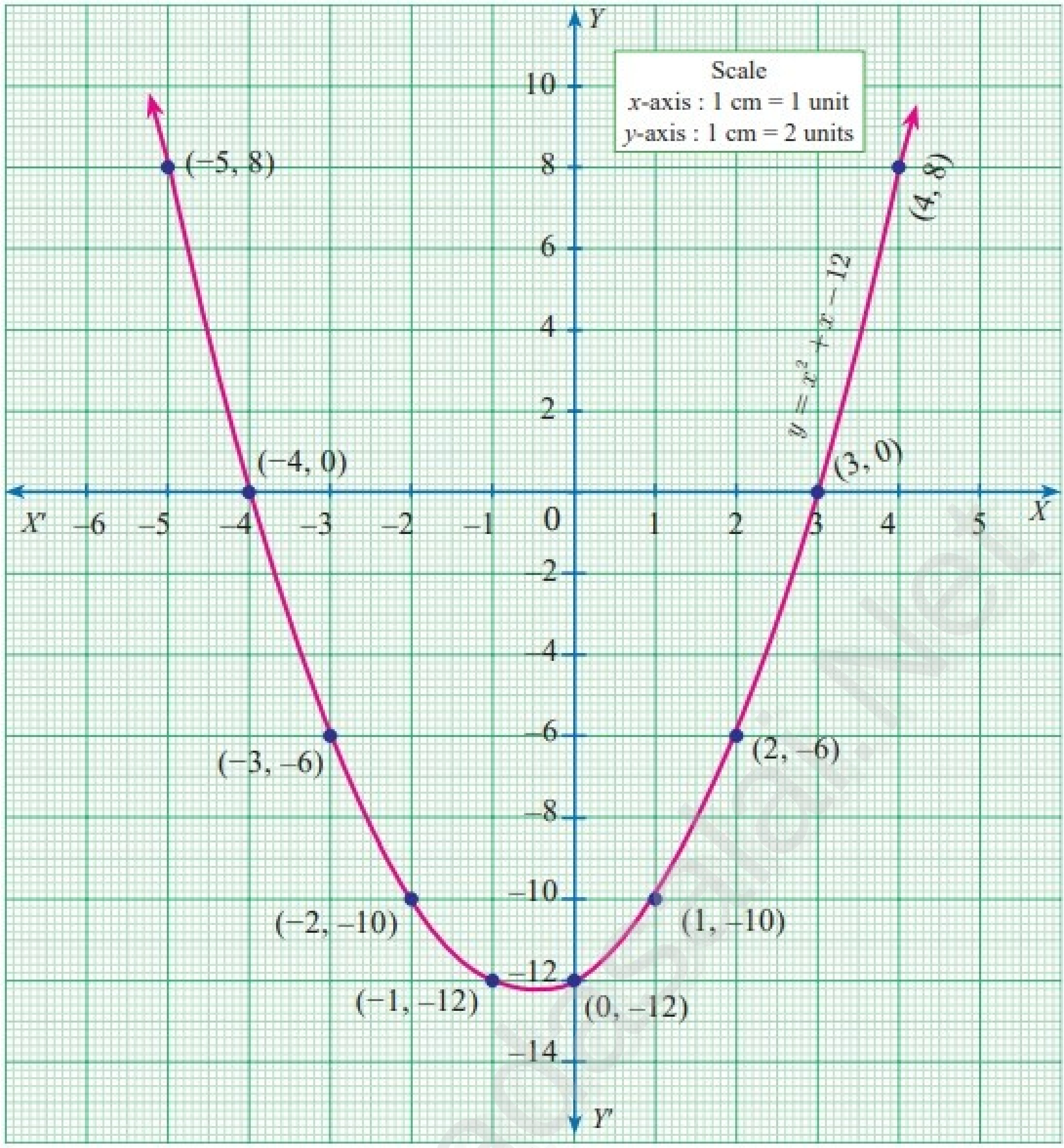


Fig. 3.12

Step 3

Draw the parabola and mark the co-ordinates of the parabola which intersect the X axis.

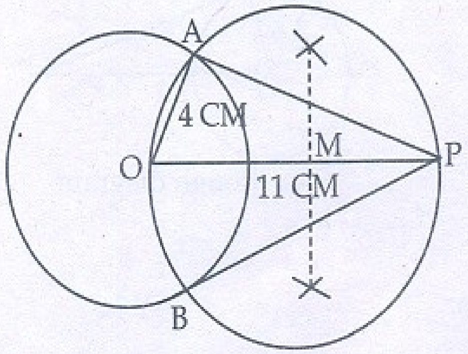
Step 4

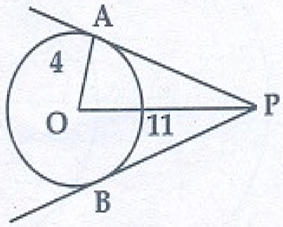
The roots of the equation are the *x* coordinates of the intersecting points (–4, 0) and (3,0)of the parabola with the X axis which are −4 and 3 respectively.

(or)

b)

Solution





Rough diagram

Construction :

1. With centre at O, draw a circle of radius 4 cm
2. Draw a line OP = 11 cm
3. Draw a perpendicular bisector of OP, which cuts OP at M
4. With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.
5. Join AP and BP. Thus AP and BP are the required tangents.

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